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[redacted]RN ACOUSTICS

MODERN ACOUSTICS

BY

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PREFACE

During the last twenty years or so the subject of acoustics has undergone a considerable change. The developments have been not so much in fundamental dynamical theory as in the introduction of electrical apparatus and methods. Consequently, Rayleigh's *Theory of Sound* and Lamb's *Dynamical Theory of Sound* remain of first importance as rigorous presentations of the theoretical bases of the subject. There has been, however, some adaptation of ideas—such as that of 'impedance'—which had proved to be of value in connection with electrical theory. There has been at the same time a marked extension of the methods of measurement, and consequently in the application of acoustics in industry and everyday life.

In the present volume the author has attempted to review these developments and to set out the essentials of acoustics as the subject is practised to-day, wherever possible in a form which could be read by anybody prepared to take mathematical formulæ for granted. At the same time he has endeavoured to include the essential theoretical basis, in detail or by reference, usually in a form suitable for application by 'impedance' methods of analysis. The book includes certain auxiliary matter which should be of value for reference purposes to workers in the subject. There are, however, some omissions—as, for instance, the theories of the vibrations of strings and bars—of material usually dealt with in elementary textbooks of acoustics, but which have little special significance in modern developments.

It is hoped that the book may be of value to students, to technical or research workers in acoustics, and to those in trade or industry who need to acquaint themselves with modern acoustical methods or to apply them.

Various parts of the work have, in the early stages, formed

the bases of special lectures the author has given to various evening students on modern and technical acoustics.

The author is indebted to Sir J. E. Petavel, Director of the National Physical Laboratory, for permission to write the book and to include certain illustrations, which have been noted in the text; to Dr G. W. C. Kaye for permission to incorporate a number of figures from Davis and Kaye's *Acoustics of Buildings*; and to H.M. Stationery Office for permission to reproduce an illustration from the Annual Report of the National Physical Laboratory.

A. H. D.

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CHAPTER I

GENERAL CHARACTERISTICS OF SOUND

Velocity of Sound. Sound waves are sent into the air by vibrating bodies, and consist of alternations of aerial condensation and rarefaction. The vibrating source, in moving forwards, compresses the air in front of it: the compression is rapidly relieved by the yielding of the adjacent layer of air; and the state of compression thus travels outwards with the velocity of sound, about 1100 ft. per second. As the source recovers and moves backwards, a region of rarefaction is sent out immediately behind the compression. This in turn is followed by another compression when the vibrating source moves forward again. The distance in the air by which the first condensation is in advance of the second condensation is called the wave-length of the sound concerned. If the frequency of the source is n complete vibrations per second, then the wave-length λ is equal to one nth part of the distance v travelled by sound in one second. We thus have the relation $\lambda = v/n$ or $v = n\lambda$.

The velocity of sound depends upon the elasticity and density of the medium in which it is travelling. The general expression for the velocity of a compressional wave of small amplitude is $c = \sqrt{E/\rho}$, where E is the appropriate modulus of elasticity and ρ the density. E is the bulk modulus of elasticity in the case of a fluid medium, but is equal to $k + \frac{4}{3}\mu$ in the case of solids, where μ is the rigidity and k the volume elasticity. For sound in free air the changes are adiabatic and the relation becomes $c = \sqrt{\gamma P/\rho}$, where P is the pressure of the gas, and γ is the ratio of the specific heat at constant pressure to that at constant volume. Since ρ varies with temperature and γ is practically constant, the velocity of sound in air varies with temperature. We have approximately $c = c_0(1 + t/273)^{\frac{1}{2}}$. Here c_0 is the velocity at 0° C. and c the velocity at t° C.

Propagation and Dissipation of Sound. Sound is a form of

energy, and when produced would persist indefinitely if it were not dissipated into heat or transformed into some other form of energy. When emitted by a point source removed from the proximity of obstacles, sound spreads equally in air in every direction, *i.e.* the wave-front is spherical. Owing to this spreading, the intensity at any point—measured in energy units—is inversely proportional to the square of the distance of the point from the source. Except for the very loudest sounds (such as those from fog-horns) and possibly also high-pitched notes, sound suffers but little dissipation into heat into the air itself, so that for all ordinary sounds the inverse-square law holds good.

This presupposes, of course, that the air surrounding the source is homogeneous. Where considerable distances in the atmosphere are concerned, various causes (such as temperature differences between adjacent layers of air) lead to apparent anomalies. For instance, the sound of great explosions is frequently inaudible at certain distances from the source, whilst it is strongly audible both at greater and at smaller distances. The inner area of audibility around the explosion may be only some tens of miles across, whilst the outer zone of audibility—which may be a complete ring or only a partial ring—is normally between 60 and 120 miles from the explosion. The time the sound takes to travel to the outer zone is abnormally great, and it is evident from the intermediate zone of silence that the sound has travelled by way of the upper atmosphere. These phenomena are accounted for as follows. Since the air is normally warmer near the ground than higher up, sound travels faster in the lower layers. The sound waves which travel outwards, at first horizontally, are therefore gradually bent upwards and the distance to which the sound is heard on the ground is restricted. However, at very great heights of 20 miles or more the atmosphere is at least as hot as it is in the surface-layers and, by deflecting the sound back to earth, accounts for the outer zone of audibility. Naturally the behaviour of sound waves in a non-homogeneous atmosphere * is of importance in connection with the audibility of fog-horns, and with the location of guns and aircraft by sound.†

* Rayleigh, *Theory of Sound*, 2, 132 ; G. Green, *Q.J. Roy. Met. Soc.*, 45, 339, 1919 ; E. A. Milne, *Phil. Mag.*, 42, 96, 1921 ; F. J. W. Whipple, *Q.J. Roy. Met. Soc.*, 57, 331, 1931.

† B. R. Hubbard, *Acous. Soc. Am. J.*, 3, 111, 1931 ; W. S. Tucker, *Q.J. Roy. Met. Soc.*, 29, 203, 1933.

In the case of sound generated within a room, the sound is absorbed at a rate depending on the properties of the various surfaces in the room; for instance, the sound waves penetrate the capillaries in the porous surfaces, and are there rapidly converted into heat by viscosity and allied effects. If porous surfaces are absent, for instance, as in an enclosure lined with steel or marble, sound echoes and re-echoes to and fro for a considerable time before it finally dies away.

Reflection and Refraction of Sound. Sound is transmitted by solids and liquids as well as by gases. As in the case of light, sound waves, on reaching the interface between two different media, are subject to reflection and refraction. In general, the laws are the same as those for light, but, since the wave-length of sound is usually large in comparison with the size of objects normally encountered, the reflected and refracted sounds spread considerably outside the limits to which light waves would be confined. When the velocity of sound in the second medium differs markedly from that in the first the sound is almost entirely reflected, and the amount refracted and so transmitted through the second medium is very small. As a consequence almost all rigid non-porous wall surfaces are very efficient reflectors of aerial sound waves—indeed, their efficiencies as reflectors of sound are higher than the efficiency of optical mirrors in the reflection of light. Sound may be focused by curved reflectors or may be screened from a given region by interposed obstacles, provided the reflector or the obstacle is of large dimensions compared with the wave-length of the sound. But, owing to the fact that the wave-lengths of ordinary sounds range from a few inches up to, say, 20 ft., focusing and screening effects by objects of moderate size are apt to be only of a partial character.

Ripple-tank Analogy. In view of the fact that sounds usually spread outside the limits of optical laws, it is useful to have some analogous case which can be studied visually or photographically to give an idea of the manner in which sound waves themselves would behave. There is no simple analogy which is completely parallel, but ripples on the surface of a liquid are rigorously analogous in certain conditions.* Strictly speaking, the analogy is two-dimensional only, and should be confined in its application to cylindrical sound waves. If the liquid is inviscid and of a depth greater than half the wave-length, and

* A. H. Davis, *Phys. Soc.*, *Proc.*, 38, 234, 1926; see Appendix, p. 331.

if the waves are small and of harmonic type, the height of the surface ripple is a measure of the condensation of air in the corresponding acoustical condition. Any obstacles introduced into the field must have dimensions which bear the correct ratio to the wave-length of the disturbance—a relation which can usually be met in a ripple tank about 3 ft. by 5 ft. in size and an inch or so deep. In Pl. I, p. 16, is a photograph of a ripple tank of these dimensions adapted for studying the behaviour of ripples within a model section of an auditorium. Experiments conducted outside the limits of strict mathematical analogy, using an impulsive disturbance instead of a maintained train of waves, reveal a striking resemblance to impulsive sound waves. This is illustrated by Pl. I, p. 16, which compares the reflection of ripples and of sound from a hemicylindrical mirror. In the case of ripples subsidiary wavelets accompany the main pulse, but apart from this the resemblance is striking.

Ripples are produced in the tank by a stylus upon the end of a vibrating tuning-fork, or, in the case of impulsive ripples, by the withdrawal of a small plunger from the water. A sloping beach at the edges of the tank suppresses the waves when they arrive, and prevents reflection. To facilitate study of the ripples light from an arc lamp some 10 ft. distant passes upwards through the glass bottom of the ripple tank, and casts a brilliant 'shadow' of the waves upon a screen suitably mounted some 5 ft. above. A translucent screen is most convenient, as the shadow of the waves may be viewed by transmitted light. Study of the ripples may be made visually, or by snapshots, or by cinematography. Intermittent illumination is a convenience in the visual study of continuous trains of waves.

Sound Pulse Photography. When a sound wave is intense, as in the sound pulse emitted from an electric spark, the wave itself may be photographed : for a sound pulse consists of compressed air followed by a region of rarefaction and when it is photographed advantage is taken of the refraction that occurs when light passes through a region in which the density of a gas is not uniform. An everyday example of the refraction phenomenon is the 'shadow' of hot-air streams that sunlight frequently casts upon a wall behind a hot radiator. In the case of a sound pulse the disturbance moves too quickly to be photographed in the ordinary way. A simple technique has, however, been developed along two lines by various physicists, notably Toepler (1867),

Dvořák (1880), Mach (1881), Boys (1891), and Foley and Souder (1912). On the one hand a direct-shadow method has been put to considerable application in acoustics in connection with the study of the reflection of sound from the boundaries of auditoriums (p. 282), and yields photographs of sound waves and of solid bodies at the same time. On the other, the 'Schlieren' method, which has been put to practical application in connection with the photography of flames and of explosion waves,

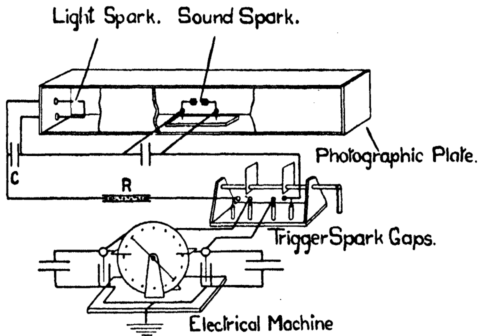


FIG. 1.—Apparatus for sound-pulse photography

depends upon an optical arrangement whereby nothing is visible in the camera except where the sound pulse has deflected light into it. In photographing a sound pulse by either method an 'instantaneous' exposure is obtained by using an electric spark as the source of illumination.

In the direct-shadow method the shadow is received direct upon a photographic plate. The actual arrangement used at the National Physical Laboratory in studying auditoriums * is illustrated in fig. 1. A sound pulse is produced within a model section of an auditorium by the 'sound spark.' A few hundred thousandths of a second later it is illuminated from a point several feet away by means of the light from the 'light spark.' The electrodes of the latter are of magnesium, so that a brilliant flash of high actinic value is produced. The light emitted casts upon the screen not only a silhouette of the model, but also a shadow of the instantaneous position of the sound pulse within it (Pl. VII (*b*), p. 257). The method adopted for controlling the interval between the two sparks is to arrange them in series as shown, and to delay the light spark by connecting a few Leyden

* A. H. Davis and N. Fleming, *J. Sci. Insts.*, 3, 393, 1926.

jars across its electrodes.* When a photograph is required, a battery of several Leyden jars is charged to a potential of some 100,000 volts by means of an electrical machine, and is then discharged through the circuit, by the operation of trigger spark gaps. The sparks pass in the trigger gaps when the glass plates shown are moved in or out of the gaps. When the sparks occur in the circuit the light spark is delayed momentarily behind the sound spark by the action of the delay condenser. The magnitude of the delay, which depends upon the size of the condenser and upon the length of the spark gap, can be adjusted as desired within certain limits. Constancy in the behaviour of the apparatus is increased by the resistance R, and by passing a series of sparks at regular intervals for some time before the photograph is taken. To prevent light from the sound spark from falling upon the screen or photographic plate a small button is placed upon the right-hand electrode to act as a screen. Fogging of the plate by scattered light is prevented by enclosing the whole in a long wooden 'light-tight' box, the interior of which is blackened, and which is fitted with a series of suitable diaphragms.

The Principle of Superposition. (Interference and Beats.) When the pressures in a sound wave rise and fall in a simple pendular manner, the motion at any point is said to execute simple harmonic motion. Such motion is represented by an equation of the form

$$y = \mathrm{A} \cos (2\pi nt + \epsilon)$$

where y is the displacement or excess pressure at a time t, n is the frequency of vibration of the motion, and ϵ is a constant. The quantity $2\pi n$ is often denoted by ω, and is sometimes called the 'pulsatance' of the motion.

If the air is excited by two or more harmonic disturbances y_1, y_2, etc. at once, and if the displacements are not individually excessive, the total displacement or pressure is given by the sum of the displacements or pressures due to the separate effects. Thus

$$y = \Sigma y_1 = \mathrm{A}_1 \cos (2\pi n_1 t + \epsilon_1) + \mathrm{A}_2 \cos (2\pi n_2 t + \epsilon_2) +$$

This principle of superposition fails if the disturbances are very large, such as those arising from very loud sounds.

Many cases of the principle of superposition are of interest in

* A smaller condenser across the sound spark advantageously increases the intensity of the sound pulse.

acoustics, and their interpretation depends largely upon the summation of a number of trigonometric terms (see Appendix). When two simple harmonic disturbances of the same amplitude ($A_1 = A_2$) and frequency ($n_1 = n_2$) are superposed the result in general is a simple harmonic disturbance of the same frequency. The amplitude may, however, be zero if the constituents are in opposite phases (*i.e.* if $\epsilon_1 = 0$ and $\epsilon_2 = \pi$). In acoustics this particular property leads to phenomenon known as interference. If two continuous sources of sound of identical amplitude and pitch—that is, the same frequency of alternations—are situated near each other, there will be regions in the sound field where at any moment the condensation from one source will be exactly neutralised by a rarefaction due to the other. Half a cycle later the conditions will be reversed, but the waves will still neutralise each other. In such regions, in spite of the proximity of the sources, no sound will be audible. At certain other points the disturbances may be additive and specially loud sounds will be heard. This distribution of regions of comparative silence and of enhanced loudness constitutes an interference field. Interference arises also when a single source is near a reflecting wall, for the reflected waves proceed as from a virtual or image source, similar to the real source. In the case of plane waves impinging normally upon a perfectly reflecting wall, maximum loudness will be perceived at multiples of half a wave-length from the wall, while no sound will be noticeable at odd multiples of a quarter of a wave-length. The regions of maximum and of minimum sound are called nodes and antinodes respectively.

When two disturbances differ slightly in frequency we have

$$y = a \cos 2\pi n_1 t + a \cos (2\pi n_2 t + \epsilon) = 2a \cos \{\pi(n_1 - n_2)t - \tfrac{1}{2}\epsilon\} \cos \{2\pi\left(\frac{n_1 + n_2}{2}\right)t + \tfrac{1}{2}\epsilon\}$$

The response is equivalent to that of a harmonic motion of an intermediate frequency $\frac{1}{2}(n_1 + n_2)$ the amplitude of which fluctuates from $2a$ to zero $n_1 - n_2$ times per second. Thus two notes of nearly equal pitch give rise to a note of intermediate pitch but fluctuating intensity. If the difference $n_1 - n_2$ is not too great, then fluctuations can be observed by the ear, and are known as 'beats.'

CHAPTER II

VIBRATING SYSTEMS

Free Vibrations. Sound arises from vibrations. The simplest vibrating system is completely defined in configuration when the displacement of one point is known. Such a system may be represented by a mass m moving in a straight line and attached to the free end of a weightless spring having the other end fixed. If the strength of the spring is such that the force tending to restore the mass to its equilibrium position is equal to s times the displacement, s is known as the stiffness constant of the system.

In mathematical symbols the equation of motion is

$$m\ddot{\xi}+s\xi=0 \tag{1}$$

where ξ represents the displacement at any moment from the equilibrium position.

If, in addition, the mass experiences a resistance $r\dot{\xi}$ proportional to its velocity $\dot{\xi}$, the equation becomes

$$m\ddot{\xi}+r\dot{\xi}+s\xi=0 \tag{2}$$

The solution of this differential equation indicates that the natural oscillation of the system, when disturbed and left free to oscillate, is of three types depending upon the relative importance of the resistance term. When the resistance is small, as is the case with a pendulum in free air, and with most acoustical examples, oscillations occur which die away slowly in amplitude as time advances. When the resistance is large, as would be the case with a rigid pendulum suspended in treacle, all real oscillations are suppressed, and the mass merely moves slowly to its equilibrium position, passing at most once through this position before coming to rest.

In setting out the actual solutions of the differential equation in the various cases it is convenient to write Δ for $r/2m$, and n^2

for s/m. The solutions for the cases of small, large and critical damping are then as follow.

Case i, $\Delta < n$, *i.e.* damping small:

$$\left.\begin{aligned} \xi &= \mathrm{A}e^{-\Delta t}\sin(n_1 t+\theta) \\ \dot{\xi} &= n_1\mathrm{A}e^{-\Delta t}\cos(n_1 t+\theta+\epsilon) \\ \text{where}\quad n_1 &= \sqrt{n^2-\Delta^2} \\ \epsilon &= \tan^{-1}(\Delta/n) \end{aligned}\right\} \quad (3)$$

and the quantities A and θ depend upon initial conditions, *i.e.* upon the displacement and velocity of the system at the instant from which the time t is measured.

In the complete absence of resistance forces the equation becomes $\xi = \mathrm{A}\sin(nt+\theta)$ and $\dot{\xi} = n\mathrm{A}\cos(nt+\theta)$. The oscillations are then of the type known as simple harmonic, and have the frequency $n/2\pi$ known as the natural undamped frequency of the system. The oscillations would continue for ever. However, with dissipative forces present the amplitude of the oscillations falls asymptotically to zero as the time t increases. The frequency $n_1/2\pi$ is slightly lower than the frequency $n/2\pi$ of the undamped system.

It is convenient to have a method of expressing the rate of decay of the vibrations of a vibrating system. From the formula (3) it is seen that the difference of the natural logarithms of successive extreme excursions is nearly constant. It is equal to $\frac{1}{2}\Delta\tau$ where $\tau(=2\pi/n_1)$ is the periodic time of a complete oscillation. This quantity is often called the 'logarithmic decrement,' and applies to the logarithm of the ratio of successive maximum amplitudes on opposite sides of zero displacement. It should be noted that the base of the logarithms concerned is e. Where logarithms to the base 10 are employed in the definition we have

$$\log_{10}\mathrm{dec} = 0{\cdot}217\Delta\tau$$

Some writers use the term 'modulus of decay' for the time $1/\Delta$ taken for the amplitude to fall in the ratio $1/e$. The time taken for a tenfold decay of amplitude is $2{\cdot}303/\Delta$, and the energy $(\frac{1}{2}m\dot{\xi}^2)$ decays tenfold in a time $1{\cdot}15/\Delta$. Since a tenfold change of any quantity can be called a change of 1 brig (p. 241) and a tenfold energy change is known as 1 bel, it may be said that the amplitude of a damped vibrating system decays by $\Delta/2{\cdot}30$ brigs per second, and the energy by $\Delta/1{\cdot}15$ brigs or bels per second.

Case ii, $\Delta > n$, *i.e.* damping large:

$$\xi = e^{-\Delta t}\left[\mathrm{A}e^{-\sqrt{\Delta^2-n^2}\,t} + \mathrm{B}e^{+\sqrt{\Delta^2-n^2}\,t}\right] \tag{4}$$

In this case the motion is the sum of two decaying motions which contain no periodic (harmonic) components. The system passes once at most through its equilibrium position, and comes asymptotically to rest without oscillations. The motion is of interest in connection with oscillographs and dead-beat instruments.

Case iii, $\Delta = n$, *i.e.* critical damping:

$$\xi = e^{-\Delta t}(\mathrm{A}t + \mathrm{B}) \tag{5}$$

The motion is a simple return to equilibrium conditions resembling that described in connection with large damping.

Forced Vibrations. So far we have considered the free oscillations of a simple system, which has been left to itself after being displaced from its equilibrium position. It is important, however, to consider the case in which the body is maintained in a state of vibration by an applied periodic force of any given frequency.

When a steady harmonic alternating force $\mathrm{F}_0 \cos \omega t$ is applied to the system referred to above the equation of motion becomes

$$m\ddot{\xi} + r\dot{\xi} + s\xi = \mathrm{F}_0 \cos \omega t \tag{6}$$

The mathematical solution of this equation shows that the application of the periodic force to the vibrating system gives rise to two effects. In the first place, on the application of the external force, transient oscillations of the type considered for a simple disturbed system are set up. These are damped and soon die away and leave a steady regular response to the steady regular excitation. This steady response which is ultimately reached may be represented by the following equation which gives the magnitude of the displacement ξ at any time t after the application of the force:—

$$\left.\begin{array}{l} \xi = \dfrac{\mathrm{F}_0}{\omega \mathrm{Z}} \sin(\omega t - \phi) = \dfrac{\mathrm{F}_0}{\omega \mathrm{Z}} \cos(\omega t - \epsilon) \\ \text{The velocity } \dot{\xi} \text{ is given by} \\ \dot{\xi} = \dfrac{\mathrm{F}_0}{\mathrm{Z}} \cos(\omega t - \phi) \end{array}\right\} \tag{7}$$

where
$$Z = \sqrt{r^2 + \left(m\omega - \frac{s}{\omega}\right)^2}$$

$$\tan \phi = \left(\frac{m\omega - s/\omega}{r}\right)$$

$$\epsilon = \frac{\pi}{2} + \phi$$

Thus the magnitude of the motion is inversely proportional to the quantity z—a function of the utmost importance in acoustics. It is to be noted also from the equations that the displacement and velocity of the driven system are harmonic and of the same frequency as the driving force, but lag behind the driving force $F_0 \cos \omega t$ by amounts ϵ and ϕ, which depend upon the frequency of the exciting force. Naturally it takes a certain time for the ultimate steady state to be set up and for the motion to settle down to lag by the stated amount behind the driving force.

This brings us back to the question of the early stages of the forced motion. It should be stated therefore that the complete solution of the fundamental equation for forced vibration involves the addition to equation 7 of terms of the form of equations 3, 4, and 5 representing vibrations arising from disturbed equilibrium. In the case of lightly damped systems ($\Delta < n$) the complete solution is

$$\xi = Ae^{-\Delta t} \sin (n_1 t + \theta) + \frac{F_0}{\omega Z} \sin (\omega t - \phi) \tag{8}$$

The first term represents a 'transient' vibration which arises from the initial application of the disturbing force, and which rapidly dies out on account of the exponential damping factor. The second term is the permanent 'steady state' which persists so long as the driving force is maintained without alteration. When the driving force is removed the forced vibration term at once disappears, and the motion is converted into a free vibration having, of course, the frequency $n_1/2\pi$ instead of the frequency $\omega/2\pi$ of the removed driving force. In the early stages of the vibration which follows the application of the driving force, the transient and steady-state motions coexist, and the phenomenon of beats may occur if the two frequencies n_1 and ω differ only slightly. There is then a rise and fall in the amplitude of the

motion which is often observable in the early stages of the motion of electrically driven tuning-forks.

In a physical sense transients may be regarded in the following manner. They are the perturbations that the system passes through with its own natural frequency in its attempt (*a*) to settle into its final equilibrium position when forces are removed, or (*b*) to get into the steady-state phase relation with the driving force when an alternating driving force is freshly applied. Transients are of course most prominent in the case of lightly damped systems. Thus it is of importance in the design of loud-speakers to have adequate damping, for otherwise inharmonic transient notes would be heard each time a fresh driving note was applied to the terminals of the loud-speaker.

Velocity Relations at Resonance—Sharpness of Resonance. Reverting to the steady-state equation (7) for the vibration of a simple mechanical system, we note that amplitudes and velocities vary inversely as ωZ and Z respectively, and that

$$
\begin{aligned}
&Z \text{ is a minimum } (=r) \text{ when } \omega^2 = n^2 = s/m,\\
&Z = m\omega \text{ when } \omega \text{ is large compared with } n,\\
&Z = s/\omega \text{ when } \omega \text{ is small compared with } n.
\end{aligned}
$$

For a system vibrating under the action of a given alternating force the velocity response depends markedly upon the value of ω, and is a maximum when the frequency of the driving force equals the *undamped* natural frequency $n/2\pi$ of the driven system. This special case of forced vibration is known as ‘resonance.’ The velocity response $\dot{\xi}_{max}$ falls to $1/\sqrt{2}$ of its maximum value (and the energy $\frac{1}{2}m\dot{\xi}^2_{max}$ to half of its maximum) at frequencies given approximately by $\dfrac{\omega}{n} = 1 \pm \dfrac{\Delta}{n}$ if Δ/n is small, or more accurately by $\omega = \pm\Delta + \sqrt{\Delta^2 + n^2}$. The quantity n/Δ determines the sharpness of resonance and is the commonly accepted measure of that quantity.

A curve showing the relation between the driving frequency $(\omega/2\pi)$ and the maximum velocity of response $(\dot{\xi}_{max})$ is symmetrical when the forcing frequency is not far removed from the resonant frequency, and the velocity resonance curve is symmetrical without restriction if the forcing frequency is expressed in terms of the musical interval in octaves $[\log_2(\omega/n)]$ by which it differs from the resonant pitch.

This follows from the equations below, which represent the resonance curve in various ways.

$$\left.\begin{aligned}\frac{\xi_\omega}{\xi_n}=\frac{r}{Z_\omega}&=\frac{\dfrac{\Delta}{n}}{\sqrt{\left(\dfrac{\Delta}{n}\right)^2+\frac{1}{4}\left(\dfrac{\omega}{n}-\dfrac{n}{\omega}\right)^2}}=\frac{1}{\sqrt{1+S^2\sinh^2 x}}\\ \text{where } x=\log_e(\omega/n)\qquad&\\ &=\frac{1}{\sqrt{1+S^2x^2}}=\frac{1}{\sqrt{1+\left(\dfrac{\omega-n}{\Delta}\right)^2}}\quad\text{approx.}\end{aligned}\right\}\tag{9}$$

Since $\sinh x=x$ approx., the velocity curve tends to be a function of Sx, the form in which the curves of fig. 2 are plotted. It

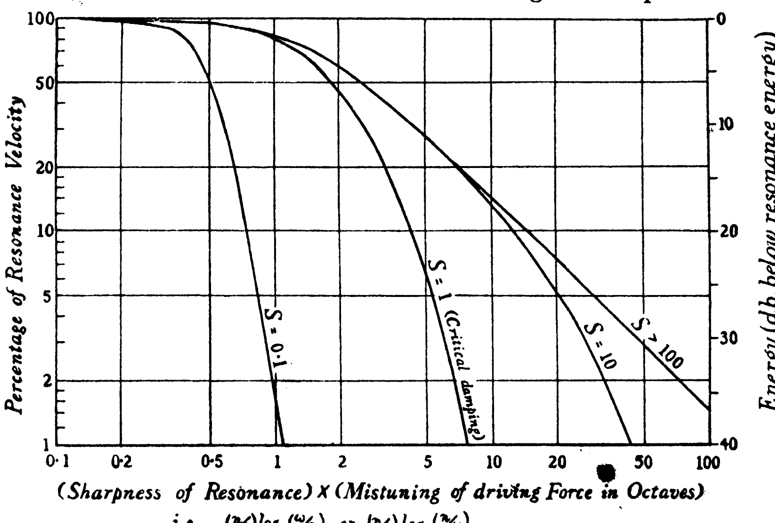


FIG. 2.—Resonance curves *

* In the equation $m\ddot{x}+r\dot{x}+sx=F_0\cos\omega t$, $r/2m=\Delta$, $s/m=n^2$, $n/\Delta=S$. The velocity response is $1/\sqrt{2}$ of its maximum value when $\frac{\omega}{n}=1\pm\frac{\Delta}{n}$ approx. When x is small $\log_2(1+x)=1{\cdot}44x$ approx.

Frequency ratio . .	1	1·007	1·07	1·15	1·41	2
Octaves	0	0·01	0·1	0·2	0·5	1

conveniently represents a wide range of data for the velocity of response, expressed as a percentage of the resonance velocity, and is useful in interpreting resonance curves and data. It will be found that the single curve (marked $S>100$) is satisfactory to 8 per cent. for all values of S provided the frequency of the

driving force does not differ from the resonance frequency by more than 1 octave. Over a somewhat more restricted range of, say, 1 or 2 musical tones, it is possible to plot results as a function of $(\omega - n)/\Delta$ to an accuracy of about 5 per cent. (fig. 3).

Damping factors (Δ) for constrained systems are best determined experimentally from the shape of the resonance curve for bluntly tuned systems, but from the decay of natural vibrations if the damping is slight.

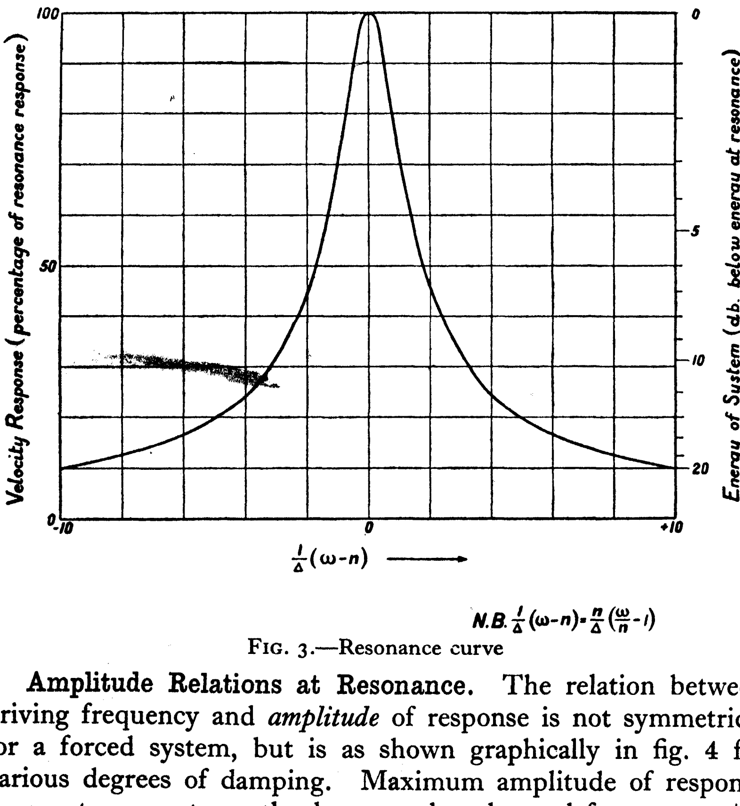


FIG. 3.—Resonance curve

Amplitude Relations at Resonance. The relation between driving frequency and *amplitude* of response is not symmetrical for a forced system, but is as shown graphically in fig. 4 for various degrees of damping. Maximum amplitude of response does not occur at exactly the natural undamped frequency $n/2\pi$ of the forced system, but when $\omega^2 = n^2 - 2\Delta^2$ as may be found by differentiating with respect to ω the expression for the amplitude of a forced system. If $2\Delta^2 > n^2$ (*i.e.* $\Delta > 0{\cdot}71n$) ω is imaginary and no real maximum occurs, as is seen in fig. 4. When a maximum occurs its magnitude is given by $\frac{F_0}{nr}\left(1 - \frac{\Delta^2}{n^2}\right)^{-\frac{1}{2}}$ which approximates to F_0/nr when Δ/n is small.

Phase Relations at Resonance. The value for the phase lag (ϕ) of the velocity behind the driving force has been given earlier. It will be observed from the equation (7) that the phase lag is zero at resonance. It is positive when the frequency of the driving force exceeds the resonant frequency of the vibrating system, and the phase of the velocity then 'lags' behind that of the force. On the contrary, ϕ is negative when the frequency

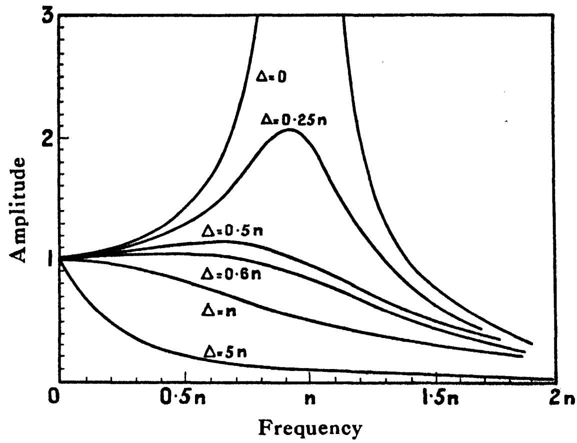


FIG. 4.—Forced vibration. Relation between frequency and amplitude of displacement for various degrees of damping

of the driving force is low compared with the resonant frequency of the system, and then the phase of the motion of the system 'leads.'

The following table summarises amplitude and phase relations for various frequencies of the driving force, on the assumption that Δ^2/n^2 is small :—

TABLE I

ω	$1/Z$	φ	ϵ
0	0	$-\pi/2$	0
$n-\Delta$	$1/r\sqrt{2}$	$-\pi/4$	$\frac{\pi}{4}$
n	$1/r$	0	$\frac{\pi}{2}$
$n+\Delta$	$1/r\sqrt{2}$	$+\pi/4$	$\frac{3\pi}{4}$
∞	0	$+\pi/2$	π

Fig. 5 shows graphically the manner in which the phase lag ϵ varies with frequency for a number of different values of damping coefficient.

Forced Vibration under the Action of two or more Simple Harmonic Driving Forces. When several simple harmonic forces act simultaneously upon a system, the resulting forced vibration

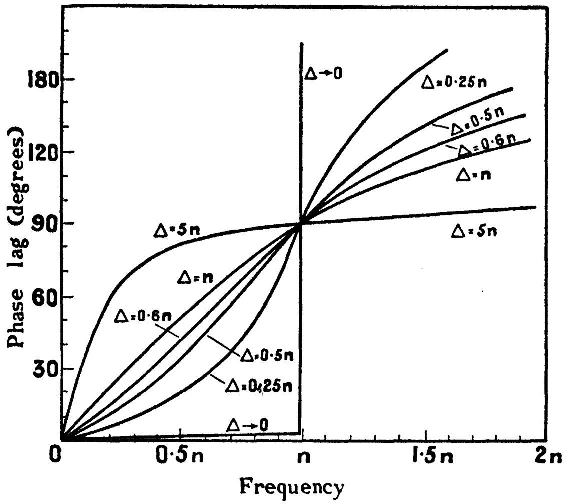


FIG. 5.—Forced vibration. Relation between frequency and phase lag (ϵ) of displacement behind impressed force for various degrees of damping

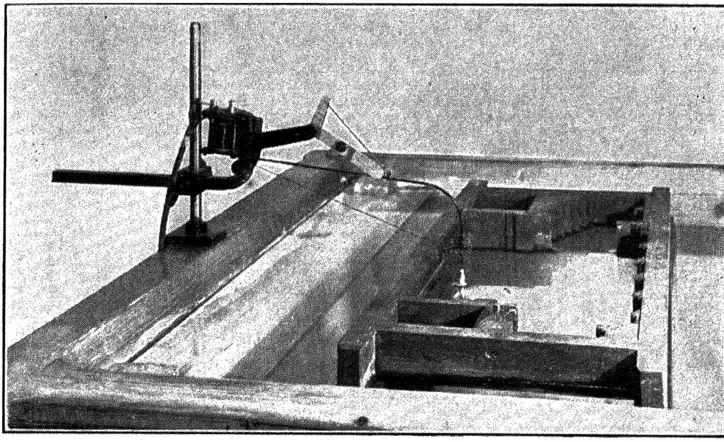
may be deduced by simple addition of the separate effect of each. Thus the composite force $F = F_1 \cos \omega_1 t + F_2 \cos (\omega_2 t + a)$ gives rise to the forced vibration

$$\left.\begin{aligned} \xi &= \frac{F_1}{\omega_1 Z_1} \sin (\omega_1 t - \phi_1) + \frac{F_2}{\omega_2 Z_2} \sin (\omega_2 t + a - \phi_2) \\ \dot{\xi} &= \frac{F_1}{Z_1} \cos (\omega_1 t - \phi_1) + \frac{F_2}{Z_2} \cos (\omega_2 t + a - \phi_2) \end{aligned}\right\} \qquad (10)$$

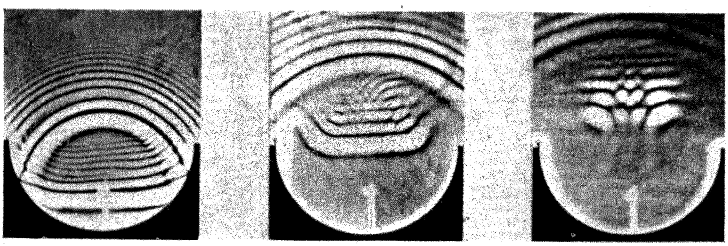
where Z_1 and ϕ_1 have the values they would have if the term $F_2 \cos (\omega_2 t + a)$ were absent from the expression for the driving force, and Z_2 and ϕ_2 the values that would obtain if the term in F_1 were absent.

It will be noted that the amplitudes of the various terms in the forced motion are not proportional solely to those of the corresponding terms in the value of the driving force, owing to the presence of the terms Z_1 and Z_2 in the denominators. Nor

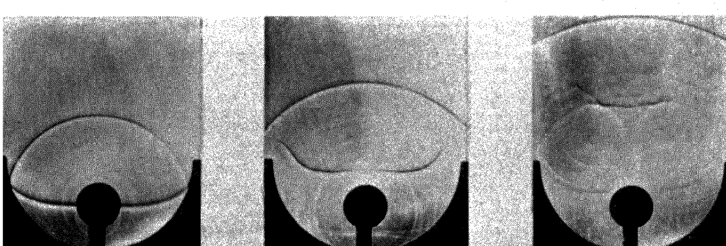
PLATE I



Ripple tank apparatus. See p. 4



Ripples



Sound Pulse

Comparison between ripple and sound-pulse photographs relating to reflection from a hemicylindrical mirror. See p. 4

are the phase relations constant. Thus in general the character of the forced vibration differs from that of the driving force and, in particular, if one of the frequencies ω_1, ω_2 is near the natural frequency n of the forced system, the vibration of this frequency will predominate in the motion set up.

It may be shown by Fourier's theorem that the most complicated driving force, if continuous and periodic—*i.e.* repeating regularly—may be regarded as made up of a series of simple harmonic constituents. Its effect upon a simple vibrating system may thus be determined by applying the above principles. The importance of the harmonic type of disturbance is manifest from this fact, and it may be mentioned further that the simple harmonic type of motion is the only type which remains unchanged in character when transmitted from one system to another.

Conditions for the Reproduction of Complex Wave-forms. Theory of the Oscillograph. Whilst in general the motion of a forced system does not reproduce the character of complex driving forces, it is possible to get reasonably faithful reproduction if the natural frequency of the system is kept well clear of the frequencies to be recorded, and if the damping is adequate to suppress transient vibration when forces are suddenly applied.

We have seen that to avoid transient oscillations and thus to obtain an aperiodic system it is necessary for Δ to be not less than n. This therefore is the first requirement in an instrument (such as vibration recorder, a microphone, or an oscillograph) designed to follow rapid fluctuations with accuracy. The second requirement is that the system shall maintain amplitude and phase relations between the various constituents. Fig. 4 shows how the amplitude of response of a system depends upon the driving frequency for various values of Δ. As it is desirable to have equal response at all frequencies the curve $\Delta = 0{\cdot}6n$, which is fairly flat for some distance from the origin, would appear to be very suitable. Since, however, Δ must be less than n for aperiodicity, the curve $\Delta = n$ is the best to employ.

By keeping the working range small, say from $\omega = 0$ to $\omega = 0{\cdot}1n$ or $0{\cdot}2n$, the error introduced by assuming amplitude response to be constant is 1 per cent. and 4 per cent. respectively. Thus to obtain a true record of relative amplitudes the natural frequency of the instrument must be 5 to 10 times the highest frequency to be recorded. For $\Delta = n$ the phase lag in degrees is seen from fig. 5 to be approximately proportional to ω. The time lag

ϕ/ω is thus approximately constant; indeed it departs from constancy only by 3 per cent. and 5 per cent. up to $\omega = 0{\cdot}1n$ and $0{\cdot}2n$ respectively.

The Use of Complex Quantities. The equations met in the study of vibrating systems and of acoustics may often be greatly simplified, and the manipulation of the equations made easier, by the use of complex quantities in the analysis.

Those acquainted with complex quantities will recall that a complex quantity z can be written in the form $x+iy$, where $i^2 = -1$, and where x is called the real part of z and iy the imaginary part. Moreover, we may write

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

where e is the base of natural logarithms and where $r^2 = x^2 + y^2$, and $\tan\theta = y/x$. The positive value of r is called the modulus of z and is written $|z|$. If two complex quantities z and z' are equal their real parts are equal, as also are their imaginary parts. That is, if $z = z'$, then $x = x'$ and $y = y'$.

Moreover, if $z = x + iy$ is a complex solution of a linear differential equation with real coefficients, then its real part x is a solution of the real part of the equation, and y a solution of its imaginary part. For instance, let the differential equation be linear and of the form

$$\mathrm{A}\ddot{z} + \mathrm{B}\dot{z} + \mathrm{C}z = \mathrm{L} + \mathrm{M}i \tag{11}$$

the coefficients being real. Let a solution be

$$z = x + iy$$

then

$$\dot{z} = \dot{x} + i\dot{y}$$
$$\ddot{z} = \ddot{x} + i\ddot{y}$$

Hence, substituting in (11),

$$(\mathrm{A}\ddot{x} + \mathrm{B}\dot{x} + \mathrm{C}x) + i(\mathrm{A}\ddot{y} + \mathrm{B}\dot{y} + \mathrm{C}y) = \mathrm{L} + \mathrm{M}i \tag{12}$$

Equating real and imaginary parts we have

$$\mathrm{A}\ddot{x} + \mathrm{B}\dot{x} + \mathrm{C}x = \mathrm{L}$$
$$\mathrm{A}\ddot{y} + \mathrm{B}\dot{y} + \mathrm{C}y = \mathrm{M}$$

i.e. x is a solution of the real part of the equation and y is a solution of the imaginary part.

Owing to the ease with which exponential complex quantities may be manipulated it is often convenient in acoustics to seek the solution of a real equation by first writing it in its complex

form and then, after finding the complex solution, taking only its real part. This will prove to be the solution sought.*

As an example consider the equation (6),

$$m\ddot{x}+r\dot{x}+sx=F_0 \cos \omega t$$

Since cos ωt is the real part of the complex quantity $e^{i\omega t}$, the corresponding complex equation is

$$m\ddot{z}+r\dot{z}+sz=F_0 e^{i\omega t} \tag{13}$$

A solution of this is readily found to be $z=Ae^{i\omega t}$, where A, found by substituting this value of z in (13), is given by

$$A(-m\omega^2+ir\omega+s)=F_0$$

i.e.

$$z=\frac{F_0 e^{i\omega t}}{(ir\omega-m\omega^2+s)}$$

and

$$\dot{z}=\frac{F_0 e^{i\omega t}}{[r+i(m\omega-s/\omega)]}=\frac{F_0 e^{i\omega t}}{|Z|e^{i\phi}}=\frac{F_0}{|Z|}e^{i(\omega t-\phi)}$$

where

$$Z^2=r^2+(m\omega-s/\omega)^2 \quad \text{and} \quad \tan \phi=\frac{m\omega-s/\omega}{r}$$

The real part of the above solution, namely,

$$\dot{x}=\frac{F_0}{Z}\cos(\omega t-\phi)$$

is the solution that we are seeking.

The procedure here applied to the case of forced vibration applies equally to free vibrations—the particular case where the applied force is zero, *i.e.* where $L=M=0$ in (11).

Mechanical Impedance. The ratio of the driving force to the velocity of a driven vibrating body is known as the mechanical impedance of the system, and is the quantity denoted by Z above. We have seen that

$$Z=\sqrt{r^2+(m\omega-s/\omega)^2}, \qquad \phi=\tan^{-1}\left(\frac{m\omega-s/\omega}{r}\right)$$

---

* If the original differential equation (11) is not linear (*i.e.* if squares and higher powers of z, $\dot{z}$, $\ddot{z}$, etc., are involved) it will be found that terms in y will occur with terms in x in the real part of (12) and the proof fails.

which may be rewritten in complex notation as

$$Z = r + i(m\omega - s/\omega)$$

The real portion (r) of a mechanical impedance written in complex form is known as the mechanical resistance, and the part $(m\omega - s/\omega)$ as the mechanical reactance. Or, in more general terms, if the impedance is separated into its real and imaginary parts,

$$Z = Z_1 + iZ_2$$

Z_1 is defined as the mechanical resistance and Z_2 as the mechanical reactance.

The mechanical impedance Z is closely analogous to impedance in electrical circuits, as is seen from the following considerations. For a circuit containing in series an alternating e.m.f. $E_0 \cos \omega t$, an inductance L, a resistance R, and a capacity C, the equation governing the current of electricity in the circuit is

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = E_0 \cos \omega t$$

where q is the quantity of electricity on the plate of the condenser at any moment. The steady state solution of this equation, expressed in terms of the quantity of electricity q, is

$$q = \frac{E_0}{\omega Z} \sin (\omega t - \phi)$$

the current $\dot{q}$ is given by

$$\dot{q} = \frac{E_0}{Z} \cos (\omega t - \phi)$$

where

$$Z^2 = R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2 \quad \text{and} \quad \tan \phi = \frac{L\omega - \dfrac{1}{\omega C}}{R}$$

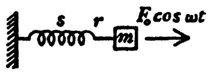
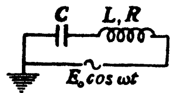
On comparing these equations with the foregoing it is seen that there is a close analogy between mechanical and electrical oscillations on the general lines that

Mass	resembles	Inductance.
Frictional resistance	,,	Resistance.
Compliance (*i.e.* 1/stiffness)	,,	Capacity.
Displacement	,,	Quantity.
Velocity	,,	Current.
Force	,,	E.M.F.

The analogy is brought out in the following table, where the mechanical and electrical cases have been set out diagrammatically, together with their appropriate differential equations and solutions.

TABLE II

Analogy between Mechanical and Electrical Systems

Meehanical	*Electrical*
	
$m\ddot{\xi}+r\dot{\xi}+s\xi=\mathrm{F}_0\cos\omega t$	$\mathrm{L}\ddot{q}+\mathrm{R}\dot{q}+\frac{q}{\mathrm{C}}=\mathrm{E}_0\cos\omega t$
$\dot{\xi}=\frac{\mathrm{F}_0}{\omega\mathrm{Z}}\cos(\omega t-\varphi)$	$\dot{q}$ (i.e. i) $=\frac{\mathrm{E}_0}{\mathrm{Z}}\cos(\omega t-\varphi)$
$\xi=\frac{\mathrm{F}_0}{\omega\mathrm{Z}}\sin(\omega t-\varphi)$	$q=\frac{\mathrm{E}_0}{\omega\mathrm{Z}}\sin(\omega t-\varphi)$
where $\mathrm{Z}^2=r^2+\left(m\omega-\frac{s}{\omega}\right)^2$	where $\mathrm{Z}^2=\mathrm{R}^2+\left(\mathrm{L}\omega-\frac{1}{\omega\mathrm{C}}\right)^2$
$\tan\varphi=\frac{\left(m\omega-\frac{s}{\omega}\right)}{r}$	$\tan\varphi=\frac{\left(\mathrm{L}\omega-\frac{1}{\omega\mathrm{C}}\right)}{\mathrm{R}}$

Owing to the marked progress which has been made during the last fifty years in the mathematics of electrical circuits it is natural that functions such as impedance, which have elegance and power when employed in electrical circuits, should be of great value when applied to mechanical circuits. It is not surprising therefore that considerable advances have been made in gramophone reproduction by matching impedances of various members of the recording system, just as one matches impedances in electrical circuits in order to get best effects. Fig. 6, due to Maxfield and Harrison, shows a mechanical system composed of a gramophone needle and sound box, together with its electrical equivalent.* The equivalent electrical circuit is a type of filter

* J. P. Maxfield and H. I. Harrison, *Bell System Tech. J.*, 5, 493, 1926; A. Whitaker, *J. Sci. Insts.*, 5, 35, 1928.

In a mechanical circuit it is sometimes helpful to remember the following rules for ascertaining whether mechanical impedances are to be regarded as equivalent to electrical analogues in series or in parallel. If various components of a system have the same amplitude impressed

(p. 183), which, when properly designed, transmits equally all frequencies between certain limits determined by the inductances and capacities involved. The function of design in the acoustical case is to adjust the analogous quantities so that as wide a frequency

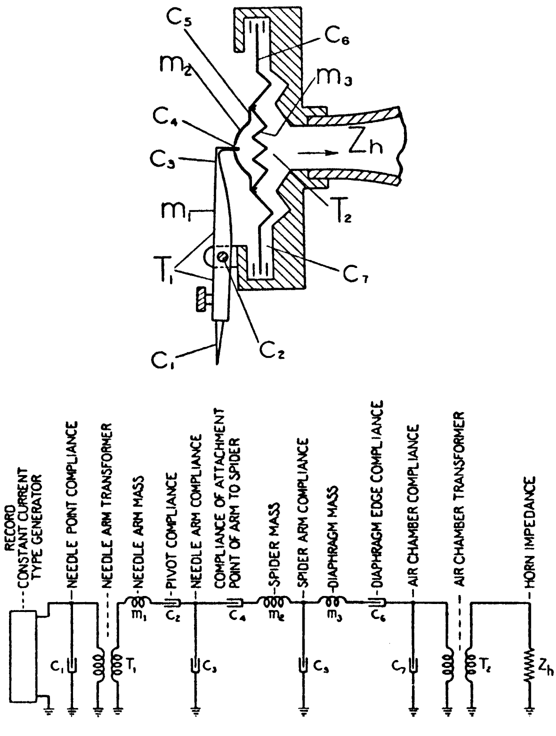


FIG. 6.—Gramophone sound box and its equivalent electrical circuit (Maxfield and Harrison)

band as possible is transmitted without distortion—a complicated problem of which the electrical solution was known from the theory of telephonic circuits.

Rate of Dissipation of Energy. It is well known that in an electrical circuit the dissipation of energy is given by Ri^2 (*i.e.* $R\dot{q}^2$),

upon them, their impedances are added to obtain the total impedance. If two or more parts of a system divide the compressed amplitude between them, they are to be regarded in parallel, and the reciprocal of the total impedance is the sum of the reciprocals of the individual impedances. Thus a stiffness between a rigid support and a moving member is represented by a series capacity and between two consecutive moving members by a shunt capacity.

where i $(=\dot{q})$ is the current through the resistance R at any moment. The dissipation of energy in the mechanical system defined by equation (6) is given * by the analogous quantity $r\dot{\xi}^2$. Where energy is dissipated in doing external work (factor R) as well as in internal friction (factor r_1), the rate of expenditure of energy is $r\dot{\xi}^2=(r_1+\mathrm{R})\dot{\xi}^2$. Here R is called the 'radiation resistance' and $\mathrm{R}\dot{\xi}^2$ is the rate at which the system does external work, or, in other words, radiates energy.

In simple harmonic motion the average value of the velocity $\dot{\xi}$ during a complete cycle is $1/\sqrt{2}$ times the maximum value $\dot{\xi}_{\max}$, and the average value of the square of the velocity is one-half of the square of the maximum velocity. In consequence the average rate of dissipation of energy from a forced system may be written $\frac{1}{2}r\dot{\xi}^2_{\max}$.

Large Vibrations. The above equations for the motion of a vibrating system apply to elastic systems in which the restoring forces are proportional to the displacement. It may be shown that this condition is approximately fulfilled in all cases of small vibrations about a position of stable equilibrium. For larger vibrations, however, it is necessary to regard the restoring force as including also the squares and even higher powers of the displacement, and it may be written as $s\xi+g\xi^2+h\xi^3+\ \ldots$ This quantity must therefore take the place of $s\xi$ in the differential equations of motion given previously.

When a term $g\xi^2$ occurs the vibrating system is asymmetrical, for this part of the restoring force does not change sign when the displacement changes from positive to negative. According to Rayleigh,† the effect of a term in ξ^2 upon the free oscillations of a system is to cause the normal frequency n of the system to be accompanied by its octave $2n$ to an extent of which the relative importance increases with the amplitude of vibration. It also introduces other slight effects.

In the case of large symmetrical vibrations, in which the term $h\xi^3$ occurs without a term in ξ^2, Rayleigh shows that the pitch of the fundamental is slightly altered and a frequency of three times the fundamental is introduced. The alteration in pitch is very small but detectable, as in the slight rise in pitch

* Rate of dissipation $=$ Force $\times$ Velocity

$$\begin{aligned}&=\text{Impedance}\times(\text{Velocity})^2\\&=\{r+i(m\omega-s/\omega)\}\dot{\xi}^2\quad\text{real part}\\&=r\dot{\xi}^2.\end{aligned}$$

† Rayleigh, *Theory of Sound*, I, 76, 1929.

which often occurs as the amplitude of vibration of a tuning-fork gradually dies away.

It is desirable to notice the effect of the simultaneous application of two simple harmonic forces to an unsymmetrical vibrating system. Neglecting damping the equation of motion is

$$m\ddot{\xi}+s\xi+g\xi^2=F_1\cos\omega_1 t+F_2\cos\omega_2 t \tag{14}$$

As a first approximation we have, neglecting the small term $g\xi^2$,

$$\xi=f_1\cos\omega_1 t+f_2\cos\omega_2 t$$

where the terms $f_1=F_1/\omega_1 Z_1$ and $f_2=F_2/\omega_2 Z_2$ are calculable by formulæ given earlier. To get the second approximation we substitute this value in the neglected term $g\xi^2$, and obtain for solution the differential equation in the form

$$\left.\begin{aligned} m\ddot{\xi}+s\xi &= F_1\cos\omega_1 t+F_2\cos\omega_2 t-\left[\frac{F_1}{\omega_1 Z_1}\cos\omega_1 t+\frac{F_2}{\omega_2 Z_2}\cos\omega_2 t\right]^2 \\ &= F_1\cos\omega_1 t+F_2\cos\omega_2 t-\tfrac{1}{2}g(f_1^2+f_2^2) \\ &\quad -\tfrac{1}{2}gf_1^2\cos 2\omega_1 t-\tfrac{1}{2}gf_2^2\cos 2\omega_2 t \\ &\quad -\tfrac{1}{2}gf_1f_2[\cos(\omega_1+\omega_2)t+\cos(\omega_1-\omega_2)t] \end{aligned}\right\} \tag{15}$$

This when solved gives the second approximation to the solution of our original equation above. We may note that it is the equation which would be obtained if a simple vibrating system were acted upon by a number of separate forces of pulsatances ω_1, ω_2, $2\omega_1$, $2\omega_2$, $\omega_1+\omega_2$, $\omega_1-\omega_2$. This point is of importance in connection with the summation and difference tones which arise when two loud pure sounds are applied to the ear together. They owe their origin to the asymmetrical response of the eardrum.

Reaction of Vibrating System upon its Support. When a spring-controlled mass is subjected to an alternating disturbing force, a certain reaction is exerted by the spring upon the support to which its fixed end is attached. The calculation of this force is of interest, particularly in view of the fact that springs are often interposed between vibrating machines and the floor with a view to minimising the transmission of vibratory forces.*

The displacement and velocity of the mass in its motion are given by $x=F_0e^{i\omega t}/i\omega Z$ and $\dot{x}=F_0e^{i\omega t}/Z$. The force transmitted by the spring to the floor is equal and opposite to the force upon

* C. R. Sodeburg, *Elec. J.*, 21, 161, 1924.

the mass, and is equal to $r\dot{x}+sx$ if the damping r is exerted by the spring. Evaluating from this we find that the force upon the floor is equal to

$$\text{Force}=r\dot{x}+sx=\frac{F_0}{|Z|}e^{i(\omega t-\phi)}\left\{r-i\left(\frac{s}{\omega}\right)\right\}=\frac{F_0}{|Z|}\sqrt{r^2+\frac{s^2}{\omega^2}}e^{i(\omega t-\phi-\theta)} \quad (1$$

where $\tan\phi=(m\omega-s/\omega)/r$; $\quad\tan\theta=s/r\omega$.

Where a spring is employed to minimise the transmission of vibration, the transmissibility ϵ of the system is defined as the maximum value of the transmitted force divided by the maximum value of the applied force, and is given as follows:—

$$\epsilon=\sqrt{r^2+(s/\omega)^2}/|Z|$$

i.e.

$$\epsilon=\sqrt{\frac{\frac{\Delta^2}{n^2}+\frac{n^2}{\omega^2}}{\frac{\Delta^2}{n^2}+\frac{n^2}{\omega^2}+\frac{\omega^2}{n^2}-2}} \quad (17)$$

where $\Delta=r/2m$, and $n/2\pi$, the natural undamped frequency, is equal to $(1/2\pi)\sqrt{s/m}$.

The calculation of the natural undamped frequency of the system $n[=(1/2\pi)\sqrt{s/m}]$ requires a knowledge of the load m and of the stiffness s. The latter quantity is the total force required to give unit displacement of the spring, and in c.g.s. units is expressed in dynes per cm. In the case of a perfect elastic support upon the floor the natural frequency can also be expressed in terms of the deflection D produced by a load of mass m; the equation is $n^2=g/4\pi^2D$, g being the acceleration due to gravity.

The curves of fig. 7 have been derived from equation (17) and show the transmissibility of the spring for three values of the viscous damping defined by $\Delta/n(=r^1)$ and for various frequencies of the impressed disturbing force. The case in which there is no damping is that defined by $r^1=0$. Considerable damping is represented by $r^1=1$. We note that in all cases when the forcing frequency $\omega/2\pi$ is equal to or nearly equal to the natural frequency $n/2\pi$ of the system, ϵ is very much larger than 1 and the spring support actually facilitates rather than decreases the transmission of vibration to the floor. The transmissibility ϵ is equal to unity when $\omega=0$ and when $\omega=\sqrt{2}n$. It is only if

the forcing frequency is, say, more than 4 or 5 times the natural frequency of the mass upon its elastic constraint that the vibratory force transmitted to the support or floor is considerably reduced by the presence of the spring. Increased damping in the spring then increases the transmission of vibration to the support; consequently damping in the spring is not really desirable. On

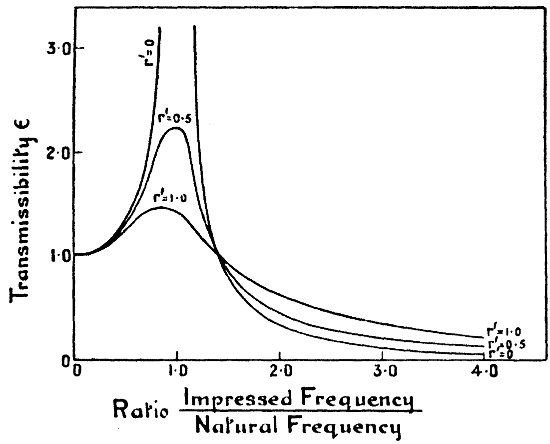


FIG. 7.—Transmissibility curves for elastic supports

the other hand, damping in the support would reduce its resonances and be beneficial.

Another case requires notice. If a massive body rests upon elastic supports and a known degree of simple harmonic vibration $\xi_0 \cos \omega t$ is communicated to the underside of the elastic support by, say, the earth upon which it rests, the equation of motion of the mass is identical in form with equation (6) above. The *force* transmitted to the mass may be shown to be related to the disturbing displacement in a manner proportional to that represented by fig. 7; the transmitted *amplitude* being related to the disturbing amplitude in the manner represented by fig. 4. Both sets of curves show the advantage of having the natural period of the system low compared with the forcing frequency, but fig. 4 shows that damping is advantageous in this case.

An aspect of the problem which arises in laboratories is the insulation of delicate instruments from vibration. Müller * recently described a method of isolating galvanometers by placing

* R. Müller, *Ann. d. Phys.*, 1, 613, 1913.

them upon a heavy base carried upon a special flexible support, and discussed other types of support which have been proposed.

Vibration Insulating Materials. Several elastic materials are available for use in isolating floors from vibration due to attached machinery. Metal springs may usually be employed, and whilst in their larger sizes they are often bulky, they have the advantage that the stiffness s is independent of the load; consequently the selection of a spring for a particular purpose presents no difficulties. Some materials, such as cork and felt, show increasing transmissibility with load which arises from increased stiffness on compression. In consequence it is necessary to stipulate that the quantity s to be used in equation (16) relates to the elastic condition under the load concerned. The variability of such materials with load and possibly with age makes them less precise than springs. B. E. Eisenhour and F. G. Tyzzer,* however, have given data for cork and various other materials, and Table III (p. 28) is largely based upon their results. They worked on samples 12 in. by 12 in. in area up to loads of 21 lb. per sq. in., and higher pressures were obtained by reducing the area of the test pieces. In the case of cork and felt, provided the thickness was not large compared with the width or breadth, the stiffness did not vary appreciably with area; it was the same for the full test specimen as for 64 separated and small specimens of the same total area. Tests on natural cork showed that the restraining band which is usually used to hold specimens together caused no appreciable change in the stiffness. With some materials under considerable load, increasing the load increased the stiffness so much that the natural frequency actually rose. The figures in the table relate to a 1-in. thickness under load as measured 24–48 hours after the load is applied. The natural frequency for any other thickness can be calculated by dividing the frequency given in the table by the square root of the thickness in inches. At the National Physical Laboratory arrangements have been devised whereby measurements may be made of the actual transmissibility of various materials under specified conditions of loading and excitation.†

Rubber compounds containing a large percentage of pure rubber are often convenient, but in this case the stiffness depends upon the area of the pad as well as upon the load. The reason

* B. E. Eisenhour and F. G. Tyzzer, *Frank. Inst. J.*, 214, 691, 1932.
† *N.P.L. Annual Reports*, 1930, 1931, 1932.

TABLE III

Natural Frequencies of Loaded Pads of Various Materials
(each 1 in. thick under Load)

Materials (Test Pieces 30 to 144 sq. in. except where stated)	Natural Frequency for Loads (lbs. per sq. in.)				
	1	5	20	50	100
Natural cork	74	54	35	24	18
Natural cork (16 sq. in.)	...	...	...	24 *	17 *
Compressed cork (light)	83	50	33	26	...
Compressed cork (medium)	99	58	36	26	24
Compressed cork (heavy)	...	63	39	26	25
Fibre board	61	42	28	22	22
Felts—Hair	31	24	21	21	24
Asbestos	56	38	26	21	21
Wool	33	25	22	21	25
Jute	36	30	30	26	21
Rubber, soft—20 sq. in.	...	...	...	14 †	...
10 sq. in.	...	...	...	12 †	...
$7\frac{1}{2}$ sq. in.	...	...	16 †	...	...
5 sq. in.	...	...	...	10 †	...

* Calculated from transmissibility measurements by Aughtie and Brown (*Engineering*, 133, 564, 1932).
† Calculated from figures given by Hull and Stewart (*loc. cit.*).
Thickness 1 in. unloaded.

Materials are unlikely to sustain the higher loads indefinitely, and arrangements should be made for occasional renewal when materials are under considerable loads.

is that rubber is nearly incompressible if entirely confined, so that when the material is compressed, any deflection is taken up by change in shape rather than in volume. When the area is large and the rubber thin the confinement is almost total, and the stiffness is very high. In the case of 1-in. soft rubber Young's modulus is three times as great for an area of 50 sq. in. as it is for 10 sq. in., the relation between area and modulus being nearly linear. For taking account of these properties fig. 8 shows the area of a pad of rubber, 1 in. thick, to be used for various loads to give two constant values (10 and 14 cycles per second) of the vertical natural frequency of the system.* If a

* E. H. Hull and W. C. Stewart, *A.I.E.E., Trans.*, 50, 347, 1931.

support were designed to have a natural frequency when loaded of, say, 14 cycles per second, it would be effective in reducing the transmission of sounds of audible frequencies above about, say, 60 cycles per second—a very low note.

As an example of the use of resilient materials to isolate against vibration of external origin, mention may be made of the isolation of rooms by insulating the floor and walls from the

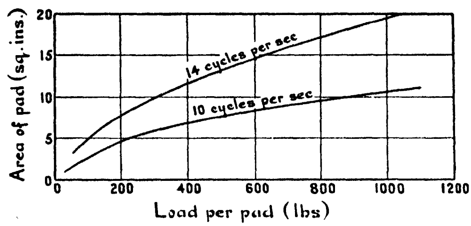


FIG. 8.—Area of soft rubber pad (1 in. thick) required to give specified natural frequencies under various loads

earth by a layer of cork or other materials. An interesting example is afforded by certain experimental rooms of the acoustical laboratories at the National Physical Laboratory (fig. 54, p. 159).* Each experimental room, of 14-in. masonry, is built upon a ferro-concrete floor ; the floor is supported only at its corners, and each corner rests upon a steel-banded layer of 2½-in. natural cork on the top of a masonry pier about 4 ft. by 3 ft. in area. The loading of the cork—about 50 lb. per sq. in.—is such that the natural frequency of the room upon its supports is of the order of 20–30 cycles per second, and extraneous vibrations of frequencies above about 30 vibrations per second are largely cut off. In view of the high loading provision is made for supporting the rooms upon hydraulic jacks when it is necessary to renew the cork insulation.

* *N.P.L. Annual Report*, p. 59, 1932.

CHAPTER III

SOURCES OF SOUND

IN experimental acoustics first importance attaches to those sources of sound which emit pure notes, consisting of a single fundamental tone, unaccompanied by harmonics or overtones. The wave-form associated with a pure tone is that known as simple harmonic, and is represented graphically by a sine curve when the disturbance of the air (pressure, velocity, or particle displacement, etc.) is plotted against time. The importance of pure tones, which are the only sounds which are unchanged in character when they are transmitted from one system to another, is due to the fact that when the behaviour of a system to pure notes is known over a wide range of pitch, its behaviour can be predicted for a more complicated sound containing specified overtones.

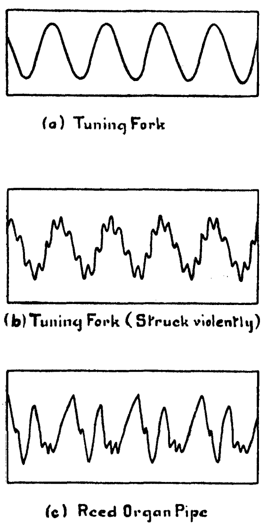


FIG. 9.—Wave-forms of sounds

As everyday examples of sources of pure notes mention may be made of the tin whistle and the ocarina, both of which are remarkably pure. Of organ pipes the flute and diapason pipes give the nearest approach to purity. Open pipes contain the full series of harmonics, but the closed pipes give only the odd members of the series. Narrow or overblown pipes favour the emission of the upper partials. Tuning-forks, when vibrating in their simplest manner, give exceedingly pure notes, the wave-form obtained being as illustrated in fig. 9 (*a*). For comparison fig. 9 (*b*) shows the wave-form associated with a violently struck fork, in which the more leisurely fundamental vibration is accompanied by

subsidiary fluctuations of higher frequency, and fig. 9 (*c*) the complicated wave-form of a certain reed-organ pipe.

The pitch of a tuning-fork depends upon its dimensions. For steel forks having prongs of length l mm. and thickness d mm. Mercadier * gave the formula for the frequency (n) as

$$n = 818270 \frac{d}{(l+3{\cdot}8)^2}$$

Auerbach † gives the following as preferable :—

$$n = 818000 \frac{d+0{\cdot}5}{(l+3{\cdot}8)^2}$$

The rate of decay of the vibration of a struck fork depends upon several factors. A slow rate of decay is ensured by the use of a rigid base between the prongs and, if the fork is to be held in the hand by the stem, by a design which reduces the amount of energy transmitted down the stem through movement of the centre of gravity of the prongs. Thus the prongs should be fairly close together, and should be exactly balanced—*i.e.* of exactly equal weight and dimensions. A rigid base also discourages the production of overtones.‡

As a source of sound of high frequency giving a series of pure notes Galton's whistle may be mentioned. Edelmann's § type of this whistle is shown in fig. 10. In reality it is a type of small

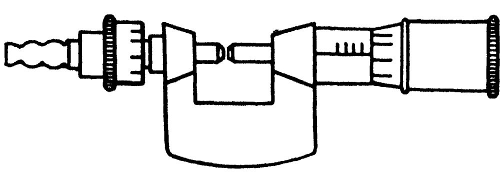


FIG. 10.—Edelmann whistle

organ pipe, consisting essentially of a very short cylindrical pipe with a sharp edge, upon which is directed a blast of air from an annular nozzle. The pitch of the note can be varied by moving a piston at the closed end of the pipe by means of the micrometer screw shown to the right of the figure. The distance of the nozzle from the pipe needs adjustment to suit sounds of different

* Mercadier, *Comptes Rendus*, 79, 1001, 1069, 1874.
† Auerbach, *Winkelmann's Handbuch d. Physik II*, 348, 1909.
‡ M. T. Edelmann, *Phys. Zeits.*, 6, 445, 1905.
§ M. T. Edelmann, *Ann. d. Phys.*, 2, 469, 1900.

pitches, and the micrometer to the left is available for accurately setting this position. In practice only a few different nozzle settings are necessary in covering the range of pitch of the instrument. The range may perhaps be taken to be from about 3500 cycles per second up to the upper limit of audibility or to, say, 20,000 cycles per second. By using the whistle with a standard air pressure it is possible to obtain reproductibility in the pitch of the note emitted if temperature conditions remain constant.

Stern's * 'tone variator,' fig. 11, is essentially a cylindrical Helmholtz resonator blown by an air blast directed over its

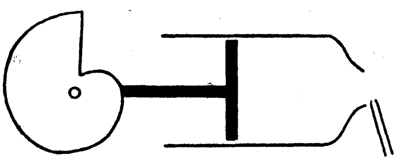


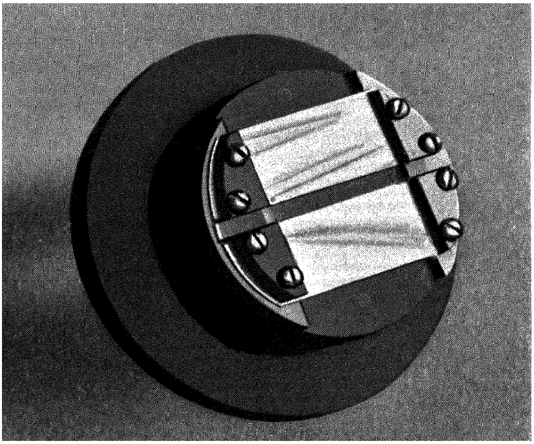
FIG. 11.—Stern 'tone variator'

orifice. The note emitted is said to be pure, and its pitch can be varied.

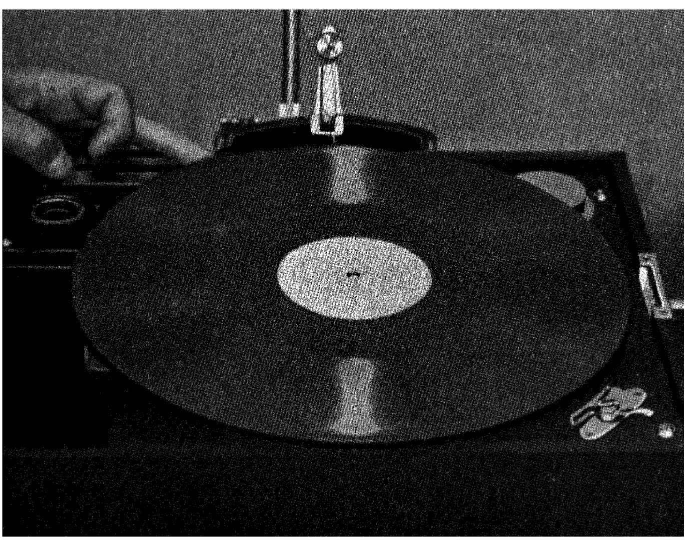
Pistonphones. For special purposes mechanically driven pistons are used as standard sources of sound, particularly for producing known audio-frequency pressure oscillations in small spaces for the purpose of calibrating microphones. In a form of pistonphone used by Wente † up to frequencies of 200 cycles per second, a small enclosure was closed at one end by the diaphragm of a microphone under calibration, and the other end was closed by a movable piston (fig. 12). The piston received a known simple harmonic oscillatory displacement through the rotation of a large cam. The pressure in the enclosure is calculable from the known piston displacements. Naturally it is possible to drive a piston electrically, by means of, say, the moving coil of a loud-speaker movement, and to measure the piston displacements by some suitable device. In an instrument (due to N. Fleming) in use at the National Physical Laboratory ‡ the amplitude of vibration of the piston is measured by means of a device in which a steel point attached to the piston bears upon an arm mounted on a torsion strip which carries a small mirror.

* W. Stern, *Phys. Zeits.*, 5, 693, 1904.
† E. C. Wente, *Phys. Rev.*, 19, 333, 1922.
‡ *N.P.L. Annual Report for 1932*, p. 93.

PLATE II



Thermophone (after Ballantine). See p. 34



Calibration of a gliding note record by an optical method (Buchmann and Meyer). See p. 51

[p. 33

It is possible to infer the amplitude of vibration from measurements of the electrical motional impedance (p. 75) of the moving system, and this gives agreement with optical measurements up to about 300 cycles per second. Above this frequency the motional impedance method fails, owing to lack of rigidity of the connections between the piston and the driving coil.

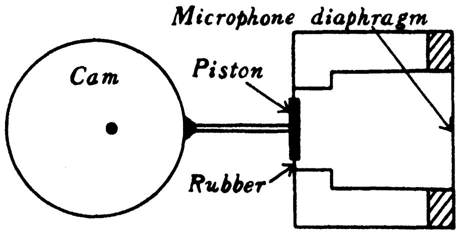


FIG. 12.—Pistonphone (Wente)

If it were not for the cooling effect of the walls of the pistonphone chamber, the pressures would be calculable from the adiabatic compression to which the contained gas is subjected.* The walls, however, are generally metallic, and when allowance is made for their cooling effect the formula for the pressure developed becomes

$$p = \frac{\gamma P_0 S\xi}{V}\left\{1 + \frac{(\gamma - 1)S_w}{aV} + \tfrac{1}{2}\left(\frac{(\gamma - 1)S_w}{aV}\right)^2\right\}^{-\frac{1}{2}} \quad \text{(Ballantine †)} \quad (1)$$

or

$$p = \frac{\gamma P_0 S\xi}{V}\left\{1 - \frac{S_w}{aV} + \frac{S_w^2}{2a^2V^2}\right\}^{\frac{1}{2}} \quad \text{(Wente ‡)} \quad (2)$$

the two formulæ differing only in the bracketed term, which represents the allowance for cooling by the walls of the chamber. The difference is a matter of approximations, and is only two or three per cent. even at frequencies as low as 20 cycles per second where the difference is greatest.

* If an enclosure, in changing from volume V to $V - dV$, compresses the contained gas adiabatically, the pressure P_0 changes to $P_0 + p$ where $p/P_0 = \gamma(\partial V/V)$. This yields $p = \rho c^2(\partial V/V)$ where $c(=\sqrt{\gamma P_0/\rho})$ is the velocity of sound in the gas.

† S. Ballantine, *Acous. Soc. Am. J.*, 3, 319, 1932.

‡ E. C. Wente, *Phys. Rev.*, 19, 333, 1922.

In the formulæ

P_0 = ambient pressure.
γ = ratio of the specific heats (= 1·4 for air).
$a = (\omega\rho C_p/2k)^{1/2} = 3{\cdot}7f^{1/2}$ for air.
S_w = area of metallic walls.
S = effective area of piston.
ξ = amplitude of piston motion.
V = volume of enclosure.
k = thermal conductivity of the enclosed gas.
C_p = specific heat of the gas at constant pressure.
ρ = density of gas.

Thermophones. The fact that acoustic effects accompany the passage of alternating current through a thin conductor was noted by Braun (1898), and considered in greater detail by Weinberg (1907). De Lange * employed the effect in a device known as the thermophone. Arnold and Crandall † developed the theory of the instrument, and subsequently Wente,‡ who was concerned with using it as a precision source of sound for the calibration of microphones, developed formulæ which were found to agree closely with such experimental tests that it was possible to apply.

A simple form of thermophone is indicated in Pl. II, p. 33. It consists simply of a piece of platinum strip (0·00007 cm. thick), clamped at the ends in flat terminals. It may be actuated in two ways. If an alternating current $I \sin \omega t$ is passed through the strip the heating effect is proportional to $RI^2 \sin^2 \omega t = \frac{1}{2}RI^2(1 - \cos 2\omega t)$, so that the frequency of the acoustic waves sent out from the strip in consequence of this alternation of local heating of the air near the strip is double the frequency of the applied alternating current. In the second method of excitation the foil is heated by alternating current (say 0·06 amp.) superposed upon a larger direct current (1 amp.) I_0. The heating of the strip is proportional to

$$R(I_0 + I \sin \omega t)^2 = R(I_0^2 + \tfrac{1}{2}I^2) + 2RI_0I \sin \omega t - \tfrac{1}{2}RI^2 \cos 2\omega t \quad (3)$$

In this case the octave term is negligible, if I_0/I is large, and the acoustic waves sent out have the frequency of the exciting alternating current. The equations lead to the following practical

* P. de Lange, *Roy. Soc., Proc.*, 91, 239, 1915.
† H. D. Arnold and I. Crandall, *Phys. Rev.*, 10, 22, 1917.
‡ E. C. Wente, *loc. cit.*

points. If only a small quantity of alternating current energy is available, greatest acoustic effect is obtained if direct current energy is added up to the limit that the strip will bear. On the other hand, if an indefinite quantity of alternating current of any frequency is at hand, a pure tone of a given frequency can be obtained best by exciting the strip wholly by alternating current of half that frequency. Naturally the presence of a large direct current is obligatory where it is desired to make the sound waves reproduce the electrical waves in both frequency and form.

The thermophone has been used in calibrating microphones, and, on inserting a small thermophone in the ear canal, in determining the threshold of audibility.

As an experimental verification of the formulæ obtained for the pressures set up in a small enclosed space by the thermophone, Wente inserted a condenser microphone in one wall of a small enclosure, and calibrated it over a range of frequencies with the aid of four thermophones which differed greatly in their physical constants, the formulæ being used to compute the pressures produced in the enclosure. The four calibrations agreed closely, and were the same as an independent calibration made with a pistonphone.

A recent discussion of the theory of the thermophone, particularly in connection with its use in calibrating microphones, has been given by Ballantine.* His formula differs somewhat from Wente's—by as much as 20 per cent. at lowest frequencies (30 cycles per second). The disadvantage of the thermophone is the complicated formula involved, which includes a dozen or more physical quantities, some of which (*e.g.* thermal capacity of the foil and thermal capacity of the gas) are not known with satisfactory accuracy. Upon certain assumptions Ballantine, who included a boundary condition to take into account the fact that the walls of the enclosure are usually of metal and thus at ambient temperature, obtained the following expression for the peak value of the alternating pressure developed in a thermophone enclosure, which is small compared with the wave-length of the sounds concerned :—

$$p_1 = \frac{2S}{\omega m C V A \alpha} \cdot \frac{0{\cdot}48 I_0 I_1 R}{D^{1/2}} \tag{4}$$

* S. Ballantine, *Acous. Soc. Am. J.*, 3, 319, 1932.

where

$$D = \left(1 - \frac{4kS^2}{\omega CVA}\right)^2 + \left(1 + \frac{4S}{VA\alpha} + \frac{4kS\alpha}{\omega C} + \frac{4kS^2}{\omega CVA}\right)^2$$

and

$$A = \frac{T_a}{T_{s0}} \cdot \frac{\gamma}{\gamma - 1} - 1$$

$$m = (\gamma - 1)T_{s0}/\gamma P_0$$

$$\alpha = (\pi f C_p \rho / k)^{1/2}$$

C = total thermal capacity of the foils (mass × specific heat).
I_0 = steady component of foil current (amps.).
I_1 = peak value of *a–c* in foil (amps.).
R = total resistance of thermophone foils (ohms).
T_{s0} = mean temperature of the foil (absolute degrees).
T_α = average temperature of the gas (absolute degrees).
k = thermal conductivity of the gas.
ρ = density of the gas.
C_v, C_p = specific heats of the gas.
$\gamma = C_p/C_v$.
P_0 = average pressure in the enclosure (ambient).
S = total area of one side of thermophone foils.
V = volume of enclosure.
f = frequency of I_1.
$\omega = 2\pi f$.

The formula derived earlier by Wente did not include the boundary condition referred to, and neglecting radiation may be written

$$p_1 = \frac{2S}{m\omega CVA\alpha} \cdot \frac{0{\cdot}48 I_0 I_1 RM}{\left[1 + \left(1 + \frac{2S}{VA\alpha} + \frac{4kS\alpha}{\omega C}\right)^2\right]^{1/2}} \tag{5}$$

where

$$M = \left(1 - \frac{S_w}{V\alpha} + \frac{S_w^2}{2V^2\alpha^2}\right)^{1/2} \tag{6}$$

is a corrective factor, derived separately, to take account of the conductivity of the walls. S_w is the total area of the walls of the enclosure.

Piezo-electric Sources. A suitably cut piezo-electric crystal can be made to execute vibrations when suitably excited by a

transverse alternating electric field. The best-known crystals of this type are quartz, tourmaline, and Rochelle salt.* The latter gives the largest effects, but is mechanically fragile and inconstant. The amplitude is small except in the region of resonance of the crystal, and this is usually so high in pitch as to be supersonic. Consequently piezo-electric crystals are used as sources of sound only where very high frequencies are required, as, for instance, in directional submarine signalling. On the other hand, the vibrations of a plate of quartz subjected to a known alternating e.m.f. of frequency well removed below the natural frequencies would serve as a definite standard of amplitude.

To obtain a suitable plate of quartz, it should be cut so that its broad faces are perpendicular to one of the three co-planar electric axes, and its breadth (length) parallel to the optic axis. Its length (breadth) will then be perpendicular to both these axes, and an alternating electric field applied perpendicular to its broad faces will result in vibrations. The length l and thickness t vary together in unison with the exciting field, in such a manner that the volume of the crystal remains constant.

For directional submarine signalling in connection with location of objects and of the ocean bed by the echo method, Langevin employed a piezo-electrically driven sound generator.† Sections of quartz crystals were firmly sandwiched between two iron slabs of thickness appropriate to the frequency of the note to be emitted. One side of the generator thus produced was in contact with the water in which the signals were to be sent out, and the other was shielded by a watertight case. Frequencies of the order of 50,000 cycles per second were employed, and the necessary electrical stimulus was obtained from an electrical oscillator. A similar assembly was used to receive the echo, the alternating pressures giving rise to corresponding e.m.f.'s in the quartz slabs. The high frequencies concerned are of course above the audible limit,‡ and circuits similar to those used in radio reception are used in the receiver.

* C. B. Sawyer, *Inst. Radio Eng., Proc.*, 19, 2000, 1931.

† See French Patent 505903 (1918); British Patent 145691 (1920); and *Nature*, May 9, 1925.

Other forms of oscillator for echo depth-sounding are also employed. See F. E. Smith, *Roy. Inst., Proc.*, 24, 342, 1924; A. B. Wood's *Sound*, 1930.

‡ A survey of work on supersonic vibrations has been given by J. C. Hubbard, *Acous. Soc. Am. J.*, 4, 99, 1932.

Electrical Telephone Receivers. A very convenient and adaptable source of sound is an electrical telephone receiver or loud-speaker. For the production of pure notes a receiver should be actuated by audio-frequency alternating current of pure sine wave-form. Electrical oscillators are available in which the frequency of the current can be readily altered over the whole audio-frequency range by turning the knob of an air condenser, so that it is easy to obtain a wide range of pitch and of intensity from a loud-speaker excited by such means.

The ordinary telephone receiver is a robust sensitive instrument in which a magnetic diaphragm of steel or ferrotype is clamped rigidly at the edge in such a position that the central part of the diaphragm is supported just clear of the pole pieces of an electromagnet, fitted with a permanently magnetised core

Without the permanent magnet the telephone receiver would be much less sensitive and would emit the octave of the frequency of the exciting current. This may be seen from the following. If S represents the effective area of each pole of a receiver magnet, and if X is the length of the air gap at each pole, then the total reluctance in the two air gaps is $2X/S$. Since the rest of the magnetic circuit is metallic, this is the principal reluctance. Assuming the permanent magnet to have an active structural magnetomotive force F_0, there is in addition a cyclic magnetomotive force $F = 4\pi In$ due to the current I through the n turns of the coils. This yields a total m.m.f. of $F_0 + F$ in the circuit. The magnetic tractive force on unit area is $B^2/8\pi$, directed along the flux paths, where B is the flux density and is equal to (m.m.f.)/(reluctance × area). Thus the total pull P by the two poles of the magnet is given by

$$P = \frac{2SB^2}{8\pi} = \frac{S}{16\pi X^2}(F_0 + F)^2 = \frac{S}{16\pi X^2}(F_0{}^2 + 2F_0F)$$

approximately, if F/F_0 is small. This may be rewritten

$$P = P_0 + \frac{S}{8\pi X^2}F_0F = P_0 + \frac{nSF_0I}{2X^2}$$

In the absence of the permanent magnetic flux the vibromotive force would be $SF^2/4\pi X^2$ (*i.e.* $4\pi n^2I^2/X^2$), so that the alternating part is increased by the permanent field in the ratio $2F_0/F$, a quantity which is normally very large, say of the order of 500. Without the permanent magnet the force on the diaphragm

would depend upon the square of the exciting current I, and would thus be positive whether the current I was positive or negative. Thus as the current performed one cycle of variation from positive to negative and back to positive, the force on the diaphragm would perform two cycles from positive to zero and back to positive. The frequency of the diaphragm would thus be double that of the exciting current.

If the permanent magnet is unduly weak, or if the receiver is actuated by excessively strong alternating currents, the term F^2 cannot be neglected, the force on the diaphragm is no longer harmonic when the exciting current is sinusoidal, and even harmonics appear.

In experimental work the usual ebonite cap which clamps the diaphragm should be replaced by a metal one, to improve the constancy of the instrument, but even this does not achieve complete constancy, particularly when changes of temperature are involved.

In the Brown * reed type of receiver a strip of soft iron clamped at one end lies with its free end across the gap of the magnet. Attached to the reed is a light aluminium cone which acts as the sound-emitting surface when the reed moves under the action of an exciting electrical current in the coils of the telephone receiver.

Ordinary telephone receivers exhibit pronounced diaphragm resonance at a frequency of about 800 cycles per second, together with others at higher frequencies. The efficiency is very small at frequencies below, say, 250 cycles per second and above 3500 cycles per second. The sharpness of the resonance peak can be reduced considerably by filling the air space behind the diaphragm with wax and reducing the clamping projection in the ear-cap so that only 10 mils separate the inner surface of the cap from the diaphragm ; this procedure also raises the frequency of the resonance slightly. Wente and Thuras † have described the design and construction of a telephone receiver, on the moving-coil principle, which is largely free from resonance defects. Fig. 13 shows a section of the receiver. It resembles in many respects a loud-speaker unit which had been described previously by the same authors, and which is referred to later. The response characteristic of the receiver was determined experimentally (as well as by calculation) by using a calibrated

* S. G. Brown, Patent Specification 29833/1910.

† E. C. Wente and A. L. Thuras, *Acous. Soc. Am. J.*, 3, 45, 1931.

condenser microphone to measure the sound pressures set up by the receiver in a small enclosure. It was found to be very uniform up to a frequency of, say, 2000 cycles per second, and moderately uniform up to 9000 cycles per second, but having a depression in the region of 4000 cycles per second.

The wave-front of the sound emitted by a telephone receiver is spherical except within a centimetre or two of the diaphragm itself. In effect the receiver acts as a simple point source. Thus the acoustical pressure set up at a point in the air is

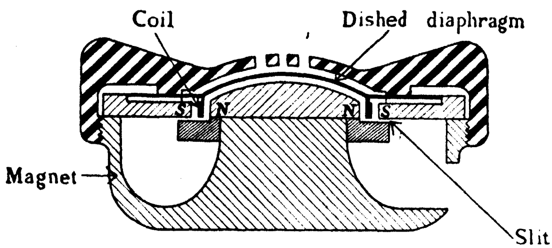


FIG. 13.—Moving-coil telephone receiver

inversely proportional to the distance of the point from the diaphragm. It should be remembered, however, that the particle velocity of the air does not obey an inverse law near the source (p. 58). W. West has made considerable use of ordinary telephone receivers in acoustical measurements, increasing the output of the receiver by resonance.* At frequencies below 1500 cycles per second he uses the receiver in an open-ended tube, and adjusts the frequency—or length of tube—to resonance. The sound emitted from the end of the tube (7 cm. diameter) agrees closely with that calculable for a point source at the mouth of the tube, at least for distances not less than 6 cm. from the mouth. At frequencies above 1500 cycles per second he forms an acoustical resonator by means of an adjustable cap over the front of the telephone, a central hole in the cap being about 1 cm. in diameter. In this case the hole acts practically as a point source.

Electrical Loud-speakers. Electromagnetic Loud-speakers. Many loud-speakers, particularly early types, were of the form in which an iron diaphragm moves under the action of the varying strength of an electromagnet. The type is not very constant for laboratory purposes, for its sensitivity usually varies appreciably

* W. West, *I.E.E., J.*, 67, 1137, 1929.

with temperature owing, for instance, to alteration in the gap between the magnet and the diaphragm.

A moving-coil type is preferable for measurement purposes. In this, as in the moving-coil receiver, a non-magnetic diaphragm carries a coil of wire moving in a constant magnetic field (fig. 14). The type can easily be so designed that the coil moves always in a region of uniform magnetic field, and then the loud-speaker has, over a wide amplitude range, an amplitude response proportional to the magnitude of the exciting current.* The moving-coil

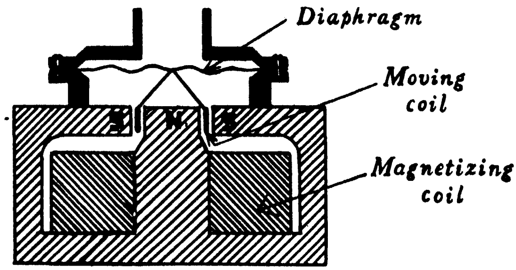


FIG. 14.—Movement of moving-coil loud-speaker

type has also a resistance which varies but little with frequency, and a practically negligible reactance.

Moving-coil loud-speaker movements are available for use with horns, and also as the central driving unit of large light diaphragms. The former type is advantageous for the reproduction of high frequencies, but the air chamber between the diaphragm of the loud-speaker and the throat of the horn sets a limit to the highest frequency attainable. For efficiency at high frequencies the chamber must be thin, but for working at low pitches the separation must clearly be sufficient to permit the somewhat large vibrations of the diaphragm associated with them.

A loud-speaker movement of high efficiency working on the moving-coil principle has been described by Wente and Thuras (fig. 15).† It was intended primarily for public address in large halls and for use in conjunction with a large horn. By the use of a special type of dished aluminium diaphragm and an annularly flared unit for acoustically coupling the diaphragm to the horn, it is ensured that pressure variations set up by the inner and

* A. H. Davis and N. Fleming, *Phil. Mag.*, 2, 51, 1926.
† E. C. Wente and A. L. Thuras, *Bell Sys. Tech. J.*, 7, 140, 1928.

outer portions of the diaphragm reach the throat of the horn approximately in phase up to high frequencies. An experimental test showed that for a particular size of diaphragm and area of throat, this type of coupling raised the cut-off frequency from about 3500 to 6000 cycles per second. With a unit attached to an exponential horn of 115 cycle cut-off, the response was sensibly uniform over the range 115 to 7000 cycles per second. The coil employed in the movement consisted of a single helix of

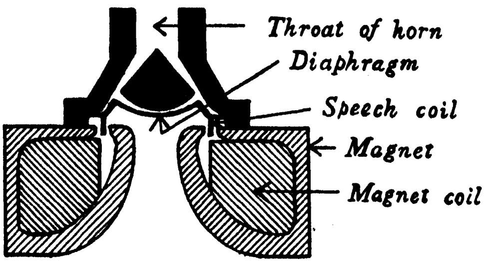


FIG. 15.—Moving-coil loud-speaker (Wente and Thuras) with dished diaphragm and annularly flared coupling to horn

aluminium ribbon wound on edge, adjacent turns being separated by insulating lacquer. This type of coil is self-supporting, no spool being required ; practically 90 per cent. of the volume is metal, heat is readily dissipated, distributed capacity between the turns is small, so that the impedance varies but little with frequency, and the coil can be made accurately to dimensions so that small clearances suffice between the coil and the pole pieces.

L. G. Bostwick * designed a horn type moving-coil receiver which, having a specially thin coupling-chamber (1/40 cm.), is efficient up to frequencies as high as 12,000 cycles per second. The chamber is so thin that large vibrations of the diaphragm such as are usual at low pitches are not possible. The instrument is therefore used in a circuit which filters low frequencies from the input, and it is only of value for supplementing an ordinary loud-speaker. The diaphragm is of thin duralmin, dished to provide rigidity. It carries a moving coil composed of an edgewise wound aluminium ribbon, the total weight of the movement being only about 1/6 of a gram. A suitable horn for the movement has a throat area of about 1/5 sq. cm. An annular opening is employed having a diameter slightly greater than half the

* L. G. Bostwick, *Acous. Soc. Am. J.*, **2**, 242, 1930.

diaphragm diameter in order to reduce interference effects in the chamber.

The German 'Band-Sprecher' employs, in conjunction with a horn, a movement composed of a strip of thin aluminium foil about 12 cm. by 1 cm. lightly clamped at the ends and placed in the field of a strong electromagnet.* Currents in the strip cause it to vibrate and generate sound waves.

In hornless loud-speakers, where 'moving coil,' 'moving iron,' or 'moving conductor' movements are used as the driving movements of large light diaphragms, the diaphragms may be stiff paper in the form of a shallow cone,† but R. W. Paul and B. S. Cohen ‡ have contrived one in which the diaphragm is a light disc of Balsa wood; it is said to be relatively too powerful in the upper register. If a diaphragm of 10 inches diameter were perfectly rigid the pressure waves sent out from different parts of the diaphragm would interfere at higher frequencies and the output of sound above 1000 cycles per second would therefore fall off. In practice, however, diaphragms 'break up' into complicated centre-moving symmetrical modes of vibration, and exhibit numerous resonances in the upper frequency range. N. W. MacLachlan § has shown that the action of the moving-coil loud-speaker in the upper register depends upon these resonances, and depends in the lower regions of pitch upon resonances determined by the controls of the diaphragm. Owing to interference in the surrounding air, and to other causes, considerable attenuation occurs above, say, 5000 cycles per second. For paper diaphragms the conical form is preferable to that of a flat plate, because the curved surface is more rigid to flexural vibrations, and breaks up into resonant modes of vibration at higher frequencies. Aluminium has been used as the material of the conical diaphragm in order to provide greater mechanical stiffness, and thus to raise the pitch of the resonances still further and to give greater output in the region of 10,000 cycles per second. Conical diaphragms may be supported at the base by thin leather or rubber in a manner which imposes but little

* Gerlach, *Phys. Zeits.*, **25**, 675, 1924; *Zeits. f. tech. Phys.*, **5**, 576, 1924; Shottky, *Zeits. f. tech. Phys.*, **5**, 574, 1924; *Phys. Zeits.*, **25**, 672, 1924; *E.N.T.*, **2**, 157, 1925.

† C. W. Rice and E. W. Kellogg, *A.I.E.E.*, *Trans.*, **44**, 461, 1925.

‡ R. W. Paul and B. S. Cohen, *J. Sci. Insts.*, **7**, 50, 1930.

§ N. W. MacLachlan, *Nature*, **128**, 517, 1931. See also *Phil. Mag.*, **7**, 1011, 1929; **11**, 1, 1931.

restraint. For precision in acoustical work in which constancy from day to day is advantageous, diaphragms and controls of paper or leather would appear to have disadvantages. Often, however, the variation is of small magnitude, possibly because the chief controlling feature in such loud-speakers is the mass of the moving parts. Among recent papers on loud-speaker diaphragms may be mentioned one by MacLachlan and Sowter * on the acoustical and mechanical behaviour of conical diaphragms, another by Strutt † on the radiation from circular diaphragms oscillating with nodal lines, and one by MacLachlan ‡ on the sound pressures set up by such diaphragms.

The advantage of the large diaphragm type of loud-speaker lies in the fact that it is relatively easy to obtain substantial output at frequencies as low as 40 cycles per second with an instrument of convenient shape and dimensions, whereas horns capable of dealing with such pitches would be of prohibitive length for many purposes. To obtain a large output of sound at very low frequencies a diaphragm of relatively large area is necessary, or a bank of smaller diaphragms moving in phase. The former requirement is evident when theoretical equations for the output per unit area of a piston source are considered (p. 61), and the latter is a related consequence which has been dealt with by Wolff and Malter. § In each case the diaphragms are supposed to be mounted in an infinite baffle. For practical purposes the baffle should be large enough (outside the diaphragm area) to prevent circulation of air from front to rear of the diaphragm, with consequent reduction of radiation.

The difficulty with diaphragm type loud-speakers is to produce uniform amplitude at different frequencies. The horn type is more efficient within the range of the horn, since a greater amount of acoustic energy is utilised. The greater acoustic loading leads to a more uniform relation between frequency and response.||

Eddy-current Loud-speaker. C. W. Hewlett (*Phys. Rev.*, 19, 52, 1922) devised a flat diaphragm loud-speaker in which the diaphragm moves as a whole, the forces on the various parts of the diaphragm being nearly in phase. It consists of a thin,

* N. W. MacLachlan and G. A. V. Sowter, *Phil. Mag.*, 12, 771, 1931.

† M. J. O. Strutt, *Ann. d. Phys.*, 11, 129, 1931.

‡ N. W. MacLachlan, *Phys. Soc., Proc.*, 44, 540, 1932.

§ I. Wolff and L. Malter, *Phys. Rev.*, 33, 1061, 1929.

|| H. Stenzel, *Zeits. f. tech. Phys.*, 12, 621, 1931.

non-magnetic metallic diaphragm between two flat coils through which a constant direct current I_0 flows in such a way that the magnetic fields tend to oppose each other on the axis. The result is to produce a radial magnetic field in the diaphragm; then when a simple harmonic alternating current I of the frequency $\omega/2\pi$ is superposed upon the direct current, circular currents are induced in the diaphragm, which thereupon is acted upon by a simple harmonic electrodynamic force and vibrates with the frequency of the alternating current. The forces on the various parts of the diaphragm are nearly in phase; particularly so for those places where the force is large. Consequently, the diaphragm may be considered to vibrate as a whole. For low frequencies the electrodynamic force is approximately proportional to $\omega I_0 I \sin(\omega t+\theta)$ and the amplitude of vibration is approximately proportional to $I_0 I/\omega$. The absence of overtones is due to the absence of ferromagnetic material, and to the fact that the radial magnetic field is constant. The aperiodicity of the diaphragm renders the calculation of the performance of the instrument practicable, and eliminates distortion, due to resonance, in the wave-form of the emitted sound when the instrument is excited by a complex alternating current.

The actual instrument described by Hewlett was used successfully at frequencies from 500 to 25,000 cycles per second. For lower frequencies the instrument should be wound with a larger number of turns, while for higher frequencies a smaller number of turns of coarser wire should be used. In the quantitative study of the performance the distribution of the magnetic field between the coils was determined experimentally, the forces on various parts of the diaphragm were calculated, and thence the amplitude of vibration and the sound energy output were determined. With an aluminium diaphragm 0·0025 cm. thick and 10 cm. in diameter, a direct current of 1 ampere, an alternating current of 0·085 ampere, and a frequency of $10^5/2\pi$, the amplitude and the output were respectively 7×10^{-7} cm. and 9 ergs per second. By increasing both direct and alternating currents fivefold, the output could be increased over six hundredfold. Measurements of the amplitude for various frequencies agreed well with the calculated values.

Since the instrument gives a pure tone of constant and measurable pitch and intensity over a wide range, it should serve as a useful source of sound for research and lecture purposes.

Condenser Loud-speaker. In the condenser loud-speaker, which may become practicable, audio-frequency voltages vary the attraction between a diaphragm and a plate, and cause the diaphragm to move in response, and incidentally to move as a whole. The elementary theory has been given by Hanna.* We shall see later that the condenser principle produces a satisfactory microphone, when employed with adequate electrical amplification. Unfortunately, however, with condenser loud-speakers, the considerable diaphragm movements which are necessary for radiation of sound at low frequencies give rise to non-linear distortion if the design does not allow for large amplitudes and considerable alternating voltages. In the Kyle † form of condenser loud-speaker the fixed electrode is of corrugated metal, and the moving electrode consists of a metal film carried on a flexible dielectric of rubber compound. This lies upon the fixed corrugated electrode, the rubber being between the two metal electrodes.

Horns. At the narrow end of a horn sound energy enters in the form of plane waves of considerable amplitude. It would not be satisfactory, however, for the sound to have access to the atmosphere through a tube of small diameter. For the energy transmitted to the atmosphere by unit area of a vibrating rigid piston in an infinite baffle (p. 61) increases considerably as the size of the piston is increased, and indeed reaches a more or less steady value when the piston circumference exceeds twice the wave-length of the sound concerned. Consequently if frequencies down to, say, 110 cycles per second ($\lambda = 10$ ft.) are to be dealt with, the piston diameter should be of the order of $6\frac{1}{2}$ ft. It is essential, therefore, for the sound to be actually communicated to the atmosphere over a fairly large area. The function of a horn is thus to receive energy from a movement of small size, and to deliver it to the atmosphere over a much larger area. The matter is, however, somewhat complicated by the possibility of resonances in the air in the horn, due to reflection of sound from end to end along its length.

Naturally the shape of the horn affects the resonances. Conical horns have been used, and in these the increase of radius from throat to mouth takes place by equal increments for constant

* C. R. Hanna, *Acous. Soc. Am. J.*, 2, 143, 1930.

† V. F. Greaves, F. W. Kranz, and W. D. Crosier, *Inst. Radio Eng., Proc.*, 17, 1142, 1929.

intervals along the axis. Calculations indicate that a conical horn is much inferior to an exponential horn in which sectional areas of the horn increase by a constant percentage for constant intervals along the axis. In exponential horns the area of cross-section (S) at a point distant x from the throat is given by the equation $S = S_1 e^{mx}$, where S_1 is the area at the throat and m is a constant.

If a simple source of strength A * is situated at the small end of a truncated horn where the area of cross-section is S_1, and if the mouth is so large (area S_2) that its impedance matches that of the infinite medium and the reflection from the opening may consequently be neglected and associated resonances ignored, it is calculable (on certain assumptions) that the average rate of energy transmission per unit area from the mouth is given by †

$$| w |_{av} = \frac{\rho c A^2}{2 S_2 S_1} \frac{k^2 x_1^2}{(1 + k^2 x_1^2)}$$

for a conical horn of solid angle $\Omega(= S/x^2)$, and

$$| w |_{av} = \frac{\rho c A^2}{2 S_2 S_1} \sqrt{1 - \frac{m^2 c^2}{4\omega^2}}$$

for an exponential horn of equation $S = S_1 e^{mx}$. In the equations ρ is the density of the air, c the velocity of sound in air, $\omega/2\pi$ the frequency of the sound concerned, x_1 the distance of the small end of the cone from the true apex, and k is equal to ω/c.

It is to be noted that for frequencies below $\omega = \frac{1}{2}mc$ the exponential horn transmits nothing. However, when the factors for the two types of horn are plotted it is found that above this critical frequency the exponential horn rapidly reaches its full radiating effect, whereas a conical horn does so very gradually. The superiority of the exponential horn lies in this fact.

From the formulæ given above for the output of a *conical horn* having an opening large enough for resonances to be neglected (*i.e.* of circumference $> 2\lambda$) the following comments may be made. (1) The intensity at the mouth is inversely proportional to the solid angle $(\Omega = S_1/x_1^2)$ of the cone, *i.e.* to the inverse square of the angle of the cone. (2) At low frequencies $(kx_1 < 1)$ the intensity rises with the square of the frequency,

* *I.e.* a small source which alternately injects fluid into the medium and withdraws it, the volume displacement being A cos ωt at any time t.

† I. Crandall, *Vibrating Systems and Sound*, p. 161.

and at high frequencies reaches a constant value inversely proportional to the sectional area of the cone at the source. For distortionless transmission x_1 should clearly be as large as possible.

In the case of an *exponential horn* in which the cross-section S at a distance x from the throat is given by $S = S_1 e^{mx}$, we infer that (1) the rate of expansion m must not exceed $4\pi f/c$ (*i.e.* $2\omega/c$ or $4\pi/\lambda$), where λ is the wave-length of sound of the lowest frequency (f) it is desired to radiate. (2) The mouth should be large enough to ensure passage of sound without appreciable reflection, *i.e.* the circumference should exceed 2λ.

These requirements lead to a long horn with a wide opening. Sometimes, to reduce the rather great length of horns required to deal with low frequencies, folding is resorted to. Indeed, ingenious forms have been used in which, to promote compactness, the sound conduit is bifurcated once or twice, and subsequently rejoined at the mouth. In the design of folded horns, however, the tendency for resonant transverse vibrations of the air to be set up at bends must not be overlooked. This is indicated by some experiments made with a curved conduit in a ripple tank,* when it was observed that pronounced stationary waves occurred at a certain bend, the direction of vibration being transverse to the conduit, and that transmission through the conduit was much reduced at the frequencies at which such transverse resonance occurred.

Kellogg † has considered devices for obtaining, in as reasonable a space as possible, horns capable of dealing with frequencies as low as, say, 30 cycles per second. For this frequency the area of the horn should double in about every 2 ft., and the bell should be about 18 ft. diameter. Kellogg suggests that the walls and floor or walls and ceiling at one corner of the room could be made to serve as the final bell. The rate of expansion of the pyramidal bell thus formed should not exceed that of the part of the horn it replaces and, in order that it should match the expansion of a horn which doubles its cross-section every 2 ft. ($\lambda/18{\cdot}1$), the transition from horn to room surfaces should be made at about 6 ft. ($\lambda/2\pi$) from the corner. In effect therefore the corner of the room should be cut off at about 6 ft. from the vertex, and the corner replaced by the exponential horn. The area at this

* A. H. Davis, *Phys. Soc., Proc.*, 40, 90, 1928.

† E. W. Kellogg, *Acous. Soc. Am. J.*, 3, 94, 1931.

junction would be about 58 sq. ft., so that the flare of the exponential part of the horn would need to be of this area—a circumstance which fixes the length of horn for a given throat area.

Within the working range of a horn the acoustic impedance at the throat is substantially the same as that of an infinite straight tube of the same diameter. It should be mentioned that the reaction of a horn upon a driving diaphragm may be given any desired value by choosing the proper ratio for the areas of the throat of the horn and of the diaphragm (p. 203).

The theory of horns has been stated by Rayleigh (*Sound*, **2**, 66), and was the subject of special investigation by Webster (*Proc. Nat. Acad. Sci.*, 5, 275, 1919). Other articles on horns include those by Kellogg (*Gen. Elec. Rev.*, 37, 556, 1924), Hanna and Slepian (*A.I.E.E., Trans.*, 43, 393, 1924), and Goldsmith and Minton (*Inst. Radio Eng., Proc.*, 42 B, 1924). Horns may of course be used to concentrate incoming sound upon microphones (Olson and Wolff, *Acous. Soc. Am. J.*, 1, 410, 1930).

Gramophones and Gramophone Records. Gramophone records are now constructed which are of value when it is desired to produce pure tones by simple means. In conjunction with a gramophone or with an electrical gramophone pick-up they may be used as a substitute for an expensive electrical oscillation generator for giving electrical alternating currents of good wave-form and known frequency. The records are also of considerable value in the rapid and accurate comparison of the relative merits of various types of complete gramophones, of sound boxes or of horns, as well as for the purely electrical test of 'pick-ups,' amplifiers, loud-speakers and associated equipment.

It is possible to obtain a set of 100 different frequencies, each of 50 seconds duration, the set covering the range 25–8460 cycles per second. Some care is necessary in adjusting the speed of the turn-table, but with the speed accurately set it is claimed that the frequencies on the record are correct to within 1 per cent. Records are made to produce a constant output of a.c. energy from 250 cycles upwards, but below 250 cycles there is a gradual decrease of energy owing to the fact that the considerable amplitudes otherwise involved would result in waves in one groove cutting into the waves of the adjacent groove. The amplitudes are such as to give a more or less constant value of about 2½ in. per second for the maximum needle velocity at

frequencies above 250 cycles per second, but only about a quarter of this value at 25 cycles per second. The double amplitude associated with these velocities ranges from 0·008 in. at 25 cycles per second to about 0·0001 in. at 8000 cycles.

In addition to constant frequency records other types are made for acoustic purposes. One record useful for testing frequency characteristics of 'pick-ups,' complete gramophones, etc., gives a tone which, starting with a pure note of high frequency—say 6000 cycles per second—glides gradually downwards in pitch during the course of the record until, at the end, a low tone of, say, 100 cycles per second is reached. In this record the velocity of the needle point is approximately constant at all the frequencies during running, variations lying within limits of $\pm$ 30 per cent. ($\pm$ 2 decibels), a variation which is often negligible for acoustic purposes.

Another useful type of record produces a tone of which the frequency varies, say, 10 times per second by a definite percentage. It thus produces a frequency band which is traversed about 10 times per second whilst the mean frequency remains constant. For some records the width of the band is comparatively small, as, for instance, in the case of the tone of approximately 5000 $\pm$ 50 cycles per second, or even of 150 $\pm$ 50 cycles per second. Thus a series of band notes of different average pitches are available. Such records are useful for acoustical work in enclosed rooms, since a note of varying pitch minimises the tendency for stationary waves to develop in an enclosed space, and tends to give rise to an average distribution of sound everywhere. Again, records having a large frequency band (say 950 $\pm$ 650 cycles per second, or 1800 $\pm$ 1600 cycles per second) are available. They are useful in tests—such as those of the output of batches of telephone receivers—where it is sufficient to measure a mean value of behaviour over a large frequency range. Records are also constructed in which an oscillatory frequency change is superimposed upon a note of steadily gliding pitch.

Calibration of Gramophone Records. Gramophone records of pure notes may be calibrated for *frequency*, on the assumption of a given rotational speed of the turn-table, by observations of the length of the record wave by means of a travelling microscope. Alternately measurement may be made of the frequency of the alternating e.m.f. produced in an electrical pick-up used in conjunction with the record. Particularly where the note is of

gliding pitch it is convenient to use a cathode ray oscillograph for observing the frequencies, obtaining upon the screen a Lissijous' figure (p. 166).

To some extent it is possible to measure the *amplitude* of the excursions of the record groove in a direction perpendicular to the normal line of groove by means of a travelling microscope. Greater magnification is required in measuring the amplitude of the high-frequency records than in measuring low, because the amplitude in high-frequency records is so small.

E. Meyer and P. Just * employed an electrical arrangement to determine the relative amplitudes in a pure-note gramophone record which gave a note gliding slowly in pitch from 100–6000 cycles per second. The velocity of rotation of the turntable was so regulated that the e.m.f. from the electrical pick-up was of the same frequency at all points of the record : the e.m.f. given by the pick-up was then a measure of the amplitude at the corresponding point of the record. G. Buchmann and E. Meyer † have described a simple optical method of calibration. Parallel light is allowed to fall obliquely on the plate, and on reflection from the grooves, is observed across the plate as a luminous band, the breadth of which is directly proportional to the velocity amplitude. By observation of a gliding-note record in this way it is readily possible to deduce from the uniform breadth or otherwise of the band, whether the record is uniformly recorded at all frequencies ; and by means of photographs measurements can be made and absolute values of the velocity amplitude deduced (Pl. II, p. 33).

* E. Meyer and P. Just, *E.N.T.*, 6, 264, 1929.
† G. Buchmann and E. Meyer, *E.N.T.*, 7, 147, 1930.

CHAPTER IV

THEORETICAL RELATIONS FOR SOURCES OF SOUND

General Wave Relations. The general relation for the propagation of sound waves of small amplitude in three-dimensional space is

$$\frac{\partial^2\phi}{\partial t^2}-c^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial^2 z}\right)=0 \qquad \text{or} \qquad \ddot{\phi}-c^2\nabla^2\phi=0 \tag{1}$$

where ϕ is the velocity potential at a time t at a point of co-ordinates x, y, z, and where c is a constant, equal to κ/ρ, where κ is the volume elasticity of the medium, and ρ its density. We shall see that c is equal to the velocity of sound waves in the medium concerned. In sound waves fluctuations are rapid and κ has therefore its adiabatic value γP, where P is the mean pressure of the gas and γ the ratio of its specific heats.

The velocity potential is an important hydrodynamical quantity from which the component velocities $\dot{\xi}$ $\dot{\eta}$ $\dot{\zeta}$ of a particle at the point x, y, z may be deduced as follows :—

$$\dot{\xi}=-\frac{\partial\phi}{\partial x} \qquad \dot{\eta}=-\frac{\partial\phi}{\partial y} \qquad \dot{\zeta}=-\frac{\partial\phi}{\partial z} \tag{2}$$

Moreover, the excess pressure 'p' $(=\kappa s)$ at the point due to the acoustical disturbance is given by the equation

$$\dot{\phi}=c^2s\equiv\frac{p}{\rho} \qquad \text{or} \qquad p=\rho\frac{\partial\phi}{\partial t} \tag{3}$$

where ρ is the density of the medium at the point x, y, z, and 's' is the condensation defined by the relation $\rho=\rho_0(1+s)$.

If P is the average pressure in a gaseous medium, we have for adiabatic expansion

$$\frac{\mathrm{P}+p}{\mathrm{P}}=\left(\frac{\rho}{\rho_0}\right)^{\gamma}=(1+s)^{\gamma}=1+s\gamma \quad \text{approximately} \tag{4}$$

Consequently $p/\mathrm{P}=s\gamma$ and $c^2=\sqrt{\gamma\mathrm{P}/\rho}$.

The general procedure in solving a three-dimensional acoustical problem—including that of determining the radiation from a source—is to find a solution of (1) which satisfies the boundary conditions. With ϕ thus determined, the excess pressure ('p') the 'particle velocity' ($\dot{\xi}$, etc.)—and indeed other quantities as well—may be deduced from equations such as (2) and (3).

In the case of plane waves with the fronts perpendicular to the axis of x the equation (1) reduces to

$$\frac{\partial^2\phi}{\partial t^2} - c^2\frac{\partial^2\phi}{\partial x^2} = 0 \tag{5}$$

A similar expression can be obtained in terms of the particle displacement ξ,

$$\frac{\partial^2\xi}{\partial t^2} - c^2\frac{\partial^2\xi}{\partial x^2} = 0 \tag{6}$$

Naturally in solving problems relating to spherical propagation of sound it is often desirable to employ polar co-ordinates ($r\theta$) rather than cartesian co-ordinates. In the particular case of spherical symmetry about the origin—such as arises with a small source of sound at the origin—ϕ is a function of r and t independent of θ, and we have

$$\ddot{\phi} - c^2\left(\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r}\right) = 0 \tag{7}$$

which may be written

$$\frac{\partial^2}{\partial t^2}(r\phi) - c^2\frac{\partial^2}{\partial r^2}(r\phi) = 0 \tag{8}$$

and the velocity, $-\dfrac{\partial\phi}{\partial r}$ in the direction of the radius, is uniform over any spherical surface having the origin as centre.*

General Solutions of Simple Wave Equations. The general

* The general expression for $\nabla^2\phi$ in polar co-ordinates (r, θ, ψ) is

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial^2\psi}$$

$$\text{where } x = r\cos\psi\sin\theta,\quad y = r\sin\theta\sin\psi,\quad z = r\cos\theta.$$

solutions of equations (1) and (8) are respectively

$$\left.\begin{aligned}\phi &= f(ct-x)+\mathrm{F}(ct+x)\\ r\phi &= f(ct-r)+\mathrm{F}(ct+r)\end{aligned}\right\} \qquad (9)$$

where the functions f and F are arbitrary, and are determined by the initial conditions. The term $f(ct-x)$ represents a wave travelling outwards with velocity c in the direction of x positive, for the value of ϕ is unchanged when x and ct are increased by equal amounts. This identifies c with the velocity of sound waves in the medium. The term $\mathrm{F}(ct+x)$ represents a wave travelling in the opposite direction.

In the case of waves of simple harmonic form the wave proceeding in the outward direction may be written

$$\phi = f(ct-x) = \mathrm{A}\cos\left\{\frac{2\pi}{\lambda}(ct-x)+\epsilon\right\} = \mathrm{A}\cos(\omega t - kx + \epsilon) \qquad (10)$$

or in complex form

$$\phi = \mathrm{A}e^{\iota(\omega t - kx + \epsilon)} = (\mathrm{C}+\mathrm{D}i)e^{\iota(\omega t - kx)} \qquad (11)$$

where $k = 2\pi/\lambda$, and λ = the wave-length, or distance between two successive maxima ; $\omega = kc = 2\pi n$, where n is the frequency or number of oscillations per second at any point.

As an illustrative solution we may take the case of waves proceeding outwards from a simple harmonic point source. At such a source fluid is supposed to be introduced periodically at a rate given by $4\pi\mathrm{A}\cos\omega t$, *i.e.* in complex notation, by the real part of $4\pi\mathrm{A}e^{i\omega t}$.

The solution which satisfies the boundary condition * may be expressed in a complex form and also in a real form thus :

Complex Form *Real Form*

$$\phi_r = \frac{\mathrm{A}}{r}e^{i(\omega t - kr - \theta)} = \frac{\mathrm{A}}{r}\cos(\omega t - kr - \theta) \qquad (12)$$

where

$$\tan\theta = kr$$

whence, remembering that $\xi = -\dfrac{\partial\phi}{\partial r}$ and $p = \rho\dfrac{\partial\phi}{\partial t}$, we have

* Rate of introduction of fluid = rate of flux through surface of sphere of small radius $= 4\pi r^2\xi_r = 4\pi\mathrm{A}\cos\omega t$ when r is vanishingly small.

$$\dot{\xi}_r = \left(\frac{1}{r} + ik\right)\frac{A}{r}e^{i(\omega t - kr)} \qquad = \frac{A}{r}\sqrt{k^2 + \frac{1}{r^2}}\cos(\omega t - kr) \tag{13}$$

$$p_r = \frac{ick\rho A}{r}e^{i(\omega t - kr - \theta)} \qquad = \frac{Ac\phi k}{r}\sin(\omega t - kr - \theta) \tag{14}$$

Specific Acoustical Impedance—Average Rate of Energy Flux. An important quantity is the ratio $p/\dot{\xi}$ that the pressure p at a point bears to the resultant particle velocity $\dot{\xi}$ of the medium. It is known as the 'specific acoustical impedance' of the medium at the point, and is of special importance when it is evaluated at the driving surface of a source of sound.

Generally it is a complex quantity which may be written

$$z = r + ix = \sqrt{r^2 + x^2} \angle\, \theta \quad \text{where} \quad \tan\theta = \frac{x}{r}$$

r is known as the specific acoustical resistance, and x as the specific acoustical reactance. A positive reactance is in phase with the acceleration and is an inertance. A negative reactance is a stiffness.

In dealing with energy flux we must leave complex quantities. The average rate of energy flux per unit area at a point is given by

$$|\,w\,|_{\text{av}} = |\,p \times \dot{\xi}\,|_{\text{av}} = p_{\text{max}} \times \dot{\xi}_{\text{max}} \times \cos\theta = \tfrac{1}{2}r\dot{\xi}^2_{\text{max}} = r\dot{\xi}^2_{\text{av}} \tag{15}$$

where θ is the angle of lag between p and $\dot{\xi}$.*

Reaction of Medium upon Source of Sound. The medium surrounding a source of sound has a reaction upon the motion of the source. This reaction can be ascertained readily if the equations of motion are written in complex form.

If the source is driven by an external harmonic force (the real part of $Fe^{i\omega t}$) the equation of motion in the absence of reaction by the medium is (*cf.* p. 10)

$$M\ddot{\xi} + R\dot{\xi} + \mu\xi = Fe^{i\omega t} \tag{16}$$

where M, R, and μ are the mass, resistance, and stiffness of the system.

The effect of the medium is to exert upon unit area of the source a pressure equal and opposite to that which the source exerts upon unit area of the medium. The latter pressure is

* If $p = A\cos\omega t$ and $\dot{\xi} = B\cos(\omega t - \theta)$ then $p\dot{\xi} = AB\cos\omega t\cos(\omega t - \theta)$, *i.e.* $(p\dot{\xi}_{\text{av}}) = AB\{\cos(2\omega t - \theta) + \cos\theta\}_{\text{av}} = AB\cos\theta$, since the average value of $\cos(2\omega t - \theta)$ is zero.

given by $z\dot{\xi}$ (*i.e.* $(r \pm ix)\dot{\xi}$ according as the specific acoustic reactance is positive or negative), where z is the average specific acoustic impedance of the medium at the surface of the source. If the reaction is uniform over the total area S of the surface of the source, the total external force upon it is $\mathrm{F}e^{i\omega t} - \mathrm{S}(r \pm ix)\dot{\xi}$. Consequently the equation of motion is

$$\mathrm{M}\ddot{\xi} + [\mathrm{R} + \mathrm{S}(r \pm ix)]\dot{\xi} + \mu\xi = \mathrm{F}e^{i\omega t} \tag{17}$$

Since in harmonic motion $-i\dot{\xi} = \omega\xi$ and $+i\dot{\xi} = \ddot{\xi}/\omega$ we may rewrite this as

$$\left(\mathrm{M} + \frac{\mathrm{S}x}{\omega}\right)\ddot{\xi} + (\mathrm{R} + \mathrm{S}r)\dot{\xi} + \mu\xi = \mathrm{F}e^{i\omega t} \tag{18}$$

or

$$\mathrm{M}\ddot{\xi} + (\mathrm{R} + \mathrm{S}r)\dot{\xi} + (\mu + \omega\mathrm{S}x)\xi = \mathrm{F}e^{i\omega t} \tag{19}$$

according as the specific acoustic reactance of the medium is positive or negative.

Thus the mechanical resistance Sr of the medium increases the resistance to the motion of the source, and the mechanical reactance Sx causes an addition either to the inertia of the system or to its stiffness, according as the sign of the reactance is positive or negative.

Alternatively equation (17) may be solved by the method of complex analysis (see p. 19) and the solution is found to be

$$\dot{\xi} = \frac{\mathrm{F}e^{i(\omega t - \phi)}}{\mathrm{Z}}$$

where

$$\mathrm{Z} = (\mathrm{R} + \mathrm{S}r) + i\left(m\omega - \frac{\mu}{\omega} \pm \mathrm{S}x\right)$$

and

$$\tan\phi = \frac{m\omega - \dfrac{\mu}{\omega} \pm \mathrm{S}x}{\mathrm{R} + \mathrm{S}r}$$

with the same interpretation as before.

Sound Waves of Finite Amplitude.* In the theory of propagation of sound waves it is assumed that the condensation is not great, and so certain second order effects are negligible.

* Rayleigh, *Sound*, **2**, 32; Lamb, *Dynamical Theory of Sound*, p. 174.

That such assumptions are normally justifiable is seen from the range of condensation given on p. 107 for normal sounds. When, however, the condensation is finite these second order effects must be taken into account. The relations are somewhat complicated, but for plane waves equation (6) of p. 53 is replaceable by

$$\frac{\partial^2\xi}{\partial t^2}=c^2\frac{\partial^2\xi}{\partial t^2}\bigg/\left(1+\frac{\partial\xi}{\partial x}\right)^{\gamma+1} \tag{20}$$

if adiabatic expansions of the gas are concerned, γ being the ratio which the specific heat of the medium at constant pressure bears to that constant volume. For isothermal expansions the equation differs only in that γ must be replaced by 1.

For small values of $\partial\xi/\partial x$—that is, for waves of small condensation or amplitude—the equation becomes identical with (6) which relates to waves travelling with velocity $c_0=\sqrt{\gamma P/\rho_0}$. The velocity for waves of large amplitude is, however, found to be

$$c=c_0(1+s)^{\frac{1}{2}(\gamma+1)} \tag{21}$$

relative to the undisturbed medium. The velocity increases with the condensation.

Summarised Acoustical Data for Simple Sources. In the following pages equations for a number of simple sources are summarised, so far as sounds of small amplitudes are concerned. In general the quantities velocity potential, velocity, pressure, and acoustic impedance are given in complex form in one column and in real form in a second column.

The symbols used in expressing the data are the following :—

ϕ = Velocity potential ; ρ = density of medium ; ξ = velocity $=-\partial\phi/\partial r$;
p = pressure $=\rho_0\partial\phi/\partial t$; c = velocity of sound in medium ;
n = frequency ; λ = wave-length of sound in medium ;
z = specific acoustical impedance $=p/\xi=r+ix$,
where r = specific acoustic resistance,
where x = specific acoustic reactance ;
$|w_s|_{av}$ = average rate of energy flux per unit area at a surface $=\frac{1}{2}r_s\xi^2_{\max}=r_s\xi^2_{av}=p_{\max}\times\xi_{\max}\times\cos\theta$ where θ is the phase angle between p and ξ ;
$\omega=2\pi n=kc=2\pi c/\lambda$.

Point Source of Strength $4\pi A$ in infinite medium ($4\pi A$ is the maximum rate of emission of fluid at source and $4\pi A \cos \omega t$ is the momentary rate at a time t).

At a distance r *from the point source.*

$$\phi_r = \frac{A}{r} e^{i(\omega t - kt)} \qquad = \frac{A}{r} \cos(\omega t - kr) \, *$$

$$\dot{\xi}_r = + \frac{\left(\frac{1}{r} + ik\right) A}{r} e^{i(\omega t - kr)} = -\frac{A}{r}\sqrt{k^2 + \frac{1}{r^2}} \sin(\omega t - kr - \theta) \text{ where } \tan\theta = \frac{1}{kr}$$

$$p_r = + \frac{ick\rho_0 A}{r} e^{i(\omega t - kr)} \qquad = -\frac{Ac\rho_0 k}{r} \sin(\omega t - kr)$$

$$| w_r |_{av} \qquad = \frac{\rho_0 k \omega A^2}{2r^2}$$

$$| E |_{av} \qquad = 2\pi\rho A^2 k \omega$$

Infinite Plane radiating into semi-infinite medium, the normal velocity $\dot{\xi}_s$ of the plane being $\dot{\xi}_0 \cos \omega t$.

At a distance r *from the plane.*

$$\phi = \frac{\dot{\xi}_0}{\iota\kappa} e^{\iota(\omega t - kr)} \qquad = \frac{\dot{\xi}_0}{k} \sin(\omega t - kr)$$

$$\dot{\xi}_r = \dot{\xi}_0 e^{\iota(\omega t - kr)} \qquad = \dot{\xi}_0 \cos(\omega t - kr)$$

$$p_r = \rho c \dot{\xi}_0 e^{i(\omega t - kr)} \qquad = \rho c \dot{\xi}_0 \cos(\omega t - kr)$$

At the surface of the plane.

$$\dot{\xi}_s = \dot{\xi}_0 e^{i\omega t} \qquad = \dot{\xi}_0 \cos \omega t$$

$$p_s = \rho c \dot{\xi}_0 e^{i\omega t} \qquad = \rho c \dot{\xi}_0 \cos \omega t$$

$$z_s = \rho c \text{ (pure resistance)} = \rho c$$

$$| w_s |_{av} \qquad = \tfrac{1}{2} \rho c \dot{\xi}_0^2$$

---

* Also $\phi_r = \frac{A}{r} \sin(\omega t - kr + \pi/2)$, a form which bears a close relation to the form convenient in connection with a pulsating sphere.

Also $\dot{\xi}_r = \frac{A}{r}\sqrt{k^2 + \frac{1}{r^2}} \cos(\omega t - kr + \theta_1)$ where $\tan\theta_1 = kr$, but the above form reveals the relation between $\dot{\xi}$ and p most clearly.

Double Source in infinite medium. A double source consists of two point source components equal in strength ($\pm 4\pi A$) but opposite in phase, separated by a vanishingly small distance.

At a distance r, *in a direction inclined at an angle* α *to the axis of the doublet,*

$$\phi = \frac{\left(\frac{1}{r} + ik\right)A}{r} \cos \alpha e^{i(\omega t - kr)}$$

The radial and transverse components of the velocity are to be found by the formulæ $-\partial\phi/\partial r$ and $-\partial\phi/r\partial\alpha$ respectively. Near the origin they are of the same order of magnitude. At a distance the lateral velocity is less in the ratio $1/kr$.

$$| E |_{av} = \tfrac{2}{3}\rho k^4 c\pi A^2$$

Near the source, where kr is small, $\phi \propto \cos \alpha/r^2$. At a distance, where kr is large, $\phi \propto \frac{ikA}{r} \cos \alpha$, and conditions are not very different from those due to a point source. In a direction perpendicular to the axis of the doublet, $\phi = 0$ at all distances.

Pulsating Sphere in infinite medium.

Mean radius $=a$. Normal surface velocity $\dot{\xi}_s = \dot{\xi}_0 \cos \omega t$. Let

$$A'_{(\text{complex})} = \frac{a^2\dot{\xi}_0(1-ika)}{1+k^2a^2}; \qquad A_{\text{real}} = \frac{a^2\dot{\xi}_0}{\sqrt{1+k^2a^2}}*$$

$$\tan\theta = 1/kr; \qquad \tan\theta_0 = 1/ka$$

At a distance r *from the centre of the sphere.*

$$\phi_r = \frac{A'}{r}e^{i(\omega t - k\overline{r-a})} \qquad = \frac{A}{r}\sin(\omega t - k\overline{r-a}+\theta_0)$$

$$\dot{\xi}_r = \frac{\left(\frac{1}{r}+ik\right)A'}{r}e^{i(\omega t - k\overline{r-a})} = \frac{A}{r}\sqrt{k^2+\frac{1}{r^2}}\cos(\omega t - k\overline{r-a}+\theta_0-\theta)$$

$$p_r = \frac{i\rho c A' k}{r}e^{i(\omega t - k\overline{r-a})} \qquad = \frac{\rho c A k}{r}\cos(\omega t - k\overline{r-a}+\theta_0)$$

At surface of source.

$$\dot{\xi}_s = \dot{\xi}_0 e^{i\omega t} \qquad = \dot{\xi}_0\cos\omega t$$

$$p_s = \frac{\rho c a k\dot{\xi}_0}{1+k^2a^2}(ka+i)e^{i\omega t} \qquad = \rho c\dot{\xi}_0\frac{ak}{\sqrt{1+k^2a^2}}\cos(\omega t+\theta)$$

$$z_s = \rho c\left(\frac{a^2k^2}{1+k^2a^2} + i\frac{ak\dagger}{1+k^2a^2}\right) \qquad |z_s| = \rho c\frac{ak}{1+k^2a^2} \qquad \angle\theta = \tan^{-1} 1/ka$$

$$\begin{pmatrix} r_s = \rho c \text{ when } ka \text{ is large} \\ x_s = i\rho cka \text{ when } ka \text{ is small} \end{pmatrix}$$

$$|w_s|_{\text{av}} \qquad = \tfrac{1}{2}\rho c\frac{k^2a^2}{1+k^2a^2}\dot{\xi}_0^2$$

$$|E|_{\text{av}} \qquad = 4\pi a^2 |w_s|_{\text{av}}$$

---

* When the source is very small $4\pi A = 4\pi a^2\dot{\xi}_0 \equiv$ strength of source.

† At low frequencies the 'added mass' for the whole sphere is $4\pi a^3\rho$, and equals the mass of fluid occupying 3 times the volume of the sphere. This can be large in water. In general the added mass per unit area is $\rho a/(1+k^2a^2)$.

If ka is greater than π (*i.e.* $a/\lambda > 0{\cdot}5$) the source can be considered as sending out practically plane waves, the pressure and velocity being in phase.

Piston, in Infinite Baffle, radiating into semi-infinite medium.
Diameter $2a$. Velocity of Piston $\dot{\xi}_s = \dot{\xi}_0 \cos \omega t$.

At any point.

$$\phi = -\frac{\dot{\xi}_0}{2\pi} e^{i\omega t} \iint e^{-ikr_1} dS,$$

where r_1 is the distance of the point from an element dS of the surface of the piston.

At a point on axis, at a distance r *from the piston.*

$$\phi_r = \frac{i\dot{\xi}_0}{k} e^{i\omega t}(e^{-ik\sqrt{a^2+r^2}} - e^{-ikr}) \quad = -\frac{\dot{\xi}_0}{k}[\sin(\omega t - k\sqrt{r^2+a^2}) - \sin \omega t - kr)]$$

$$p_r = -\rho c \dot{\xi}_0 e^{i\omega t}(e^{-ik\sqrt{r^2+a^2}} - e^{-ikr}) = -\rho c \dot{\xi}_0 [\cos(\omega t - k\sqrt{r^2+a^2}) - \cos(\omega t - kr)]$$

$$|w_r|_{av} \quad = 2\rho c \dot{\xi}_0^2 \sin^2 \frac{k}{2}(\sqrt{r^2+a^2} - r)\ *$$

$$= 2\rho c \dot{\xi}_0^2 \sin^2 \frac{\pi m^2 \lambda}{2r} \quad \text{if } a = m\lambda,\ r >> m\lambda$$

At the surface of the piston.

$$\dot{\xi}_s = \dot{\xi}_0 e^{i\omega t} \quad = \dot{\xi}_0 \cos \omega t$$

$$\iint p_s dS = \pi a^2 . z_s . \dot{\xi}_0 e^{i\omega t} \quad = \pi a^2 \dot{\xi}_0 |z_s| \cos(\omega t - \theta)$$

(Total force on piston)

$$z_s \dagger = \rho c\left[\left(1 - \frac{J_1(2ka)}{ka}\right) + i\left(\frac{K_1(2ka)}{2k^2a^2}\right)\right]$$

$$= \rho c\left[1 + i\frac{2}{\pi ka}\right] \text{ if } ka \text{ is large,} \quad |z_s| = \rho c\sqrt{1 + \frac{4}{\pi^2 k^2 a^2}}, \quad \angle\theta = \tan^{-1}\frac{2}{\pi ka}$$

$$= \rho c\left[\frac{k^2a^2}{2} + i\frac{8ka}{3\pi}\right]\ddagger \text{ if } ka \text{ is small,} \quad |z_s| = \frac{\rho cka}{6\pi}\sqrt{9\pi^2k^2a^2 + 256}, \quad \angle\theta = \tan^{-1}\frac{16}{3\pi ka}$$

$$|w_s|_{av} \quad = \tfrac{1}{2}\rho c\left[1 - \frac{J_1(2ka)}{ka}\right]\dot{\xi}^2$$

(mean value over surface).

---

* *High-frequency Radiation.* If $m^2\lambda > r > m\lambda$, the intensity is large, and passes through a series of maxima and minima.

Low-frequency Radiation. If $r > m^2\lambda$, the intensity declines rapidly without oscillations. It is approximately equal to $2\rho c\dot{\xi}_0^2\pi^2 m^4\lambda^2/4r^2$ and varies inversely as the square of the distance. The radiation outwards from the distance $m^2\lambda = r$ is in fact confined approximately to a cone of angle $\theta = \pi/m^2$ with its apex at the piston centre.

† The specific acoustic impedance is not constant over the surface of the piston and the average value is given.

$J_1(z)$ and $K_1(z)$ are Bessel functions, details of which are given on p. 318.

‡ The added mass per unit area is $8a\rho/3\pi$ when ka is small.

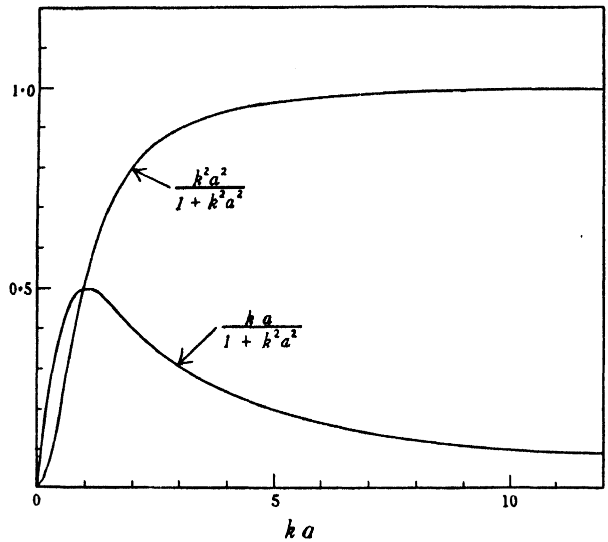


FIG. 16.—Data relating to a pulsating sphere source

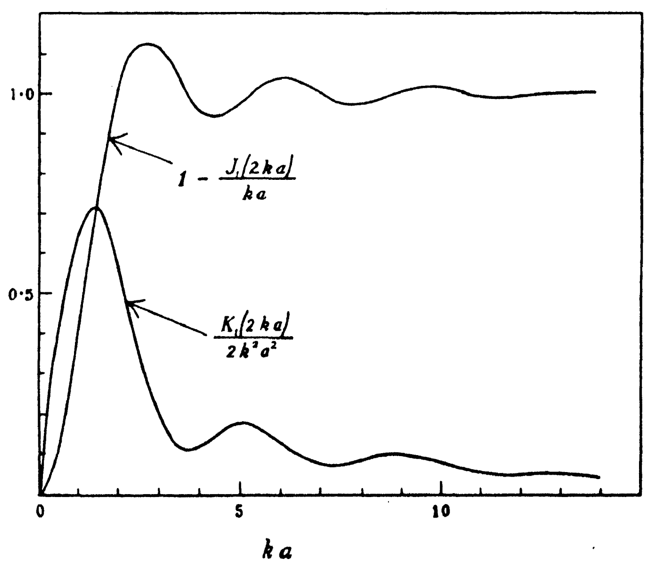


FIG. 17.—Data relating to a piston source in an infinite baffle

Line Systems of Point Sources, all in Phase. The directional radiation from sources of sound has been the subject of a paper by I. Wolff and L. Malter,* who considered various systems of point sources, line sources, and plane surface sources, and gave references to earlier work by H. Stenzel.†

In the case of a line of point sources or a continuous line source, the relative intensities on a circle in a plane passing through the points or line and having the centre of the source as centre are as follows :—

For equally spaced point sources on a straight line,

$$R_a \propto \frac{\sin nZ}{n \sin Z}$$

For straight line sources,

$$R_a \propto \frac{\sin Z_1}{Z_1} \tag{22}$$

where a = angle between the normal to the source and the line joining the source and observer.
d = distance between point sources.
n = number of point sources.
λ = wave-length.
$Z = (\pi d/\lambda) \sin a$.
$Z_1 = (\pi l/\lambda) \sin a$, where l is the length of the linear source.
R_a = directional characteristic at angle a, is the ratio between the pressure observed at a point and the pressure which would exist there if the vectors from the sources all arrived in phase.‡

There is supposed to be no phase difference at different points of the source, and the amplitudes at all points of the source are supposed to be equal.

Generally speaking such sources give directions of maximum radiation, with a number of subsidiary maxima between the large ones. A special case given by Wolff and Malter is illustrated in fig. 18.

Line Systems of Point Sources, Phases Different. The characteristic of a line of equally spaced point sources, when there

* I. Wolff and L. Malter, *Acous. Soc. Am. J.*, **2**, 201, 1930.
† H. Stenzel, *E.N.T.*, June 1927 and May 1929.
‡ See Wolff and Malter's paper for exact definition.

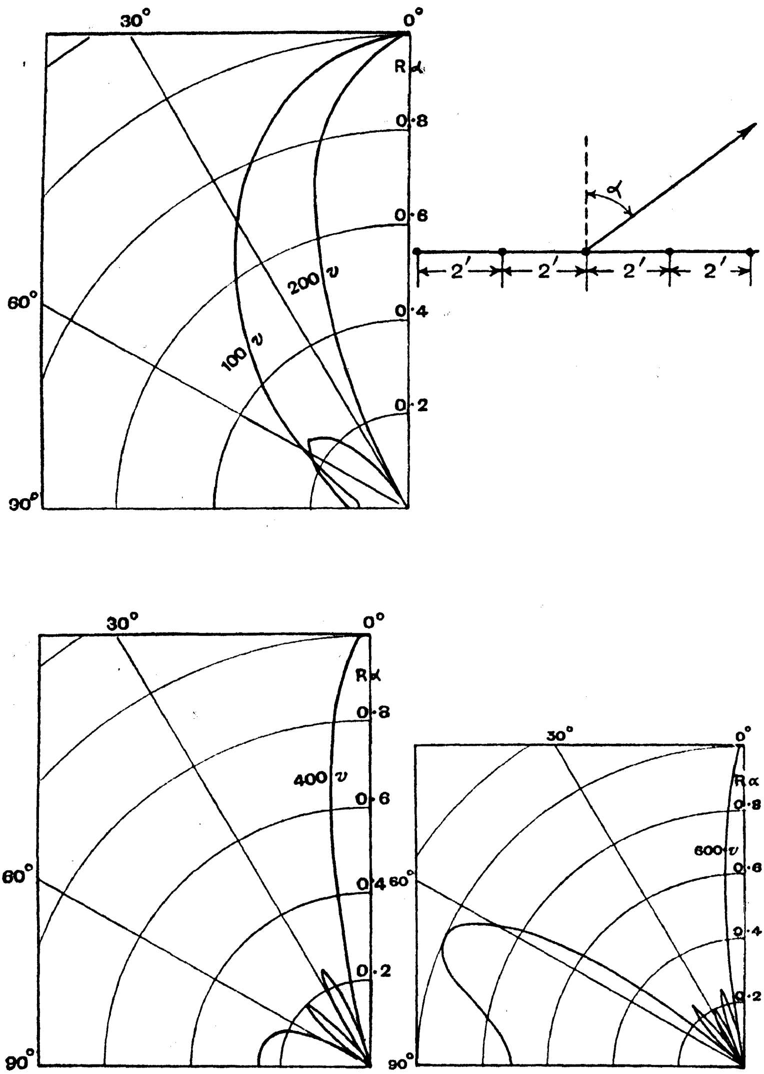


FIG. 18.—Theoretical radiation distribution characteristics of five point sources on line (special case)

is a progressive phase shift ϕ between the successive point sources, is

$$R_a = \frac{\sin n\pi\left[\dfrac{d \sin a}{\lambda} + \dfrac{\phi}{2\pi}\right]}{n \sin \pi\left[\dfrac{d \sin a}{\lambda} + \dfrac{\phi}{2\pi}\right]} \tag{23}$$

the symbols having the meaning indicated on p. 63. Many references and curves showing the characteristics of various sources of this type have been given by H. Stenzel * and Foster.† This characteristic differs from that of a system of point sources with no phase shift in that the direction of the principal maximum is at an angle γ to the normal of the line of sources instead of being on the normal, where $\sin \gamma = \phi\lambda/2\pi d$.

The characteristic of an odd number ($n = 2m + 1$) of point sources all of the same intensity and phase equally spaced upon the arc of a circle of radius R is

$$\mathrm{R}_a = \frac{1}{2m+1}\left|\sum_{k=-m}^{k=+m} \cos\left[\frac{2\pi \mathrm{R}}{\lambda} \cos(a + k\theta)\right] + i\sum_{k=-m}^{k=+m} \sin\left[\frac{2\pi \mathrm{R}}{\lambda} \cos(a + k\theta)\right]\right| \tag{24}$$

where θ is the angle subtended at the centre of the circle by the arc joining any two successive point sources, and a is the angle between the radius drawn through the central point and the line joining the source and the distant point. The expression for an even number of sources is similar.

The analyses have been extended to the cases where the points in a system of sources or the elements of a line source are not all radiating with the same intensity. An interesting case is presented by a linear source with a distribution of velocity of the type $f(v) = e^{-mx}$ between $x = 0$ and $x = l/2$, and $f(v) = e^{mx}$ between $x = 0$ and $x = -l/2$, where $f(v)$ represents the distribution in intensity along the source, m is a constant and l is the length of the source. By changing the value of m, a wide variety of sources can be studied. When m is positive the intensity drops off from centre to edge, when m is negative it increases, when m is zero a source with uniform distribution of intensity is obtained. The larger

* H. Stenzel, *E.N.T.*, 4, 239, 1927; 6, 165, 1929.
† Foster, *Bell Sys. Tech. J.*, 5, 292, 1926.

m is, as a positive number, the more rapid the decrease in intensity towards the edges and as a limiting case for $m = -\infty$, the single point source is obtained. On the other hand, the larger m is, as a negative number, the more rapidly the intensity increases towards the edges and as a limiting case for $m = -\infty$, two point sources are represented. When all the points are in phase, the relative sound intensity, at a distance, along a circle in the plane of the line for this exponential distribution is

$$R_a = \frac{\omega}{1 - e^{-\omega}}\left[\frac{e^{-\omega}(Z \sin Z - \omega \cos Z) + \omega}{\omega^2 + Z^2}\right] \tag{25}$$

In this equation $\omega = lm/2$ and $Z = (\pi l/\lambda) \sin \alpha$.

Surface Systems of Point Sources. In general actual sources are not lines or points, but consist of a single surface or a number of surfaces. Wolff and Malter * show that many results obtained for points and lines may be used to determine the radiation from surfaces by a consideration of the two following general ideas:—

(1) A great many surface distributions are of the form $f(v) = f_1(x)f_2(y)$ where $f(v)$ represents the distribution of intensity, and the phase distribution is represented by $F(v) = F_1(x) + F_2(y)$. In this case the directional characteristic is represented by

$$R_d = \frac{1}{\int f_1(x)dx \int f_2(y)dy} \left| \int f_1(x) e^{2\pi i\{(x \sin \alpha/\lambda) - F_1(x)/2\pi)\}} dx \int f_2(y) e^{2\pi i\{(y \sin \beta/\lambda) - (F_2(y)/2\pi)\}} dy \right| \tag{26}$$

the integration being carried out over the entire surface. α is the angle which the line connecting the point and source make with the plane normal to the x axis, and β is the angle between the same line and the plane normal to the y axis. It is seen that the radiation characteristic is equal to the product of the characteristics of two mutually perpendicular line sources with intensity and phase distributions given respectively by

$$f_1(x) \text{ and } F_1(x)$$
$$f_2(y) \text{ and } F_2(y)$$

(2) Very often it is desirable to consider the directional characteristics of a combination of identical line or surface sources

* I. Wolff and L. Malter, *loc. cit.*

linearly arranged and equally spaced. It can be shown that the directional characteristic in this case, or in the general case of a spatial distribution in which the sources are held parallel to each other, is equal to the product of the characteristic of a similar arrangement of point sources, by the individual characteristic of each separate source.

Because of its interest for the approximate calculation of the directional characteristics of cone radiators, which are in common use, the radiation from a plane circular surface source is given as developed by Stenzel.*

In this case

$$R_\alpha = \frac{2J_1\left(\frac{2\pi r}{\lambda}\sin\alpha\right)}{\frac{2\pi r}{\lambda}\sin\alpha} \tag{27}$$

where J_1 is Bessel's function of the first order, r is the radius of the circle, and α is the angle between the line connecting the point and the source and the normal to the plane of the circle.

Circular Membrane. So far as low-frequency excitation is concerned, a stretched circular membrane, vibrating in its fundamental mode, may be treated dynamically as a piston of which the mass is one-third of the total mass of the membrane, and of which the stiffness constant is $2\pi\tau$ where τ is the tension (stress per unit length) of the membrane. The natural frequency is given by $n = \frac{1}{2\pi}\sqrt{\frac{2\pi\tau}{m}} = \frac{1 \cdot 22}{\pi a}\sqrt{\frac{\tau}{\rho}}$, where '$m$' is the mass of the diaphragm of radius 'a,' ρ the mass per unit area. This result differs by only about 1 per cent. from a more rigid calculation of the natural frequency of an undamped membrane.

For a piston to be a source of sound of the same strength as a diaphragm of equal area the velocity of the piston must be equal to 0·306 times the velocity of the centre of the diaphragm.

It is well known that circular membranes may vibrate in several natural modes in which there are certain *nodal lines* where no motion takes place. The lines may be 'nodal circles' or 'nodal diameters,' and a number of modes are illustrated to scale in fig. 19. The positions of these nodal lines, and the membrane frequencies associated with them, are calculable as follows. The typical solution of the differential equation for the normal free

* H. Stenzel, *E.N.T.*, 7, 87, 1930.

vibration displacement ξ of a point on a membrane may be written

$$\xi = AJ_s(kr) \,.\, \cos(s\theta + \alpha)\cos(\omega t + \epsilon) \qquad (28)$$

where (r, θ) are the polar co-ordinates of the point measured from the centre of the membrane as origin of co-ordinates,

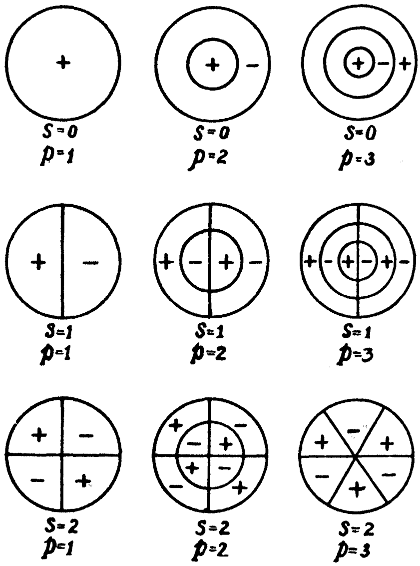


FIG. 19.—Modes of vibration of a circular membrane

s = number of nodal diameters
p = number of nodal circles, including the clamping circle

$k = \omega/c$, A, α and ϵ are constants, and s is an integer. $J_s(z)$ is a Bessel function of the s^{th} order of the first kind (p. 318).

If the membrane is fixed at the periphery, where $r = a$, we have $J_s(ka) = 0$, a relation which determines $k\left(= \frac{2\pi n}{c}\right)$ and thus the natural frequencies of the membrane. From the equation the position of nodal lines may be deduced. Concentric *nodal circles* have radii given by

$$J_s(kr) = 0 \qquad (29)$$

Nodal diameters are given by

$$s\theta + \alpha = \pm \tfrac{1}{2}\pi \,;\; \pm \tfrac{3}{2}\pi \,.\,.\,. \qquad (30)$$

The diameters, s in number, are equally spaced at intervals of π/s.

It will be noted that the order of the Bessel function $J_s(kr)$ determining the radii of the nodal circles and the form of the membrane displacement depends upon the number s of nodal diameters. The forms of the first two functions $J_0(z)$ and $J_1(z)$ are given in fig. 20. They reveal the characters of the normal modes which occur with no nodal diameters, and with one nodal diameter respectively. The curves may be taken to represent a section through the centre of the membrane, normal to the

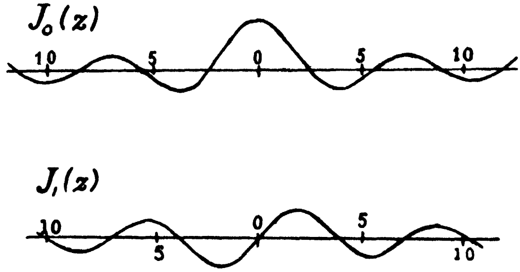


FIG. 20

nodal diameter in the latter case. To a first approximation, when ka is small, $J_0(kr)$ is paraboloidal $(=1-\frac{1}{4}k^2r^2)$.

The natural frequencies corresponding to various modes of vibration are calculable from the values of kr for which $J_s(kr)=0$. In the case of no nodal diameters $s=0$, and the first three roots of $J_0(kr)$ are $0{\cdot}766\pi$, $1{\cdot}757\pi$, $2{\cdot}755\pi$ corresponding to frequencies $0{\cdot}766c/2a$, $1{\cdot}757c/2a$, and $2{\cdot}755c/2a$. The frequencies do not form a harmonic series. In the case of one nodal diameter $s=1$ and the first three roots of $J_1(kr)$ are $1{\cdot}220\pi$, $2{\cdot}333\pi$, and $3{\cdot}238\pi$, corresponding to frequencies $1{\cdot}220c/2a$, $2{\cdot}333c/2a$, and $3{\cdot}238c/2a$. It will be realised that the natural frequencies of a circular membrane are closely spaced in any frequency range above the fundamental owing to the variety of possible forms of vibration. Some 44 natural frequencies occur in the first three octaves.

If a driven membrane vibrates in these higher modes it is clear that adjacent sections moving in opposite phases will emit sound waves which immediately interfere with each other and result in a considerable degree of neutralisation. It is not then possible to treat the membrane as a piston moving with all its area in phase. It is desirable, therefore, when using a circular membrane as a standard source, to drive the membrane at frequencies below its fundamental, or to ensure that the driving

force is of such a distributed character that the various parts of the surface shall move in phase.

When a harmonic force $F\cos(\omega t+\alpha)$ per unit area acts uniformly over the surface of the membrane the displacement is given by

$$\xi=\left\{\frac{J_0(kr)}{J_0(ka)}-1\right\}\frac{F}{\omega^2\rho} \tag{31}$$

and the amplitude becomes very great whenever ka approaches to a root of $J_0(ka)=0$, *i.e.* whenever the impressed frequency approaches that of one of the symmetrical modes. When ka is small the displacement is approximately

$$\xi=\tfrac{1}{4}\frac{F}{r}(a^2-r^2) \tag{32}$$

and corresponds to the static deflection that would be caused by a steady force F.

Circular Plate. A thin flexible disc clamped at the edges has a fundamental mode of vibration in which it takes a concave or convex form as a whole, and also certain other higher modes of vibration in which nodal lines are present. For a circular disc of radius r, and thickness t, the material of which has density ρ, Young's modulus E and Poisson's ratio σ, the fundamental frequency (n) of vibration is given by (Rayleigh, *Sound*, I, 366)

$$n=\frac{2{\cdot}56t\sqrt{e}}{\pi r^2\sqrt{3\rho(1-\sigma^2)}} \tag{33}*$$

For steel $\sigma=0{\cdot}31$ and the equation becomes approximately

$$n=1{\cdot}5\frac{t}{\pi r^2}\sqrt{\frac{e}{\rho}}$$

and $\sqrt{e/\rho}$ is the velocity of sound in the material—about 4700 metres per second. Diaphragms of ordinary telephone receivers have a thickness of, say, 0·025 cm. and a diameter of 5 cm., whence the natural frequency should be in the region of 940 cycles per second (p. 305).

Kennelly and Taylor have explored the motion of a telephone diaphragm in various points of its surface. The exact theoretical calculation of the shape of the diaphragm vibrating in its

* For the second gravest mode of vibration the constant of proportionality is approximately 10 instead of 2·56.

fundamental mode is a matter of difficulty, but a simple empirical formula which agrees well with experimental results is

$$\xi_x = \xi_0 \cos^2\left(\frac{\pi}{2}\frac{x}{r}\right) \tag{34}$$

where ξ_x is the displacement at any distance x from the centre of the diaphragm of radius r, and ξ_0 is the displacement at the centre.* Thus if $x=0$, $\xi_x=\xi_0$, and if $x=r$, $\xi_x=0$, which agrees with boundary conditions. It is found from this equation that the volume displacement of air in front of the diaphragm due to movement from flat to concave is equal to that which would result from the movement of a piston, of 0·29 of the diaphragm area, through a distance equal to the displacement of the centre of the real diaphragm.

Membranes and Plates (General).† The general analysis of the vibrations of membranes and plates is somewhat complicated. Rayleigh † deals with stretched membranes, rectangular and circular, and shows that when the boundary is rectangular natural frequencies are given by the equation

$$n = \sqrt{\frac{\mathrm{T}}{\rho}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

where T is the tension, ρ the superficial density, a and b the lengths of the sides of the rectangle, and m and n are integers. The mathematical difficulties in the case of the vibration of plates are even greater than with membranes. Reference may be made to standard treatises on the theory of sound, and also to a recent paper by Francke,‡ who, dealing with the vibration of a stretched circular plate clamped at the edges, gives diagrams showing the modes of vibration. R. C. Colwell § deals with the vibration of membranes and plates having at the edges one degree of freedom—a condition which is approximately fulfilled in the Chladni plate.

References to papers dealing with the effective mass of and the radiation from diaphragms and flexible discs are given elsewhere (pp. 44, 79) in sections on loud-speakers.

* J. A. Fleming, *I.E.E., J.*, 61, 613, 1923.

† Rayleigh, *Theory of Sound*, 2nd ed., 1, 306, 352, 1929; H. Lamb, *Theory of Sound*, 2nd ed., 141, 1925.

‡ G. Franke, *Ann. d. Phys.*, 2, 649, 1929.

§ R. C. Colwell, *Phil. Mag.*, 12, 320, 1931; *Frank. Inst. J.*, 213, 373, 1932.

CHAPTER V

ELECTRO-ACOUSTICAL RELATIONS FOR ELECTRICAL SOURCES OF SOUND

The Electrical Impedance of Telephone Receivers. When a source of sound is driven electrically, the acoustical reaction exerted by the medium upon the source has certain repercussions upon the apparent electrical impedance of the source. The effects are exemplified by the variations which are observable in the electrical impedance of a telephone receiver as the frequency of the exciting current is varied, and the principles are effectively illustrated by the theory of the receiver.

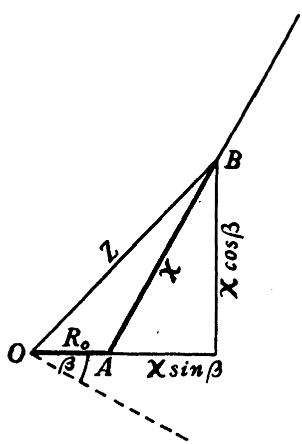
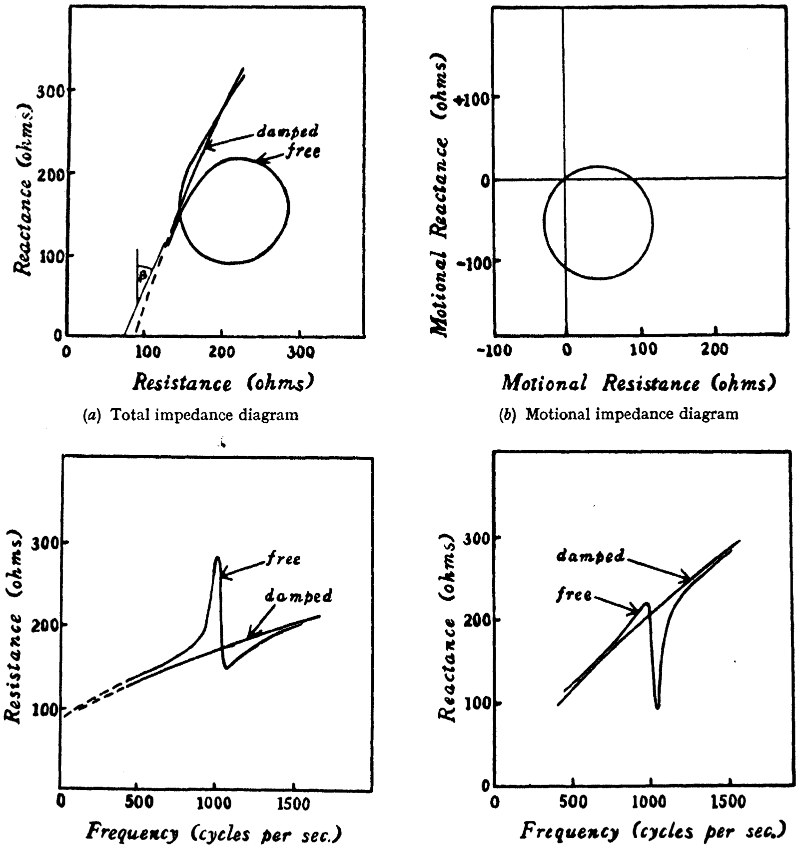


FIG. 21.—Vector diagram of the impedance of a telephone receiver

In the study of the electrical impedance of a telephone receiver there are two main characteristics to notice. Firstly, the existence of hysteresis in the magnet gives rise to an apparent resistance component which is not constant, but which depends upon the frequency. Secondly, the diaphragm resonance leads to marked variations in the electrical impedance of the instrument as the frequency is varied in the vicinity of the resonance region. It is therefore desirable to consider the impedance of the instrument when the diaphragm is clamped so that it cannot move ('damped impedance'), as well as the impedance ('free impedance') when the diaphragm is free to move.

Damped Impedance. Fig. 21 is a vector diagram of the damped impedances of a telephone receiver measured at constant current as the frequency is varied. The length OA represents the direct current resistance of the receiver. Owing to hysteresis and to skin effect (assumed constant), the magnetic flux lags behind the

vector OA by the angle β. The vector reactance is therefore developed from A in a direction perpendicular to the flux, *i.e.* in the direction AB. At any frequency $(\omega/2\pi)$ the vector reactance is represented by a length $x=\omega L$ along this line, where L is the



(*a*) Total impedance diagram

(*b*) Motional impedance diagram

(*c*) Variation of resistance with frequency

(*d*) Variation of reactance with frequency

FIG. 22.—The electrical impedance of telephone receivers

damped inductance of the telephone receiver. The impedance of the receiver as measured is then the vector $Z=OB$, having a real component $R=R_0+|x|\sin\beta$ and an imaginary component $X=|x|\cos\beta$. These are respectively the apparent resistance and reactance of the receiver at the frequency concerned. The damped impedance diagram for an actual receiver is shown in the straight line in fig. 22 (*a*), this being the locus traced out, as

the frequency is varied, by the point defining the resistance-reactance relation.

Free Impedance. When the free impedance diagram is plotted a line similar to AB is obtained, but modified by a loop (fig. 22 (*a*)). If at any frequency we find the vector difference between the damped impedance and the free impedance we obtain the impedance due to the motion of the diaphragm ('motional impedance') at that frequency. When the components of the motional impedance are plotted on an impedance diagram at a series of frequencies we obtain, as was first pointed out by Kennelly and Pierce,* a curve which is normally very nearly a circle passing through the origin of co-ordinates (fig. 22 (*b*)). Each point on the circle corresponds to some definite frequency, and shows the resistance and reactance components of the motional impedance at that frequency. The diameter of the circle which passes through the origin of co-ordinates—*i.e.* the diametral vector—is depressed below the resistance axis by an angle 2β, which is twice the angle of lag of the magnetic flux behind the phase of the exciting current. The frequency at the point at which the diametral vector intersects the circle is the resonant frequency of the diaphragm under the conditions imposed by the construction of the receiver.

The representative curves for a telephone receiver shown in fig. 22 have been compiled from data given by Kennelly.† The first two illustrations show respectively the curves obtained when, at a series of frequencies, the resistance and reactance components of the total and motional impedances are plotted against each other. The others show the damped and free resistance and reactance as a function of frequency.

Theoretical Equations of the Telephone Receiver. When the diaphragm of a telephone receiver is vibrating in its fundamental mode, it may be regarded as a certain equivalent mass subject to elastic restraint, and moving under the action of impressed magnetic forces. Equations of motion were derived on this basis by Poincaré, who laid the foundation of the mathematical equations for a simple receiver.‡

* A. E. Kennelly and G. W. Pierce, *Am. Acad. Sci., Proc.*, 48, 113, 1912.

† A. E. Kennelly, *Electrical Vibration Instruments.*

‡ H. Poincaré, *Éclairage Électrique*, 50, 221, etc., 1907.

The equations, when second order effects are neglected, may be written *

$$\left.\begin{aligned} L\frac{di}{dt}+Ri+M\frac{du}{dt}&=e \\ m\frac{d^2u}{dt^2}+r\frac{du}{dt}+su-Mi&=0 \end{aligned}\right\} \qquad (27)$$

Here L is the damped inductance, R the damped resistance, M the e.m.f. induced in the winding for unit velocity of the diaphragm and also, by the principle of reciprocity, the magnetic force on the diaphragm for unit current. In the second equation m is the effective mass of the diaphragm, r the damping force per unit velocity of the diaphragm, s the force required at the centre of the diaphragm to produce unit deflection, u the deflection at the centre of the diaphragm, i the current, and e the applied e.m.f., a function of time.

The first of the equations is an equality between applied and counter e.m.f.'s in the circuit, and the second is a simple statement of the mechanical and magnetic forces on a constrained mass. It should be mentioned that since hysteresis and eddy currents give rise to a phase lag between the current and the force on the diaphragm (p. 72), the quantity M which expresses the relation between them is complex.

If e is a simple harmonic function of time of frequency $\omega/2\pi$ the solution of the equations (27) † is, in complex notation,

$$i=\frac{e(s-m\omega^2+j\omega r)}{(s-m\omega^2+jr\omega)(R+j\omega L)+j\omega M^2}=\frac{e}{Z} \qquad (28)$$

where Z, equal to e/i, is known as the electrical impedance of the receiver. We find on inspection

$$Z=R+j\omega L+\frac{M^2}{r+j(m\omega-s/\omega)}=R+j\omega L+\frac{M^2}{z} \qquad (29)$$

where $z\{=r+j(m\omega-s/\omega\}$ is the mechanical impedance of the diaphragm under its mechanical restraints.

Clearly $R+j\omega L$ is the ordinary damped impedance of the receiver. The remainder (M^2/z) is the motional impedance, and will be denoted by $Z_m(=R_m+jx_m)$.

* See L. C. Pocock, *Electrician*, 87, 708, 1922.

† The steady state solution of these equations may be obtained by substituting $j\omega$ and $j^2\omega^2$ for d/dt and d/dt^2 respectively, and then solving by ordinary algebra (see p. 324).

It can be shown that when ω varies, the locus of the end of the vector Z_m represented in a complex plane is a circle (as indicated on p. 74) passing through the origin, and having a diameter equal to M^2/r, the angle made by the diameter with the real axis being twice the angle of the vector M. The resonance (undamped) frequency at which Z_m is a maximum ($=M^2/r$) is given by $\omega^2 = s/m$.

The displacement u and the velocity $\dot{u}$ of the centre of the diaphragm are best expressed in terms of the current i, and are given by

$$u = Mi/j\omega z, \qquad \dot{u} = Mi/z \tag{30}$$

Although the locus of u is not a circle, the locus of the velocity $\dot{u}$ is circular when ω varies, the diameter of the circle being Mi/r.

The total mechanical impedance of the diaphragm depends of course upon the nature of the space into which the receiver radiates sound and upon the nature of the medium. The equations above apply to any type of conditions, and the effect of altering the acoustical conditions is chiefly to alter r, and to change the values of s and m to a lesser extent. In fact, we may write

$$z = z' + z_a = r' + r_a + j(x' + x_a) = r + jx$$

where z' is the part of the mechanical impedance of the diaphragm arising from its mechanical and magnetic restraints, and z_a is the acoustical impedance of the medium into which the receiver radiates. There are well-known equations (see p. 61), due to Rayleigh, for x_a and r_a as functions of diaphragm size and frequency, assuming the diaphragm to radiate to free air and to be replaceable by a piston in an infinite wall.

When a telephone receiver is working under normal conditions near to an ear, the motion is so considerably damped that the motional impedance circle has very small dimensions. This indicates a relatively large value for r under these conditions.

The following power relations may be noted: The total input power (W_i) is equal to $\frac{1}{2}i^2(R_m + R)$. The power (W) transmitted to the diaphragm is given by $W = \frac{1}{2} \mid Mi\dot{u} \mid \cos(\frac{1}{2}\pi - \theta)$, this being the vector product of the magnetic force Mi into the diaphragm velocity $\dot{u}$, and $(\frac{1}{2}\pi - \theta)$ the angle between force and velocity. Clearly $\tan\theta = \omega r/(s - m\omega^2)$. The power radiated is $r_a\dot{u}^2$, and is equal to the power supplied to the diaphragm if r' is small.

Speaking roughly, it may be said that an ordinary telephone

receiver is excited by electrical energy of the order of 1 milliwatt, and emits acoustical energy of the order of 1 microwatt.

Experimental Determination of Constants of a Receiver. Kennelly * has pointed out that the fundamental constants of a receiver may be determined from the d.c. resistance of the windings, the motional impedance circle, and one other factor. Of the many additional factors tried, only two were found to give satisfactory results :

(*a*) Loading the diaphragm by a mass at the centre and repeating the impedance measurements.

(*b*) Measuring the maximum amplitudes over the poles at resonance.

The former measurement is troublesome, as the loading alters the equivalent mass of the diaphragm unless the added mass is quite small, say less than ½ gm. For measuring the amplitude of vibration of the diaphragm at any point, A. E. Kennelly and H. O. Taylor † used a special form of optical device. In this a very small light triangular mirror of about 1 mm. length, mounted on a short torsion strip on a suitable carrier, was placed in such a position that its apex was in contact with the diaphragm at the point. Vibration of the diaphragm set the mirror in motion and resulted in the deflection of a beam of light reflected from it. The carrier was adjustable in position by means of screw controls, so that the motion of any point on the diaphragm could easily be explored. Useful measurements may be made, however, without this latter elaboration.

Methods of Damping Telephone Diaphragms. For the purpose of making measurements of the damped impedance of a telephone receiver, the diaphragm may be damped by placing a finger upon it. This procedure is not altogether satisfactory, however, not only because any pressure of the finger alters the air-gap over the poles of the receiver, but also because the finger is liable to raise the temperature of the diaphragm, and thus to affect its mechanical properties. Other methods rely upon the use of wax. In one a metal rod carries a mass at one end to provide inertia, and the other end has a coating of wax. The whole is supported so that the waxed end is just in contact with the diaphragm of the receiver. The rod is then lightly heated so that the wax on the face of the rod melts and adheres to the diaphragm. When the wax is cool

* A. E. Kennelly, *Electrical Vibration Instruments*, p. 115, 1923.
† A. E. Kennelly and H. O. Taylor, *Am. Phil. Soc.*, April 1915.

this procedure results in satisfactory clamping. In another method of damping a bridge of paraffin wax is prepared which adheres to the cap of the receiver, and carries a central wax stem which presses lightly upon the diaphragm. A hot wire applied momentarily to the wax stem melts the surface and removes the pressure upon the diaphragm. When the wax solidifies again it adheres and thus damps the diaphragm. Kennelly states that another plan is to insert a wooden plug in the mouth of the receiver, so that it is only just clear of the diaphragm. The thin air film then supplies acoustic damping such that the amplitude of vibration of the diaphragm is negligibly small for many purposes.

It should be noted, however, the damped impedance can often be approximately inferred from the free impedance diagrams.

Theoretical Equations of the Moving-coil Diaphragm-type Loud-speaker. The fundamental equations for the telephone receiver apply also to loud-speakers. In the case of the moving-coil loud-speaker, in which a very lightly constrained diaphragm is driven by a coil moving in the annular gap of a pot-shaped magnet, the quantity M may be evaluated directly. It is equal to Nl where N is the field strength in the annular gap in lines per sq. cm., and l is the length of wire in the coil. The restraint s may be neglected. The electrical impedance becomes

$$Z = R + j\omega L + \frac{N^2 l^2}{r + j\omega m}, \text{ L being the coil inductance} \qquad (31)$$

The mass of the diaphragm has the effect of a capacity in the electrical circuit. This can be seen by rationalising the above. In practice r is mainly the radiation resistance of the air into which the diaphragm radiates, and its electrical effect can often be neglected. In this case

$$Z = R + j\left(\omega L - \frac{N^2 l^2}{\omega m}\right) \qquad (32)$$

and electrical resonance occurs when $\omega^2 = N^2 l^2 / Lm$.

For a coil of 1000 turns of 46 S.W.G. wire on a 2-inch former in a gap in which the strength is about 10,000 lines per sq. cm. and the total mass of the coil and disc about 20 grm., the electrical resonance would be in the region of 500 cycles per second. To increase high-frequency performance the inductance of the coil should be kept low by using a large diameter coil of few turns.

Generally speaking, results obtained by analysis on the lines of the above electro-acoustical equations resemble the results of experiments sufficiently closely to be of value in loud-speaker design. Usually suitable simplifying assumptions need to be made so that the acoustic impedance terms may be assessed; for acoustical impedances are known for only a few simple cases, such as a piston in an infinite baffle. The procedure is to work out the current i to be expected in the loud-speaker from equation (28), noting that in actual practice some of the quantities are sufficiently small to be negligible. The velocity u of the diaphragm may then be calculated by (30), and the acoustic power radiated is deducible from $r_a\dot{u}^2$. If in this latter calculation it is assumed that a large diaphragm acts as a simple piston, it will be found that the result calculated will hold up to the point where elastic vibrations of the diaphragm begin—say 200 to 300 cycles per second.

The question of the effective mass of rigid discs, diaphragms, and flexible discs has been considered by MacLachlan * and by Strutt.† MacLachlan has dealt with the measurement of the mechanical impedance of diaphragms by electrical means, and Strutt with the estimation of the equivalent mass of loud-speaker cones by exploration of the amplitude of motion of the cone over its surface, and also from observations of the effect of attaching the cone to a vibrating mechanical system.

Where the moving coil actuates a diaphragm in the throat of a horn instead of a large diaphragm in free air, appropriate formulæ are used to express the acoustic impedance in this case.

MacLachlan ‡ has dealt at length with the theory of moving-coil loud-speakers, and for details reference to original papers is desirable. H. Neumann § points out the value of strong magnetic fields for increasing damping and improving the behaviour to transients.

Theory of the Reed-driven Diaphragm-type Loud-speaker. MacLachlan and Sowter || have also dealt with reed-driven loud-speakers of the diaphragm type. These loud-speakers differ from the moving-coil type in the pronounced resonance which

* N. W. MacLachlan, *Phil. Mag.*, 11, 1139, 1931; *Phys. Soc., Proc.*, 44, 88, 1932; 44, 546, 1932.

† M. J. O. Strutt, *Wireless Engineer*, 9, 143, 1932; 8, 238, 1931.

‡ N. W. MacLachlan, *Phil. Mag.*, 7, 1011, 1929.

§ H. Neumann, *Zeits. f. tech. Phys.*, 12, 627, 1931.

|| N. W. MacLachlan and G. A. V. Sowter, *Phil. Mag.*, 11, 1, 1931.

occurs in the upper register owing to the reed. Thinner diaphragms are employed to compensate for the difference.

Loud-speaker in Valve Circuit. In practice a loud-speaker is often situated electrically in the anode circuit of a thermionic

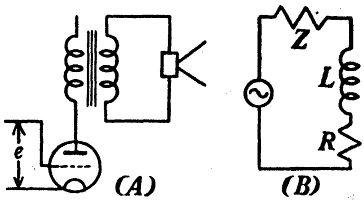


FIG. 23.—Loud-speaker circuit and its equivalent

valve. In such cases the impedance of a loud-speaker should match the impedance of the valve. If a step-down transformer is used (ratio $\rho : 1$) the impedance of the valve in the primary circuit throws an impedance into the secondary circuit equivalent to $1/\rho^2$ of valve impedance, and the coil should thus match an impedance equal to $1/\rho^2$ times the valve impedance.

For the purpose of calculating the current through the loud-speaker coils the valve circuit (*A*) (fig. 23) may be replaced by an equivalent circuit (*B*). If e is the e.m.f. applied to the grid of the valve, of amplification μ and slope-resistance (impedance) R, then $\mu e/\rho$ is the effective e.m.f. of the generator in the equivalent circuit (*B*). Z is the total impedance of the loud-speaker. $\mathrm{R} = r_s + r_p/\rho^2 + \mathrm{R}/\rho^2$, $\mathrm{L} = l_s + l_p/\rho^2$, r_p, r_s, l_p, and l_s are the primary and secondary resistances and inductances of the transformer.

CHAPTER VI

AUDIO-FREQUENCY ELECTRICAL APPARATUS

Resistances. The ordinary type of resistance box with bifilar winding is often unsuitable for use with audio-frequency alternating currents, because the low-resistance coils have appreciable inductance and the higher ones have considerable capacity. For resistances up to 100 or 200 ohms, however, self-capacity is advantageous and bifilar winding is best, whilst for medium resistances up to 1000 ohms bifilar winding is fairly satisfactory.

For a coil of 1000 ohms resistance or higher, self-capacity should be reduced and the bifilar winding avoided. The aim in design is to separate the two ends of the coil as much as possible, so as to avoid close proximity of parts of the coil which have appreciably different potentials, and between which capacity effects may therefore be appreciable. A convenient form of winding is in slots on mica sheet, notched in two opposite edges. A few turns are wound in one slot, then proceeding behind a tooth, a few turns are wound in the next slot in the opposite direction. The coils being very thin are largely free from induction for this reason, but the reverse winding of the second coil neutralises residual inductance in the first. To eliminate inductance more completely, a return to the first slot is then made and an equal number of turns added in this same new direction. A final return to the second slot follows with turns in the original direction. A similar procedure now takes place in the third and fourth slots, and so on until the required resistance is wound.

Low resistances are ordinarily wound in a plain bifilar manner, but very best results are obtained by parallelling medium resistances thus: 25 ohms is represented by 4 coils of 100 ohms each in parallel, and 1 ohm by 5 coils of 5 ohms each in parallel. For highest precision resistances wound on a mica former are not satisfactory, for the sharp bends in the wire at the edges cause the resistance to be somewhat inconstant.

Attenuator Circuits. Resistance boxes graduated in decades are not very convenient in acoustical work, where logarithmically graduated steps are frequently desired. A convenient unit is, say, a 100,000-ohm potentiometer adjusted so that each step gives a voltage increment of 5 decibrigs—*i.e.* a power step of 10 decibels—and another such potentiometer designed to give power steps of 1 decibel.* Another useful potentiometer circuit, or attenuator network, is indicated in fig. 24. It bears some

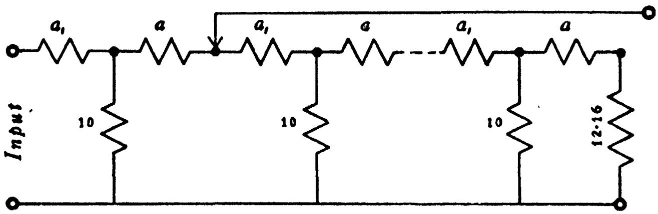


FIG. 24.—Attenuator network

resemblance to a filter circuit, and the current in succeeding sections falls off in the ratio $e^{-\lambda}=\frac{1}{2}\{(b+2)\pm\sqrt{b(b+4)}\}$, a quantity which equals $\sqrt{10}$ if $b=a+a_1=1{\cdot}479$. The power attenuation per section in this case is thus 10 decibels. The intermediate studs enable attenuations of $\sqrt[4]{10}$ in voltage, *i.e.* 5 decibels in power, to be obtained. As drawn the characteristic impedance of the network—that is, its impedance at the input terminals—is 12·16 ohms, and this resistance is used as the terminating output resistance. Another type of attenuator, for working between two sections of a line each having an impedance equal to the characteristic impedance of the attenuator, consists of a single section of a T network, similar to the first section of fig. 24 but having equal arms. By means of a suitable switch the three resistances are varied in steps in a manner which alters the attenuation without altering the characteristic impedance of the attenuator.

Thermionic Valves. The thermionic three-electrode valve is too well known to require an elaborate description. It consists essentially of an exhausted glass vessel containing an electrically heated filament which provides a supply of electrons, an anode or 'plate' to which the electrons will flow if the potential v_a of the anode is positive relative to the potential of the filament,

* For definitions of decibrig and decibel see p. 241.

and, in addition, an intermediate electrode or grid. By altering the potential v_g of this grid relative to that of the filament, the stream of electrons from filament to plate may be controlled.

If the anode voltage v_a is kept constant, a curve can be plotted (fig. 25) showing, for different values of the grid voltage v_g, the magnitude of the anode current i_a. This curve is known as the characteristic curve of the valve. If the anode voltage is altered, a somewhat different curve is obtained. Over considerable distances the characteristic curves are practically straight lines and, indeed, in this region the relation between the parameters may be expressed in the form

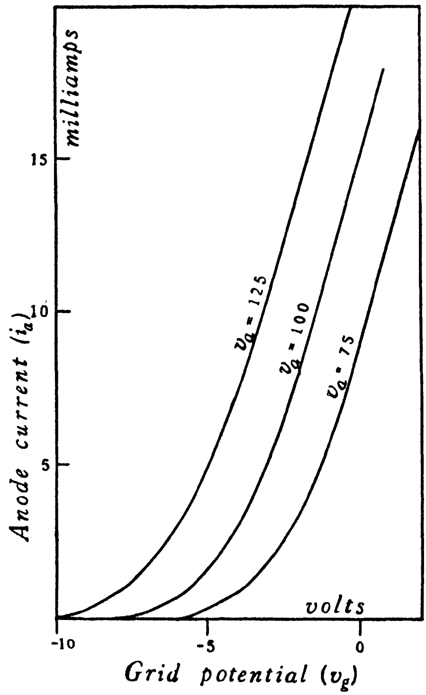


FIG. 25.—Characteristic curves of a thermionic valve

$$i_a = k_1 v_g + k_2 v_a \qquad (1)$$

where k_1 and k_2 are constants, depending upon the design of the valve, and known respectively as the 'mutual conductance' and the 'plate conductance' of the valve. The reciprocal of k_2 is often known as the 'impedance' of the valve. If v_a is kept constant, unit increase of the voltage v_g will increase the anode current by k_1 units. Similarly if v_g is kept constant, a unit increase in v_a will increase the anode current by k_2 units. The ratio $k_1/k_2 = \mu$ is called the amplification factor of the valve, as it is a measure of the voltage amplification obtainable.

Denoting the valve impedance by R, we may write the above equation in the form

$$i_a = \frac{1}{R}(\mu v_g + v_a) \qquad (2)$$

The quantity μ/R is the 'mutual conductance' of the valve and,

for various reasons, is usually a better measure of the excellence of the valve than the amplification factor.

The above equations for the current in the anode circuit of the valve relate only to a certain restricted region, namely, that over which the characteristic curves of the valve are sensibly straight, and not to the curved ends. Generally speaking, one is concerned in amplifiers mainly with the straight part of the characteristic, but in the case of rectifiers, the curved portion is of primary importance. Often the curved portion may be represented for a given anode voltage, by the equation

$$i_a = \alpha + \beta v_g + \gamma v_g^2 \tag{3}$$

No current flows in the grid circuit so long as the potential of the grid is negative relative to that of the filament, but when the grid is positive a small current flows from the filament to the grid, the amount of current increasing rapidly as the positive potential of the grid is increased.

Valve Rectifiers. The earliest valve rectifiers were of the Fleming two-electrode type, which will pass current in one direction, but not in the other. The three-electrode valve may be used as amplifying rectifier in two ways (see fig. 31, p. 99). Of these 'anode bend' rectification depends upon operating the valve in the region of the curved part of its characteristic, where the equation of the characteristic is not linear, and where in consequence the change of anode current resulting from a specified increase of grid voltage is not equal and opposite to the change due to a decrease of the same magnitude. For this purpose a suitable steady negative bias is applied to the grid either by means of a battery or through the potential drop in a resistance, and then, when an a.c. voltage is superimposed, the steady anode current through the valve changes. On the other hand, 'grid leak' detectors operate in virtue of the grid current, which flows when the grid voltage becomes positive relative to that of the filament. Thus when a.c. is applied to the grid, the grid potential becomes positive during part of the cycle, grid current flows and gives to the grid a steady charge, which accordingly alters the steady anode current through the valve. The provision of a grid leak ensures that the grid becomes discharged when the applied a.c. is removed, or varied in magnitude. If the applied a.c. fluctuates in amplitude and it is desired to obtain in the anode current a full indication of these fluctuations, clearly the

grid leak must be small enough to allow the leaking away of the additional charge on the grid (and of its associated condenser) in less than the period of one of the fluctuations concerned. For 'grid leak' rectification, therefore, the grid is given a slightly positive bias, and the grid condenser and grid leak are correctly proportioned. For minimum distortion the operation of the grid leak detector should be confined to the straight part of the characteristic curve—in other words, an adequate anode voltage should be employed.

In general 'anode bend' rectification is rather less likely to give distortion through overloading than the 'grid leak' rectifier, and it therefore has advantages where the applied a.c. is of moderate strength. A logarithmic rectifier has been described by Kirke.*

Amplifiers. If a small e.m.f. δe_g is applied to the grid of a valve, in the plate circuit of which an impedance Z_0 is situated, we find the change in conditions is represented by the equation

$$\mu(\delta e_g) = i_a(R + Z_0)$$

The valve thus acts as a source of e.m.f., $\mu\delta e_g$ having an internal resistance R. The voltage drop δe_0 across the terminals of the load Z_0 is given by

$$\frac{\delta e_0}{\delta e_g} = \mu \frac{Z_0}{R + Z_0} \tag{4}$$

When Z_0 is much greater than the impedance R of the valve, the voltage amplification is approximately equal to μ, but when Z_0 is of the same order of magnitude or smaller than the impedance of the valve, the impedance of the valve has an important effect upon the available voltage amplification.

There are certain limitations to the range of operations of valves in amplifiers. Firstly, the operation must be restricted to the range of conditions defined by the straight part of the characteristic curve. Secondly, if the grid voltage fluctuations have any considerable positive values, current will flow in the grid circuit whilst the potential is positive, but not when it is negative, and the wave-form of the grid current will differ markedly from that of the input e.m.f. Distortion will thus occur. To render the effect of grid current negligible, sufficient negative 'grid bias' must be applied to the grid to ensure that

* H. L. Kirke, *Wireless Engineer*, **9**, 369, 1932.

it remains negative throughout the input cycle. This leads to the requirement of high anode voltages and considerable negative 'grid bias,' particularly in the later stages of an amplifier where the excursions of the input voltage to the grid may amount to several volts.

Successive valves of a multistage amplifier * may be coupled in various ways. Transformer coupling is often employed, but generally speaking it is resistance-capacity coupling which is used in amplifiers designed for high-quality acoustical work. In resistance-capacity coupling a circuit of the form fig. 26 is usually employed. To attain uniform amplification over a range 50–5000 cycles per second, attention must be paid to the characteristics of the valve, to the magnitude of the intervalve components, and to certain stray capacities between the various parts, as indicated by the dotted lines in the diagram. Analysis shows that these may be lumped together into a larger capacity between the anode and the filament, so that the circuit becomes equivalent to the network given. For this network it can be shown that, if R_g is great compared with R_a, and $\omega/2\pi$ is the frequency concerned.

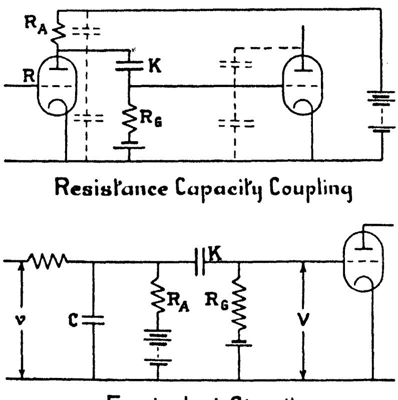


FIG. 26.—Resistance-capacity coupling. Equivalent circuit

(1) For high notes (where the effects of K and R_g can be ignored)

$$\frac{V}{v}=\frac{R_a}{R+R_a}\cdot\frac{1}{\sqrt{1+k^2}} \quad \text{where } k=\frac{\omega CRR_a}{R+R_a} \tag{5}$$

(2) For middle notes (where the effects of C and K can be ignored)

$$\frac{V}{v}=\frac{R_a}{R+R_a} \tag{6}$$

* A. C. Bartlett (*Phil. Mag.*, 10, 734, 1930) gives full equations for a multistage amplifier.

(3) For low notes (where C can be ignored)

$$\frac{V}{v}=\frac{R_a}{R+R_a}\ \frac{p}{\sqrt{1+p^2}} \quad \text{where} \quad p=\omega KR_g \tag{7}$$

Thus, compared with the middle-note amplification, high notes may be reduced in the ratio of $1/\sqrt{1+k^2}$, and low notes in the ratio $p/\sqrt{1+p^2}$.

In order that the variation from middle-note response shall be not more than 5 per cent. at extreme frequencies of 50 and 8000 cycles, it is necessary to arrange that KR_g shall not exceed 0·015 where K is in microfarads and R_g is in megohms, and that $(R+R_a)/CRR_a$ shall not exceed 0·15 where C is in microfarads and R and R_a are in megohms.

It should be noted here that R is the working impedance of the valve under the conditions in which it is employed, and not its rated impedance. The quantity C is the sum of a number of constituents. It is equal to the sum of the anode-filament capacity of the first valve and the self-capacity of its anode resistance, holder, etc. (say a total of 10 micro-microfarads), plus the grid-filament capacity of the succeeding valve when unlighted (say 10 $\mu\mu$F), plus (μ^1+1) times the anode-grid capacity (say 10 $\mu\mu$F) of the succeeding valve, where μ^1 is the working amplification factor of the valve as employed in the circuit. Thus C is often of the order of $20+10(\mu^1+1)$ micro-microfarads. Since in this expression it is the second term which predominates in present-day triode valves, C can be kept low—and high-note loss minimised in audio-frequency amplifiers—only by avoiding too high an amplification per stage.

Various 'noises' which occur in amplifiers place a limit to the extent to which amplification of feeble signals is profitable. The noises arise from electrical irregularities in the emission of electrons in the valve—the 'small-shot' and 'flicker' effects *—and from thermal agitation in resistances employed.† These disturbances often become comparable with the signal voltage applied to the grid when the latter is of the order of 1 microvolt, an input e.m.f. which thus appears to mark a critical region with amplifiers.

Towards the problem of reducing the noise caused by thermal agitation three suggestions have been made. The first is to use

* W. Shottky, *Phys. Rev.*, **26**, 71, 1926.

† J. B. Johnson, *Phys. Rev.*, **32**, 97, 1928.

a low input resistance, the second to keep the input resistance at a low temperature, the third to confine the frequency range of the system to no greater than is essential for the proper transmission of the applied input voltage.

Amplifiers with Special Characteristics. It is often possible to correct, in an amplifier circuit, for distortion introduced by, say, a resonant microphone or other component of the complete circuit of which the amplifier is part. As an illustration a resonance can sometimes be corrected by putting a series tuned circuit in parallel with the anode resistance as shown in fig. 27. The tuned circuit should have the same resonance frequency and the same damping factor as the microphone resonance for which it is desired to correct. These are determined from the shape of the peak of the microphone response curve.

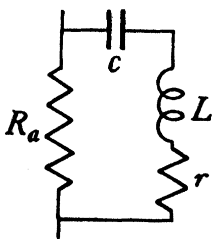


FIG. 27.—Resonance correcting circuit

The effect is to substitute for R_a in the equations above an impedance which is least at the frequency for which the microphone is most sensitive, so that the reduced amplification of the amplifier at this frequency compensates for the enhanced sensitivity of the microphone. The constants of the correcting circuit are determined approximately as follows. The product Lc is determined by the resonance frequency of the microphone; the damping factor $r/2L(=\Delta)$ is determined by the sharpness of resonance of the microphone; and the ratio r/R is determined by the extent to which the normal response of the microphone (as measured away from the resonant frequency) is enhanced at its resonant frequency. Quite simple calculations will indicate the approximate circuit required, which should then be the subject of experimental trial and adjustment.*

Sometimes it is desired to arrange that the amplifier shall

* The impedance of the complete anode circuit indicated above is $R_a r/(R_a + r)$ at resonance, and tends to R_a at high and low frequencies well removed from the resonance region. Actually the impedance at any frequency $\omega/2\pi$ is

$$\frac{R_a\sqrt{r^2+\left(\omega L-\frac{1}{\omega c}\right)^2}}{\sqrt{(R_a+r)^2+\left(\omega L-\frac{1}{\omega c}\right)^2}}$$

have a certain amount of high-frequency loss and also some degree of low-frequency loss. Such a case arises when the characteristics of the amplifier are intended to resemble as far as possible those of the human ear, an organ which is less sensitive at high and low frequencies than it is for notes of medium pitch. The actual characteristics of the ear will be dealt with later, as they are rather complicated and are not the same for sounds of widely different loudness, but the methods of obtaining specified high- or low-frequency loss in an amplifier may be deduced from the equations given above. To attain loss at high frequencies the quantity C (fig. 26) may be increased by putting a suitable condenser in parallel with the anode resistance R_A; low-frequency loss is attained by reducing the capacity of the coupling condenser K.

The above outlines the simple essentials of valve amplifiers, but in the design of modern amplifiers various special devices are incorporated. For instance, with an amplifier in which several valves are employed with a common H.T. battery, the fluctuations which occur in the anode circuit of one of the valves (say an output valve) are liable to pass through the common battery, and thus react upon other valves in the amplifier. This coupling gives rise to instability and oscillation. To avoid it an 'anode feed' resistance is inserted between each anode resistance and the H.T. battery. A 'decoupling condenser' of, say, 2 microfarads capacity is connected between earth (filament) and the junction of the two resistances in the anode circuit. With this arrangement the fluctuations of current in the anode resistance pass through the condenser rather than through the battery and its protecting anode feed resistance, and thus do not react upon the circuits of the other valves.

Endeavours to provide from electrical mains the H.T. supply for an amplifier, lead to special units for smoothing the mains supply; and when, in addition, a resistance in series with the mains is used for providing grid bias, 'grid decoupling' units also are incorporated. The grid leak is in effect divided into two, and a condenser is connected between the mid-point and the filament.

In electro-acoustical work, owing to the smallness of the e.m.f.'s involved, effective shielding from external influences is often necessary.* Apparatus can be shielded against leakage

* H. L. Curtis, *A.I.E.E.*, *J.*, 48, 453, 1929; J. G. Ferguson, *ibid.*, 48, 517, 1929.

currents by a proper arrangement of earths and guards; from electrostatic fields by enclosure in a metal case; and (with difficulty) from magnetic fields by surrounding it with a thick case of magnetic metal. Electrostatic leaks are the most frequent, and it is often desirable to enclose an amplifier in an earthed metal case, and to surround the lead to the grid of the first valve in a metal cover connected to the filament of the valve and to the earthed case.

Screened Grid and Pentode Valves. In recent years there have been considerable improvements in the design of thermionic valves, and special valves with more than three electrodes have been introduced. It probably suffices to say here that a *screened grid valve* is a four-electrode valve. In addition to the filament, the control grid, and the anode which constitute the usual triode, it contains an additional screening grid which surrounds the control grid but is insulated from it. In use a fairly high positive potential (60–80 volts) is applied to the screening grid, and as a result the inter-electrode self-capacity of the valve is greatly reduced and a high amplification factor is obtained. The reduced inter-electrode self-capacity is of importance at radio-frequencies, and the valve is thus particularly of use as a radio-frequency amplifier giving considerable voltage amplification (equal to that of several triodes) from a single stage. Ordinarily the anode is connected to a special terminal on the top of the bulb, whilst the screening grid is connected to what would be the anode pin on an ordinary triode. There is also a metal screen within the valve.

In one form of screen-grid valve—known as the variable -mu valve—some parts of the control grid are of more open mesh than the others. The closely meshed portion cuts off the anode current in its neighbourhood at a certain value of negative bias, whilst the open-meshed portion still allows anode current to pass. To secure complete suppression it is necessary to use a much heavier bias. The net result is that anode current is but very slowly reduced as negative bias is increased, and the characteristic curve has a very extended bend at its lower end. As the curvature is slight the valve does not rectify appreciably, and gives practically distortionless amplification, but control of volume can be effected in a gradual manner by adjusting the bias of the control grid. The variable -mu valve has been employed as the essential component of a logarithmic voltmeter.*

* S. Ballantine, *Acous. Soc. Am. J.*, 5, 10, 1933.

The *pentode* is essentially a power valve for audio-frequency amplification. In its construction a grid surrounding the control grid is again employed, and is given a constant positive voltage. This permits more grid bias to be applied to the control grid, so that a greater value of the fluctuating input voltage can be handled without causing grid current to flow. A fifth electrode consists of another grid surrounding the other two and insulated from them but connected to the filament. Its function is to prevent secondary effects. As a consequence the valve has a high mutual conductance, *i.e.* gives a high variation of anode current for a given a.c. input to the grid.

Mention should be made of valves in which the filament is not heated directly by the passage of electrical current through them, but by the passage of current through a conductor situated near them. Fluctuations in the heating current are then of less importance as the potential of the cathode is unaffected, and the valves are of special use where amplifiers are operated from electrical mains rather than from batteries.

Valve Oscillators. The three-electrode valve can be arranged, in conjunction with a suitable circuit, to act as a source of alternating current over a very wide range of frequencies, and oscillators based upon this principle have practically displaced all other generators for acoustical purposes. An oscillating circuit in frequent use is shown in fig. 28. A tuned circuit in series with the anode of the valve consists of a variable capacity C in parallel with a coil having resistance R and inductance L. This coil is inductively coupled to a coil in the grid circuit, the mutual inductance between the two being M.

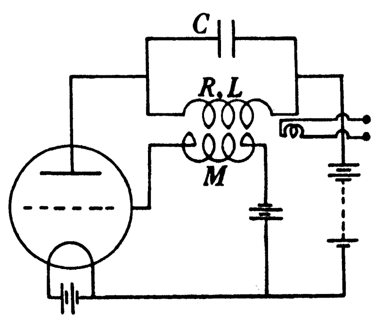


FIG. 28.—Thermionic valve oscillator

The condition of oscillation is that the power supply to the anode circuit shall exceed the thermal dissipation of power in the resistance R. This leads to a relation which may be written, if μ is the amplification factor k_1/k_2 for the valve and k_2 its plate conductance,

$$\mu M < \frac{CR}{k_2} + L \tag{8}$$

Two considerations tend to limit the amplitude to which the oscillations build up. Firstly, when the region over which the valve operates reaches one of the extremities of the valve characteristic curve, k_2 increases rapidly and the above oscillating condition is no longer satisfied. Secondly, oscillation ceases when the oscillating anode voltage approaches equality with the applied steady anode voltage V_0. The best adjustment of the oscillator is that which obtains when these two limiting factors become operative together. This may be shown to be when

$$\frac{C}{L}=\frac{I_s}{2RV_0} \tag{9}$$

where I_s is the saturation current for the valve. For maximum purity of output the coupling M between the grid and anode circuits should be as small as possible. The following adjustments are desirable when an oscillator is being set up to give maximum output: (*a*) uncouple the coils (*i.e.* make $M=0$); (*b*) adjust the grid bias potential until the anode current is equal to half of its saturation value I_s; (*c*) gradually increase M until oscillations set in; (*d*) keeping CL constant vary the ratio C/L (adjusting M to be a minimum for oscillation on each occasion) until the oscillating current is a maximum.

In some forms of oscillator the grid coil and the anode coil are wound upon the same bobbin. The coupling between the coils is thus fixed. However, a high-resistance potentiometer is so arranged, in parallel with the grid coil, that only a controllable fraction of the full voltage in the grid coil is actually applied to the grid of the oscillating valve. In oscillators constructed to cover a range of frequency the capacity C is variable. If a wide range is required the anode- and grid-inductance coils are each wound in equal halves, and may be used either in parallel or in series according as a high range or low range of frequency is required. To obtain frequencies below 100 cycles per second an iron core may be placed in the inductance, but purity suffers. It is better to wind large air-core inductances having low resistance. Thus a coil having an inductance of about $\frac{1}{3}$ henry and a d.c. resistance of 7 ohms,* is suitable for frequencies down to, say, 50 cycles per second. The frequency n of oscillations is largely

* Say 1350 turns of double cotton-covered wire (S.W.G. 16) on a bobbin of 5-in. diameter and 2-in. length. It would weigh about 35 lbs.

but not entirely determined by the inductance and capacity in the tuned circuit. It is approximately given by the usual relation $\omega = 1/\sqrt{LC}$ in absolute units, or $n = 159/\sqrt{L_1C_1}$ where L_1 is measured in henries and C_1 in microfarads. The frequency of the note emitted by an oscillator actually depends somewhat upon the supply voltage and upon the load placed upon the oscillator. To prevent the load from reacting upon the frequency of the oscillator, it is usual to interpose an amplifier between the oscillator and the load. Incidentally this amplifier may itself be tuned, and it then purifies the oscillator output. Dow * has pointed out that by using a screen-grid valve as oscillator, the frequency can be made nearly independent of changes in supply voltage. This arises because the frequency effects of changes in plate voltage, and of changes in screen-grid voltage are of opposite sign. Consequently, if the two voltages are obtained in proper proportions from a voltage divider, constancy of frequency can be ensured.

The audio-frequency oscillator may be arranged to emit a note of fluctuating pitch (warble note) by arranging for part of the tuning inductance to be variable in a cyclical manner. Where very rapid fluctuations are desired, faster than can be conveniently achieved by rotating coils, the tuning coil, if iron-cored, can be varied in inductance by passing through it alternating current of appropriate frequency in superposition upon a direct current.†

Barrow,† in a paper dealing with theoretical and experimental aspects of the warble note, has summarised the theoretical analysis of a note of varying pitch represented by the formula

$$f(t) = \sin 2\pi(f_0 + \Delta f \,.\, \sin 2\pi a t)t$$

where $2\Delta f$ is the range of the warble, and a its frequency. Provided $\Delta f << f_0$ and $a << f_0$, we have the following series for the warble note :—

$$f(t) = \sum_{-\infty}^{\infty} J_n\left(\frac{\Delta f}{a}\right) \sin 2\pi(f_0 + na)t$$

where $J_n\left(\frac{\Delta f}{a}\right)$ is the Bessel function of the first degree and n^{th} order, and n is an integer. Thus the components of the warble tone have their frequencies separated by a, lie symmetrically on

* J. B. Dow, *Inst. Radio Eng., Proc.*, 19, 2093, 1930.

† W. L. Barrow, *Ann. d. Phys.*, 11, 147, 1931.

each side of the mean frequency f_0, and have amplitudes proportional to $J_n\left(\frac{\Delta f}{a}\right)$, the amplitude of components outside the range of warble being very small. Hunt * gives reasons why $\Delta f/a$ should exceed 3, and finds that for reverberation measurements $\Delta f/f_0$ should be not less than 0·2.

Heterodyne Oscillators. The ordinary audio-frequency oscillator requires considerable inductances and capacities in order that it may cover the whole audio-frequency range of 10 octaves or so, and its output is not the same at all frequencies within that range. To overcome these defects oscillators are employed in which the audio-frequency is produced as the difference tone between two oscillators each tuned to frequencies well above the audible range. For instance, one oscillator can be maintained at a fixed frequency of, say, 100,000 cycles per second, the other being variable over the range 90,000 to 100,000 cycles per second by adjustment of the tuning condenser. As the frequency is very high a variable air condenser is suitable, and since the frequency needs to vary only by 10 per cent. no appreciable variation of output occurs over the range. A difference tone varying in frequency from 10,000 to 0 cycles per second is produced when the two high frequencies are supplied to a circuit and rectified. A suitable filter following the detector will suppress the constituent high frequencies and summation tones, and pass only the audio-frequency difference tone required.

Some points in construction may be noted. When the two high-frequency oscillators are linked together there is a tendency for one to 'pull' the other into synchronism, if the difference between them is only a few cycles per second. The tendency is minimised if each high-frequency oscillator is followed by one stage of amplification before the two outputs are mixed. It is desirable for each set of oscillator coils to be wound on a toroidal former and to be screened in a metal case. Indeed, careful screening of successive stages of the oscillator in separate compartments is desirable. Amplification is necessary after the detector to give adequate output for use, and a set of suitable transformers enables the output to be efficiently applied to a range of uses where widely different impedances are encountered.

The heterodyne oscillator is easily adapted for use when automatic recording of frequency characteristics is desired, for

* F. V. Hunt, *Acous. Soc. Am. J.*, 5, 133, 1933.

a photographic drum can be attached to the spindle of the air condenser which controls the frequency of note emitted.*

By providing one of the oscillating circuits of a heterodyne oscillator with a small rotating air condenser, it is readily possible to arrange that the note emitted by the oscillator shall fluctuate slightly or markedly in pitch in a regular manner, and so yield the 'warble' notes which are of value in certain forms of acoustic measurement.

Valve used in Dynatron Condition. When the voltage applied

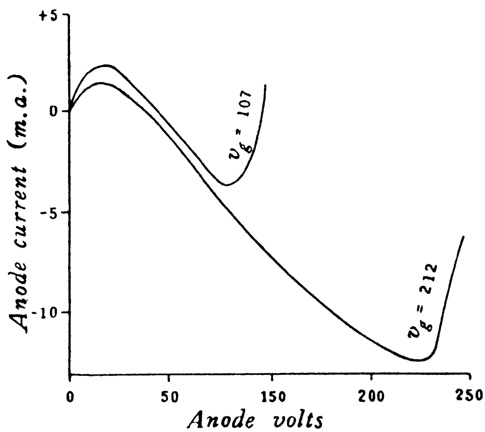


FIG. 29.—Characteristic curves of valve used in dynatron condition

to the grid of a triode is positive, and greater than the anode voltage, the secondary electrons emitted from the anode owing to the impact of the electrons from the filament are considerably increased in number, and the reverse current (anode grid) in the valve may exceed the direct anode current. Further, the negative current in certain circumstances increases in magnitude proportionately with the applied anode volts (fig. 29), and the valve acts in fact as a negative resistance.† If a screened-grid valve is employed the negative impedance (slope of characteristic curve) can be adjusted to have any value in a certain range by adjusting the voltage on the control grid.‡ Altering the filament current has some degree of effect of this kind, but alterations in the voltage on the grid of a triode have no appreciable effect on

* B. S. Cohen, A. J. Aldridge, and W. West, *I.E.E., J.*, 64, 1023, 1926.
† A. W. Hull, *Inst. Radio Eng., Proc.*, 6, 5, 1918.
‡ F. M. Colebrook, *Exptl. Wireless*, 10, 69, 1933.

the negative resistance, only on the range of anode volts for which it is obtainable (fig. 29).

In consequence of the above characteristics the dynatron circuit is valuable in the construction of oscillators * (fig. 30) and for neutralising the resistance of tuned electrical circuits, and thus for increasing the sharpness of resonance in a very marked manner.† The dynatron arrangement may also be employed as a current or voltage amplifier by using it respectively in parallel or in series with a suitable resistance. The theory of dynatrons

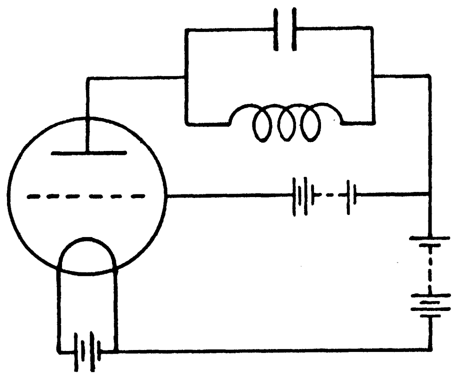


FIG. 30.—Dynatron oscillator, or, if adjusted to be stable, a sharply selective amplifier

is given by Ito,‡ who deals with various circuits in which they may be employed.

'Thyratron' Valves for Instantaneous Relays. The thyratron is a mercury vapour triode, comprising cathode, grid, and anode inside a bulb containing mercury vapour at low pressure. If the anode is at a sufficiently positive potential a heavy anode current or luminous arc (say ½ ampere) will pass between the anode and cathode.

The arc can, however, be prevented from striking if the grid voltage is more negative than a certain critical grid voltage, say about −2 volts. If, then, the grid is made slightly more positive than the critical voltage, even for a few micro-seconds, the anode current or arc immediately starts. A positive ion sheath immediately forms round the grid, which then exerts no

* F. Gabriel, *E.N.T.*, 4, 338, 1927. See also D. Hale, *Rev. Sci. Insts.*, 3, 230, 1932.

† F. M. Colebrook, *Exptl. Wireless*, 10, 69, 1933.

‡ Y. Ito, *E.N.T.*, 7, 419, 1930; 8, 23, 1931.

further control over the arc. Once started the arc continues independent of changes in grid voltage, until the anode voltage is removed. The thyratron may thus be regarded as a sensitive inertialess relay, which can be operated by a practically negligible current of extremely short duration. The change of grid voltage necessary to start the arc may be obtained from a valve rectifier, and the thyratron may thus be arranged to operate when the sound falling upon a microphone-amplifier-rectifier system reaches a certain predetermined value.

Transformers. Iron-core transformers are often employed in commercial amplifiers as intervalve, input, or output connectors. They consist essentially of two windings which, being on a magnetic core, are in close electromagnetic coupling. Cores of 'stalloy' are frequently used; cores of permalloy or nickel iron give higher inductances, but often, with nickel iron, the response fails to be proportional to the input if the magnetisation of the core is appreciable.

If a source of electrical power has an internal impedance Z_i, and is to deliver power to an output circuit of impedance Z_o, maximum transmission occurs if the two impedances are coupled by means of a suitable transformer. The impedances of the secondary and primary windings of the transformer should be in the ratio Z_0/Z_i, and each should be adequately high in comparison with the impedance to which it is connected. Since each winding has an impedance which, at low frequencies, is practically proportional to the square of the number of turns of wire in it, the number of turns in the secondary coil should be n times the number in the primary where $n^2 = Z_0/Z_i$. Within limits the voltage step-up is then n-fold. When a transformer is employed between impedances which are very different from those for which it was designed it gives a distorted response. Consequently transformers should be used and tested only in the circuit for which they are intended.

The voltage step-up is limited, in practice, by complications due to the fact that a transformer possesses leakage inductance, and a self-capacity which is effectively a shunt across the secondary terminals. Moreover the self-capacity, in association with the inductances of the instrument, gives rise to electrical resonances. The design of transformers therefore consists in so balancing the various factors, including the damping afforded by terminating impedances, that the voltage amplification of the transformer is

substantially constant over the frequency range it is desired to cover. The response falls off at lowest and highest frequencies, but a flat characteristic can be obtained nowadays over a very wide range. A large variety of transformers is commercially obtainable, and some makers will design instruments specially if the required range, step-up, and impedance are specified.

It should be mentioned that frequency-response curves obtained for an input of given magnitude do not indicate the extent, if any, to which the response fails in proportionality to the input as the strength of the input is varied. Nor does a flat characteristic necessarily imply faithful reproduction of phase relations. In general, phase relations between components of tones are not important to the ear, but faithful reproduction of transients possibly makes more stringent demands.

Instruments for the Measurement of Audio-frequency Voltage and Current. For details of instruments applicable to the measurement of the audio-frequency e.m.f.'s which occur in the electrical measurement of sound, reference should be made to electrical textbooks. An outline, however, is given below.

The power absorbed by the ordinary type of low-frequency voltmeter is so great that in audio-frequency acoustical work the circuit into which the instrument is introduced is usually considerably disturbed. It is possible, however, to use an electrostatic voltmeter for the higher ranges, say, above 40 volts, and valve voltmeters such as those devised by Moullin for ranges of from 1 to 10 volts. The Moullin voltmeter absorbs practically no power, has a negligible capacity, and its readings are unaffected by frequency. Two types of voltmeter of this kind are obtainable. One depends upon rectification due to curvature of the anode-current/grid-potential characteristic of the valve (*i.e.* anode-bend rectification), and the other on curvature of the grid-current/grid-potential characteristic.

The first type of instrument, illustrated in fig. 31, measures voltages of the order of 0–1½ volts, and does not require a separate anode battery. A conducting circuit must always exist between the voltage-input terminals of the instrument, so that the grid potential shall be correct. In consequence it cannot be used to measure the potential drop across one of two condensers in series, and the calibration is obviously upset if any steady e.m.f. exists between the input terminals. The second type of Moullin voltmeter has the range of the order of 0–10 volts, requires an

anode battery, but can be used to measure an alternating electromotive force, superimposed upon a steady electromotive force of large or small value.

Naturally, if a calibrated amplifier is used as auxiliary, very small voltages may be measured. A variable mutual inductometer or some form of calibrated attenuator is valuable for producing, from a measured current, known small voltages for comparison—*via* an amplifier and indicator—with a voltage of unknown magnitude (see p. 146).

A.C. potentiometers are available for the measurement of audio-frequency sine-wave voltages. In the one type, illustrated by the Larsen * potentiometer, measurement is made in terms of rectangular co-ordinates; in the other type, illustrated by Drysdale's instrument, measurement is made in terms of the maximum amplitude and phase angle.†

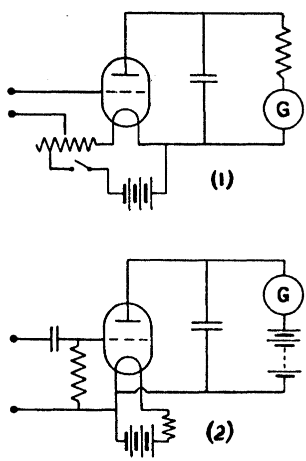


FIG. 31.—Moullin voltmeter circuits

Audio-frequency electrical current can be measured by means of:

(a) *Vibration galvanometers* giving sensitivities of, say, 1–60 mm. per micro-amp. at 1 metre for frequencies 500–50 cycles per second. An ordinary telephone receiver is a very sensitive detector of audio-frequency e.m.f.'s at all frequencies except low ones: for the latter a vibration galvanometer is more sensitive.

(b) *Thermo-galvanometers*, in which a resistance is heated by the current to be measured, and the heat thus generated affects a thermo-junction which operates a sensitive galvanometer. In Duddell's form the whole unit is self-contained. Deflections tend to be proportional to the square of the a.c. current. When the heater element has a resistance of 1000 ohms, a current of 20 micro-amps. will give a deflection of 1 cm. at a distance of 1 metre.

(c) *Galvanometers in conjunction with* separate *thermo-junctions*. These are somewhat less sensitive than the thermo-galvanometer.

* A. Larsen, *Electrotechnische Zeits.*, 31, 1039, 1910.

† C. V. Drysdale, *Phys. Soc.*, 21, 561, 1907. See also *I.E.E.*, *J.*, 68, 339, 1930.

(d) *Galvanometers in conjunction with* crystal, valve, or metal oxide *rectifiers* arranged for half-wave or full-wave rectification.

Three ways of obtaining full-wave rectification, the 'centre tap' (2), the 'bridge' (3), and the 'voltage doubler' (4), are shown diagrammatically in fig. 32, the direction in which current is allowed to pass being indicated by the arrow-heads. The 'bridge' connection has advantages over the 'centre tap,' and in the case of metal oxide rectifiers the four components are conveniently mounted in a single unit. With valve rectifiers the 'centre-tap' circuit is used as a rule, special valves with two anodes and one filament being available.

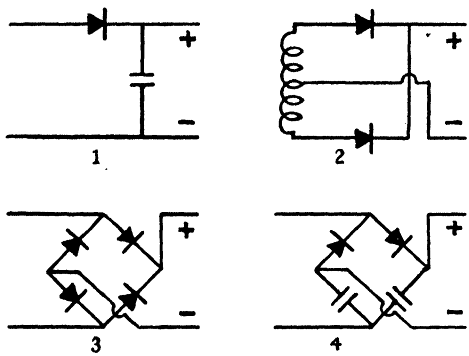


FIG. 32.—Half-wave (1) and full-wave (2–4) rectifier circuits

The indications of instruments used for measuring alternating current of complex wave-form may depend somewhat upon the nature of the detector used. Full-wave square-law detectors (such as thermal meters and some vacuum-tube detectors) give a reading which is a function of the average impressed power, not of the wave-shape. With devices following other simple laws, however, the meter indication is usually dependent somewhat upon the wave-shape, and may depend upon the polarity with which the input is connected to its terminals. The deflection due to a linear detector is independent of reversal of input terminals, but depends upon the phase relations between the fundamental and harmonics. For measuring the amplitude of the fundamental the linear detector may be best, because it gives less increase than other forms of detector when harmonics are present.* Various methods have been suggested for employing valves to give linear detection—*i.e.* detection in which the reading is proportional to the input for all positive values. In one type the valve is used as a diode; in another the plate potential is kept small, and the input is amplified sufficiently before it is applied to the detector, so that it keeps the grid potential of the detector positive over the greater part of one-half of the current cycle.

* I. Wolff, *Inst. Radio Eng., Proc.*, 19, 647, 1931.

Duddell Oscillograph. An oscillograph is an instrument devised for indicating the instantaneous value of an electrical current, and accurately following rapid fluctuations. The first oscillograph * had a heavy-moving iron system, but much lighter arrangements are necessary in order to deal with audio-frequency fluctuations. Duddell † devised an extremely light oscillograph movement of relatively high natural frequency, which is frequently used for recording acoustical wave-forms. In this the alternating current, derived, after amplification, from an electrical microphone, passes through a bifilar strip of stretched phosphor bronze, lying in the gap of a powerful electromagnet. The current passes up one strip and down the other. Fluctuations and alternations of current in the strip cause it to tilt to and fro in the magnetic field, and this motion is detected by means of a small mirror (1 mm. × 0·3 mm.) attached to the centres of both legs of the strip. Suitable optical arrangements, in conjunction with a photographic drum recorder, make it possible to take permanent records, and a time-scale can be superposed upon the record either by interrupting the light by means of a tuning-fork or phonic wheel, or by employing a second oscillograph movement.

As to detail, a narrow tongue of soft iron is arranged between the strips to reduce the magnetic reluctance of the gap. The strip is stretched by a tensioning device until the natural frequency of the system in air is about 10,000 cycles per second. To reduce the instrument to the dead-beat condition, however, the strips are mounted in oil, in channels of which the clearances at the sides are as small as possible. This damping lowers the natural frequency to about 7000 cycles per second.

Whilst the instrument may be used without correction up to about 1000 cycles per second (p. 17), the resonance becomes more and more important when higher frequencies are involved, so that the instrument then requires calibration and the records need careful interpretation.

The instrument is very convenient for making permanent records of acoustical wave-forms and fluctuations, especially in the many cases where absolute accuracy and analysis is not required. It is, however, rather insensitive, so that its use is limited to fairly loud sounds unless specially effective valve amplification is employed.

* Blondel, *La Lumière Elec.*, 41, 401, 1891.
† W. Duddell, Brit. Assoc., 1897; *Electrician*, 39, 636, 1897.

Einthoven String Galvanometer. The Einthoven string galvanometer is more sensitive than the Duddell oscillograph, but at the expense of a lower natural frequency. In it the current is carried by a fine conducting wire or fibre stretched tightly between the poles of a strong electromagnet of field strength about 20,000 lines per sq. cm. The deflections of the fibre when a current passes through it depend upon the direction and magnitude of the current. The motion is observed, by an optical system, through holes in the poles of the electromagnet, the magnification obtained being about 500 times. The optical system consists of a microscope objective in one pole, and in the other pole a second objective serving as a condenser lens to illuminate the fibre for observation, or for projecting an image upon a photographic film.

The frequency of the fibre is controlled by a tensioning screw. Sensitivity depends upon the tension and upon the thickness of the fibre. Silvered quartz or glass fibres with a diameter of 1 or 2 μ ($1\ \mu = 10^{-4}$ cm.) give the highest frequencies ; a quartz fibre of 10 cm. length and 2 μ diameter stretched to its maximum safe tension has a period of about 3500 cycles per second, and is almost dead-beat.

Fibres of copper, aluminium, phosphor bronze, or tungsten may have thicknesses of, say, 20 μ. They are heavier than the quartz, and have less tensile strength, so that the resonant frequencies are much lower, and the fibres are not dead-beat. However, a method of damping fibres has been described by Irwin,* in which a 'resonant shunt'—a condenser C, an inductance L, and a resistance R in series—is employed. When a shunt is properly designed,† records quite free from signs of resonant oscillation have been obtained with even 14 μ phosphor-bronze fibres.

Cathode-ray Oscillograph. Oscillographs—such as the Duddell or Einthoven instruments—which have mechanical moving parts, have resonances in the audio-frequency range, and they are not accurately applicable without troublesome corrections, to frequencies above about 1000 cycles per second. In the cathode-ray oscillograph, however, the moving system is a beam of cathode rays, and is equally sensitive at all frequencies from zero to the highest obtainable electrically.

* Irwin, *Oscillographs*.

† S. Butterworth, A. B. Wood, and E. H. Lakey, *J. Sci. Insts.*, 4, 8, 1926.

The first cathode-ray oscillograph was due to Braun,* who achieved visual observation of the deflections of the rays, and succeeded in getting photographic records by arranging for the cathode rays to continue automatically to retraverse exactly the same path for a considerable time. Dufour,† with an intensified beam of cathode rays, succeeded in getting photographic records with a single traverse of the film. The voltages employed for producing the cathode stream in these instruments were, however, very high, and the instruments, on account of the very high velocity of the cathode stream, were insensitive. Sir J. J. Thomson ‡ therefore employed an electrically heated lime-coated filament as a source of the cathode stream, and reduced the necessary voltage by some twentyfold to 3000 volts.

Very convenient cathode-ray oscillographs for visual observation—and some degree of photographic recording—are now obtainable (Pl. III, p. 112), operating on 300–3000 volts. In these the vacuum tube consists of a pear-shaped bulb a foot or so in length. At the small end of the bulb is located the electrically heated filament which acts as cathode and supplies electrons for the cathode stream. A small perforated shield near the filament protects it from bombardment and disintegration by positive electrical particles which travel in a direction opposite to the cathode stream. A tubular anode or perforated disc, maintained at a potential of 300–3000 volts, attracts electrons from the cathode. Some of these pass right through the tube and, emerging as a well-defined stream on the other side, form the essential cathode stream of the oscillograph. This stream falls on the flattened end of the bulb which acts as a viewing screen. For this purpose it is coated internally with a mixture of calcium tungstate and zinc silicate, which fluoresces where the cathode stream impinges upon it. The pattern traced by the electron stream thus becomes visible and, to a certain extent, photographically active. As a detail it may be mentioned that the tube is highly evacuated, and the air replaced by a small quantity of argon gas which concentrates the electron stream to a point on the viewing screen.

Deflection of the stream by the electrical e.m.f. to be studied is effected by a pair of parallel metal deflecting plates between

* F. Braun, *Wied. Ann.*, 60, 552, 1897.
† A. Dufour, *Comptes Rendus*, 158, 1339, 1914.
‡ J. J. Thomson, *Engineering*, 107, 543, 1919.

which the stream passes. These deflect the stream at any instant by an amount depending upon the sign and the magnitude of the e.m.f. applied to them, and thus cause the fluorescent spot on the screen to move. The sensitivity decreases as the anode voltage increases, and, for an anode voltage of 300 volts, is often about 1 mm. per volt applied to the deflecting plates.

A second pair of deflecting plates in the oscillograph arranged at right angles to those already described can be used to deflect the stream in a direction perpendicular to the other deflection. They are convenient for providing a time-scale. It is now possible with cathode-ray oscillographs, using a specially active screen and having, say, 3000 volts on the anode, to photograph records of frequencies as high as 10,000 per second. A film camera of high speed and a lens of wide aperture are essential. Previous to this improvement A. B. Wood * devised an arrangement in which a drum of photographic plates could be introduced into a robust cathode-ray bulb. It was of course necessary to exhaust the bulb after introducing the plates. References to the history of the cathode-ray oscillograph are given in a paper by J. B. Johnson,† and Wood ‡ has discussed recent developments.

Where a repeated time-scale is required, the deflecting field must increase uniformly with time and then return to zero instantaneously. Such a scale can be obtained from the voltage fluctuations across a neon discharge tube in the following manner. The neon tube, in parallel with a variable condenser C, is connected to a 300-volt battery in series with a high resistance R. When the connections are made the condenser begins to charge up at a rate determined by CR. The rate is linear with respect to time in the early stages of the charging at any rate. When a voltage of about 160 volts is reached across the plates of the condenser and thus between the terminals of the neon lamp, the lamp 'flashes,' and the condenser is discharged. The operation then repeats itself. It is preferable to use a constant current device (such as a voltage saturated diode) instead of the high resistance, as the charging of the condenser is then linear with time throughout. By altering the capacity of the condenser the frequency of the repetitions may be adjusted to be the same as the fundamental frequency of any wave-form under study.

* A. B. Wood, *Phys. Soc., Proc.*, 35, 109, 1923.
† J. B. Johnson, *Frank. Inst. J.*, 42, 687, 1931.
‡ A. B. Wood, *I.E.E., J.*, 71, 41, 1932.

For the study of periodic phenomena against such a linear time-scale it is frequently necessary to introduce a stabilising arrangement which, when the frequency of the time-base has been approximately adjusted, will hold the frequency of the time-base equal to that of the phenomenon under study. This can be done by feeding some of the energy of the wave to be studied into the oscillator providing the time-base. In spite of minor fluctuations of the frequency under observation, the pattern upon the viewing screen then remains stationary.*

With a cathode-ray oscillograph it is not necessary to have a time-base in order to observe whether an audio-frequency e.m.f. is substantially pure. For this purpose the circuit shown in fig. 57 (*b*), p. 167, may be employed. The test e.m.f. is applied to a resistance R and capacity C in series. The potential drop across the resistance is applied to one pair of the oscillograph plates, and that across the capacity to the other pair. If the e.m.f. is of pure sine-wave form these two potentials will differ by 90° in phase, and can be made equal (or approximately equal) in magnitude by suitable choice of R and C. The resultant trace on the viewing screen is a perfect circle or a perfect eclipse. If the wave is not pure, distortion occurs and other shapes are obtained. The presence of a harmonic having an amplitude 10 per cent. of that of the fundamental is normally detectable by visual observation of the oscillograph circle. A similar impurity is detectable when the sine-wave form is observed.

Use of Cathode-ray Oscillograph for obtaining Frequency Response Curves. Oscillographs are normally used for recording the variation, as time changes, of the response of some electrical system which is under observation. Since, however, cathode-ray oscillographs are now part of the normal equipment of acoustical laboratories it is advantageous to adapt them to various uses for which special equipment would otherwise be required. For instance, a frequency scale can be attained on a cathode-ray oscillograph instead of a time-scale, and used in obtaining curves showing the relation between the response of a loud-speaker and the frequency of the exciting current.† The essential features of the arrangement are indicated in fig. 33. An oscillator supplies the loud-speaker with an electrical e.m.f. of constant magnitude

* See also G. I. Finch, R. W. Sutton, and A. E. Tooke, *Phys. Soc., Proc.*, 43, 502, 1931.

† A. H. Davis, *Phil. Mag.*, 16, 408, 1933.

but variable frequency, and the acoustical output of the loud-speaker is measured by a microphone circuit which is uniformly responsive at all frequencies. The rectified output from the microphone circuit is applied to two plates of a cathode-ray

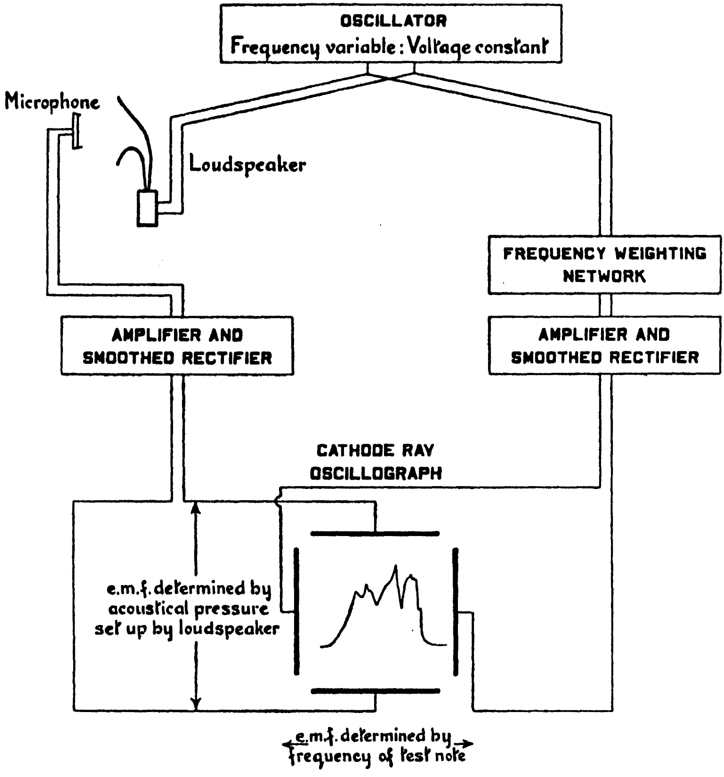
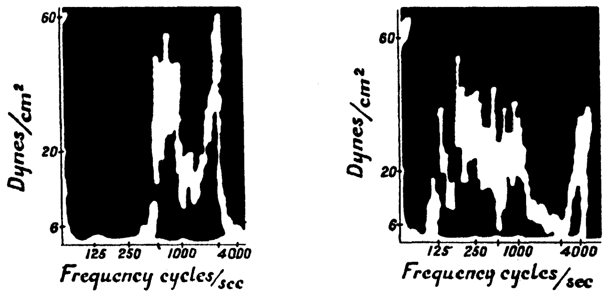


FIG. 33.—Circuit for obtaining frequency response characteristics of loud-speaker by means of a cathode-ray oscillograph

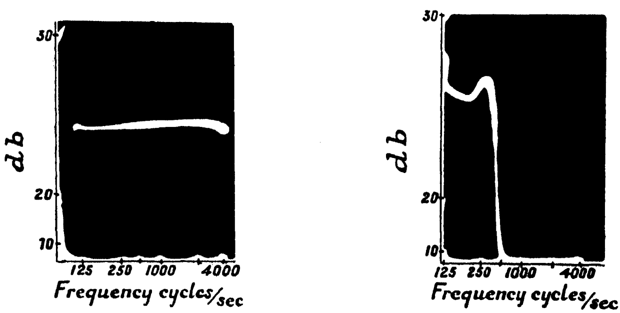
oscillograph, and gives a proportionate vertical deflection to the 'spot' of the oscillograph. The same oscillator supplies current to a weighting network so adjusted that the output, when amplified, rectified, and smoothed, is closely related to the logarithm of the frequency of the exciting current. This output, applied to the other plates of the oscillograph, deflects the 'spot' horizontally to an extent determined by the frequency of the note concerned. As the frequency of the oscillator is varied the oscillograph spot thus traces out a curve showing the relation between the pitch of the note and the output of the loud-speaker as observed at the

position of the microphone. Two such curves are shown in fig. 34. The frequency scale is calibrated by causing the spot to retraverse the screen horizontally (the loud-speaker being switched off) with pauses at standard positions known from the



Small Horn Type Cone Diaphragm Type

(*a*) Acoustical response curves of loud-speakers



Without Filter Circuit With Filter Circuit

(*b*) Electrical amplifier characteristics

FIG. 34.—Frequency-response curves

scale of the oscillator. The vertical response scale is obtained by a somewhat similar procedure when the frequency network is switched off and the microphone amplification varied in a known manner.

The arrangement is not limited to the study of electro-acoustical systems, for the loud-speaker–microphone combination can be replaced by any system which can be adapted so that an electrical output results from an electrical input. As an illustration, fig. 34 (*b*) shows the frequency response curve of an electrical amplifier, and also the response of the same amplifier after it had been fitted with a low-pass electrical filter.

CHAPTER VII

MEASUREMENT OF SOUND INTENSITY

I. General

Physical Relations for Sound Waves in Air. The propagation of a sound wave in air depends upon rapid alternating displacements of the air particles, and is associated with oscillatory changes in the pressure, velocity, temperature, and density of the air. Methods of measuring the intensity of sounds * are therefore largely based on the determination of the amplitudes † of one or more of these fundamental oscillatory phenomena which, for ordinary sounds of a given frequency in air, are proportional to the amplitude of vibration and thus to each other.

The intensity of sound at a point is often expressed in terms of the amplitude of the pressure, displacement, or velocity. It is more formally expressed in energy units, in terms of the rate of flow of sound energy across unit area. The intensity in energy units is proportional to the square of the displacement, pressure, or velocity amplitudes.

In addition to the alternating effects of sound, there is a small steady radiation pressure exerted upon any reflecting object placed in the path of the waves, and tending to drive it away. The radiation pressure upon an infinite normal reflecting surface is numerically equal to twice the energy density of the incident waves, and is independent of their frequency.

The following table gives numerical relations between the various quantities for a plane wave in air. They are deducible from the equations for the propagation of sound waves (p. 52), particularly those for radiation from an infinite plane surface. The density change is defined by the condensation s in the

* C. V. Drysdale (*Phys. Soc. Discussion on Audition*, 1931) has discussed acoustic measurements.

† Smoke particles follow closely the motion of the air in which they are suspended and, for suitable loud sounds, can be used to reveal the amplitude (E. N. da C. Andrade, *Roy. Soc., Proc.*, 134, 445, 1931).

relation $\rho=\rho_0(1+s)$, where ρ_0 and ρ are respectively the normal and disturbed densities of the air at the point of measurement. The range of intensities dealt with in the table practically covers the normal range of hearing (see p. 238). It will be noted, therefore, how very small are many of the quantities associated with a sound wave: in particular it is noteworthy that the ear can detect a sound having a pressure variation as small as one thousand-millionth of an atmosphere, or having an amplitude of vibration of the order of one-thousandth of the wave-length of light.

TABLE IV

Intensity Relations for Plane Waves in Air

Relating to the plane wave $\xi=a\sin\omega\left(t-\frac{x}{c}\right)$ at 0° C. and 760 mm. pressure

Maximum Excess Pressure (p)	Maximum Particle Velocity ($\dot{\xi}$)	Maximum Condensation (s)	Average Rate of Energy Flux per Unit Area (W)	Average Energy per Unit Volume (E)	Temperature Excess (θ)
$p=\frac{a\omega}{c}\gamma P$ $=a\omega\rho_0 c$ $=41{\cdot}7\dot{\xi}$ $=1{\cdot}42\times10^6 s$	$\dot{\xi}=a\omega$ $=\frac{1}{\rho_0 c}p$ $=6{\cdot}28na$ $=\frac{p}{41{\cdot}7}$	$s=\frac{a\omega}{c}$ $=\frac{1}{c}\dot{\xi}$ $=2{\cdot}94\times10^{-5}\dot{\xi}$ $=7{\cdot}05\times10^{-7}p$	$W=\frac{1}{2}\rho_0 ca^2\omega^2$ $=\frac{1}{2\rho_0 c}p^2$ $=p^2/83{\cdot}4$ $=20{\cdot}8\dot{\xi}^2$	$E=\frac{1}{2}\rho_0 a^2\omega^2$ $=\frac{W}{c}$ $=2{\cdot}94\times10^{-5}W$	$\theta=\frac{\gamma-1}{\gamma}\frac{T}{P}p$ $=\frac{(\gamma-1)Tp}{c^2\rho_0}$ $=8{\cdot}11\times10^{-5}p$
dynes/cm.²	cm./sec.		ergs/cm.²/sec.	ergs/c.c.	° C.
10^{-3}	$2{\cdot}40\times10^{-5}$	$7{\cdot}05\times10^{-10}$	$2{\cdot}40\times10^{-8}$	$7{\cdot}05\times10^{-13}$	$8{\cdot}11\times10^{-8}$
1	$2{\cdot}40\times10^{-2}$	$7{\cdot}05\times10^{-7}$	$2{\cdot}40\times10^{-2}$	$7{\cdot}05\times10^{-7}$	$8{\cdot}11\times10^{-5}$
10^{3}	$2{\cdot}40\times10$	$7{\cdot}05\times10^{-4}$	$2{\cdot}40\times10^{+4}$	$7{\cdot}05\times10^{-1}$	$8{\cdot}11\times10^{-2}$

Notes

(i) Maximum amplitude $(a)=\frac{\text{Particle velocity}}{6{\cdot}28\times\text{frequency}}$.

(ii) Radiation pressure $=2E$.

(iii) 1 microwatt $=10$ ergs per second.

(iv) R.M.S. values $=$(Max. values) $\div\sqrt{2}$.

(v) n (frequency) $=\omega/2\pi$.

$\gamma=$ Ratio of specific heats of air.

$P=$ Atmospheric pressure.

$\rho_0=$ Density of air.

$c=$ Velocity of sound (34,045 cm. per sec. at 0° C. and 760 mm. pressure).

(vi) 1 Atmosphere $=10^6$ dynes per sq. cm.

Refractometric Measurements of Sound Intensity. Over half a century ago Toepler and Boltzmann * indicated a refractometric method of measuring the density changes at a node in a column of air in stationary undulation. It employs an optical interference system, and consists in observing the interference between two rays of light from the same source, one of the rays having passed through still air, and the other through air at a nodal point. Since the density of air at a node changes periodically, the interference fringes themselves oscillate. By employing an intermittent source of suitable frequency—the well-known principle of stroboscopic observation—the fringes can be made to appear at rest, or to move sufficiently slowly for their passage to be counted with ease. Raps † improved the method, dispensing with the stroboscopic arrangements, and allowing the image of the fringes to fall upon a slit and thence upon the photographic paper carried by a rapidly revolving drum. In this way the wave-form could be photographically recorded, whether periodic or not. Raps applied the method not only to the node at the end of a closed pipe in stationary undulation, but also to observations in free air near the mouth of a trumpet.

The method is somewhat insensitive. In air at ordinary atmospheric density (ρ_0) there are normally $1{\cdot}695 \times 10^4$ wave-lengths of sodium light per centimetre of path length. If the air is subject to an r.m.s. oscillatory sound pressure of 'p' dynes per square centimetre, the maximum change (dn) in the number of wave-lengths of light per centimetre is given by

$$\frac{dn}{n} = \frac{\mu - 1}{\mu} \frac{\sqrt{2}}{v^2 \rho_0} p \tag{1}$$

where

μ = optical refractive index of air,
v = velocity of sound in air,

whence, substituting appropriate values for air,

$$dn = 4{\cdot}72 \times 10^{-6} p$$

In the range of human hearing, as the intensity of sound varies from the minimum audible to a painful loudness, p varies from

* A. Toepler and L. Boltzmann, *Pogg. Ann.*, 141, 321, 1870.
† A. Raps, *Wied. Ann.*, 50, 193, 1893.

about 1/1000 to 1000 dynes per sq. cm., the value for the loudness of ordinary conversational speech being about 1 dyne per sq. cm. It is clear therefore that even with an optical path length of, say, 20 centimetres, it would require a sound of extreme intensity (10^4 dynes per sq. cm.) to cause an oscillatory fringe shift equal to the distance between successive fringes. Raps refers to the possibility of reflecting the light to and fro several times through the optical path, but remarks on the additional difficulty encountered.

Non-electrical Instruments of the Diaphragm Type. Many instruments for measuring sound have been described in which the sound sets a diaphragm in motion, and the diaphragm movements are measured by mechanical, optical, or electrical means. The electrical devices are of highest importance, and are dealt with separately, in detail. Generally speaking, the non-electrical forms are of historical interest, and are not uniformly sensitive over the frequency range. The reason for this is inherent in the smallness of the motion of a diaphragm under acoustical excitation, except when the diaphragm has resonances in the audible range.

Many early forms were used for recording or reproducing sound rather than for measurement. Reference may be made to the Scott-König phonautograph (1859), and the Edison phonograph (1877). Reproductions of phonograph records on a large scale have been made photographically by Hermann * and Bevier,† and mechanically by Scripture.‡ König (1862) devised the manometric capsule, in which variations of air pressure due to the sound wave are communicated to one side of a membrane, which fluctuates a supply of gas passing the other side on its way to a lighted jet. The height of a flame is thus caused to vary in sympathy with the pressures in the sound wave. König observed the vibrations of the flame in a rotating mirror. Nichols and Merritt § and Brown ‖ photographed the flame pictures obtained when acetylene gas was used as the illuminant. Miller,¶ in his well-known phonodeik, employed a delicate mirror system in conjunction with a diaphragm for

* Hermann, *Pflügers Archiv*, 45, 282, 1899.
† Bevier, *Phys. Rev.*, 10, 193, 1900.
‡ Scripture, *Experimental Phonetics*, Washington, 1906.
§ Nichols and Merritt, *Phys. Rev.*, 7, 93, 1908.
‖ Brown, *Phys. Rev.*, 33, 446, 1911.
¶ Miller, *Phys. Rev.*, 7, 93, 1908.

obtaining photographic records of sounds, and later Dey,* Anderson,† Lapp,‡ and Barton and Browning § described other forms of phonoscope. Garten || observed the motion of a small soap film covering a small cross-shaped opening in a very small chamber to which sound was conveyed by means of a rubber tube. The Low-Hilger ¶ audiometer is a commercial form of mechanical apparatus for the recording of sounds. Essentially it consists of a thin ($2\,\mu$) celluloid diaphragm upon which sound is concentrated by a horn. A small portion of the surface of the diaphragm, on one side of the centre, is rendered reflecting by a kathodic deposit of platinum, and suitable optical arrangements are made which, together with a rotating drum, permit visual observations as well as permanent photographic records of the wave-form to be made.

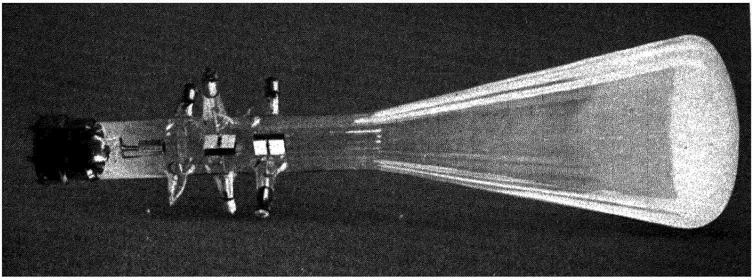
Among diaphragm instruments proposed purely for sound measurement one of the earliest was the vibration manometer due to M. Wien.** In the closed end of a resonator an elastic plate was fixed, and its movements were transmitted to a small rotating mirror, which enabled the movement of the plate to be measured. This instrument may perhaps be regarded as the basis of several later devices such as that described by Sharpe,†† and the Webster phonometer which is described below.

Rayleigh Disc. The well-known Rayleigh disc provides an absolute method of measuring the air particle velocity at a point in the sound field. It consists essentially of a small thin disc of mica or glass, about $\frac{1}{2}$ cm. diameter, suspended by a quartz fibre so that its plane faces are vertical. For convenience in observing its deflections, the disc may be silvered. When such a disc is immersed in a fluid stream it tends to turn its flat side to the direction of the motion. The direction of the displacement is unaltered when the direction of motion of the fluid is reversed, and is therefore maintained, even when the motion of the fluid is alternating.

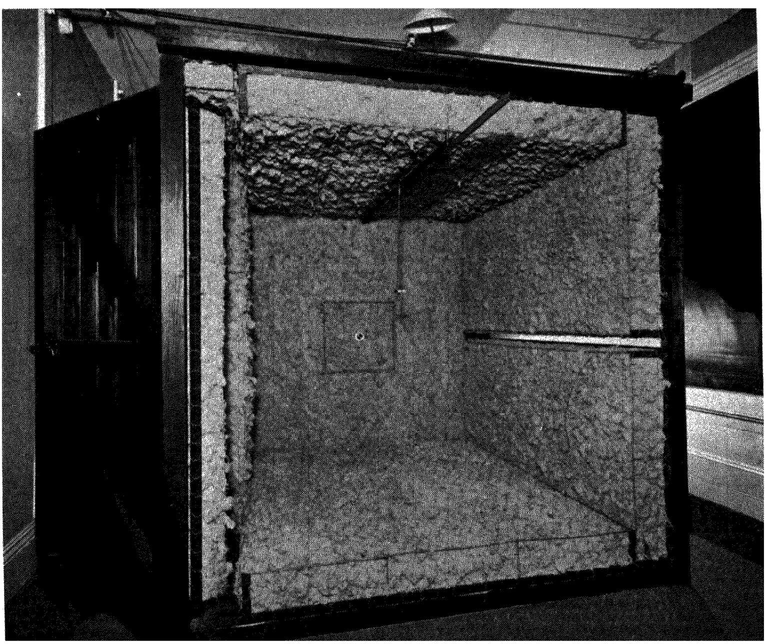
Lord Rayleigh ‡‡ suggested the use of this phenomenon for

* Dey, *Phil. Mag.*, 39, 145, 1920.
† Anderson, *Op. Soc. Am., J.*, 11, 31, 1925.
‡ Lapp, *Op. Soc. Am., J.*, 7, 661, 1923.
§ Barton and Browning, *Phil. Mag.*, 50, 967, 1925.
|| Garten, *Ann. d. Phys.*, 48, 273, 1915.
¶ *Engineering*, 117, 108, 1924.
** M. Wien, *Wied. Ann.*, 36, 834, 1889.
†† Sharpe, *Science*, 9, 808, 1899.
‡‡ Rayleigh, *Phil. Mag.*, 14, 186, 1882.

PLATE III



Cathode ray oscillograph. See p. 103



Simple lagged chamber for acoustical measurements (N.P.L.) See p. 143

One side removed to show door (left), source in end wall, Rayleigh disc support (centre), and scale for Rayleigh disc (right)

[p. 112

acoustical measurements. Grimsehl * constructed a phonometer based upon the principle. König † investigated the theory of the behaviour of the disc and his formulæ for the steady torque F exerted upon a thin circular disc in a frictionless fluid moving with velocity v is

$$F = \tfrac{1}{6}\rho d^3 v^2 \sin 2\theta \qquad (2)$$

where ρ = density of fluid.

d = diameter of disc.

θ = angle between a perpendicular to the disc and the direction of the motion of the fluid.

It may be noted that the calculation involves a knowledge of the atmospheric density, but in England variations of ρ in a warmed room in winter will rarely exceed 3 per cent. when expressed in velocity measurements. If the disc is subject to a simple harmonic alternating velocity the steady deflection is determined by the r.m.s. value of the velocity. There is also a fluctuating force as the velocity rises and falls in each half-period, but the disc is too massive to follow these fluctuations. By putting $v_0 \cos \omega t$ for the velocity in the above equation, and noting that the r.m.s. velocity v is equal to $v_0/\sqrt{2}$, and that $v_0^2 \cos^2 \omega t = v_0^2(1 + \cos 2\omega t)$ the formula becomes

$$F = \tfrac{1}{6}\rho d^3 v^2 \sin 2\theta[1 + \cos 2\omega t] \qquad (3)$$

and the fluctuating part in $\cos 2\omega t$ is revealed.‡

In measuring the steady torque the disc is usually suspended by a quartz fibre and set so that $\theta = 45°$ and $\sin 2\theta = 1$. Deflections are observed by optical methods similar to those employed in observing galvanometer deflections. From the observed angular deflection ϕ the torque F can be calculated from the formulæ $\phi = F/\mu$ if μ, the moment of torsion of the suspending fibre, is known. μ can be found by measuring the periodic time of oscillation of any body of known moment of inertia suspended on the fibre. There is a small correction due to damping. Thus

* Grimsehl, *Wied. Ann.*, 34, 1028, 1888.

† König, *Wied. Ann.*, 43, 43, 1891.

‡ In a somewhat complicated method of using the disc described by Sivian (*Phil. Mag.*, 5, 615, 1928) the amplitude of the sound wave to be measured is modulated with a frequency equal to that of the natural vibration of the suspended disc. The oscillating part of the torque is then involved and swinging excursions of the disc are measured.

if T = periodic time, I = moment of inertia, and D = ratio of two consecutive swings in the same direction, we have

$$\mu = \frac{1}{T^2}\{4\pi^2 + (\log_e D)^2\} \tag{4}$$

Again, if the modulus of rigidity 'n' of the material of a cylindrical fibre of diameter 'a' is known, μ can be calculated from the formula $\mu = \frac{1}{2}\pi a^4 n$. The sensitiveness of the instrument may be inferred from a concrete case. For a disc of 1 cm. diameter in a sound field having a particle velocity of 1 cm. per sec. (r.m.s.), the torque, $2{\cdot}04 \times 10^{-4}$ dyne cm. in magnitude, would cause a deflection of 5 mm. at 1 meter when observed optically, if a suspending quartz fibre of 1 cm. length and 2 μ diameter was employed.

Experiments have been carried out to calibrate the Rayleigh disc by means of continuous air stream or by means of a low-frequency alternating air stream. Good results may be obtained with a continuous stream with fairly heavy and insensitive discs, but any slight lack of symmetry in the disc appears to be serious. In the calibration by means of low-frequency alternating currents of air, Zernov * and also Barnes and West † have carried out work in which a small container surrounding the disc was maintained in oscillatory motion either by being attached to the arm of an electrically maintained tuning-fork or by a motor-driven rocker. In both cases the amplitude of motion was observed by means of a microscope.

It appears from the work of Barnes and West that, as long as resonant frequencies are avoided, low-frequency alternating air currents give results in close agreement with theory, if comparatively heavy discs are used.

A small piece of mirror attached to the mica disc produced no appreciable effect, but resonances at audio-frequencies sometimes occurred owing to vibrations of the disc in its natural modes as a free circular plate. The errors associated with resonance were of the order of 12 per cent., and occurred, for instance, with a mica disc 1·6 cm. in diameter at a frequency of about 1830 cycles per second.

The theoretical formula applies to discs of infinitesimal thickness, and a correction for thickness may be required. If x

* W. Zernov, *Ann. d. Phys.*, 26, 79, 1908.

† A. J. Barnes and W. West, *I.E.E., J.*, 65, 871, 1927.

denotes the ratio of thickness to the diameter of the disc, König deduced the following corrected formula as a result of his experiments:—

$$\text{Torque} = \tfrac{1}{6}\rho d^3 v^2 \sin 2\theta(1 - 0{\cdot}3x) \qquad (5)$$

Zernov derived the following empirical correction $(1 + 2{\cdot}78x - 9{\cdot}05x^2)$ in place of König's bracketed correction term. He used two discs of 1 cm. diameter and thickness 0·04 and 0·1 cm., and one disc of $\frac{1}{2}$ cm. diameter and 0·02 cm. thickness. His formula therefore is based on somewhat meagre information.

For wave-lengths shorter than five times the diameter of the disc errors in velocity measurements appear liable to occur, but for wave-lengths even as low as four-thirds of the disc diameter such errors probably do not exceed about 12 per cent.

The sensitive discs such as are at present under consideration cannot be used except in an atmosphere which is normally practically motionless. Difficulties occur even in small testing cabinets, say 6-ft. cube, possibly owing to convection currents. Such effects are not serious when the disc is quick in movement and nearly dead-beat, but they are troublesome with silvered glass. The difficulty can be overcome by the use of a suitable draught-screen of butter muslin of, say, 10 cm. × 10 cm. × 20 cm. in size, but there is some slight uncertainty as to the effect of the screen on the sound field.

Meyer * has compared thermophone and Rayleigh disc calibrations of a condenser microphone, and obtained agreement. N. Fleming (unpublished) has carried out a similar comparison between calibrations based upon the Rayleigh disc and upon a pistonphone.

Since in practice a Rayleigh disc is often employed for studying sound waves in a tube, it should be noted that transverse resonances may occur in tubes when the diameter is greater than about three-fifths of the wave-length. Skinner † has conducted experiments on the anomalous action of large Rayleigh discs confined in tubes having diameters somewhat greater and somewhat less than a wave-length of the sound concerned.

Webster Phonometer. Webster ‡ evolved a very compact portable tuned phonometer which can be tuned over two octaves, and which, over a limited range of frequency, will compete in

* E. Meyer, *Zeits. f. tech. Phys.*, 7, 609, 1926; *E.N.T.*, 4, 86, 1927.

† C. H. Skinner, *Phys. Rev.*, 27, 346, 1926.

‡ A. G. Webster, *Engineering*, 111, 763, 1921.

sensitiveness with the human ear. It consists essentially of a large tunable resonator in the mouth of which a diaphragm is held by means of three coplanar stretched wires symmetrically disposed (fig. 35). Under the influence of sound of the correct pitch the resonant excursions of air at the mouth set the diaphragm in oscillation, the magnitude of the oscillation being greatest when the supporting wires are also correctly tuned. The motion of the diaphragm is observed through the medium of a steel point attached to the back of the diaphragm, which bears against

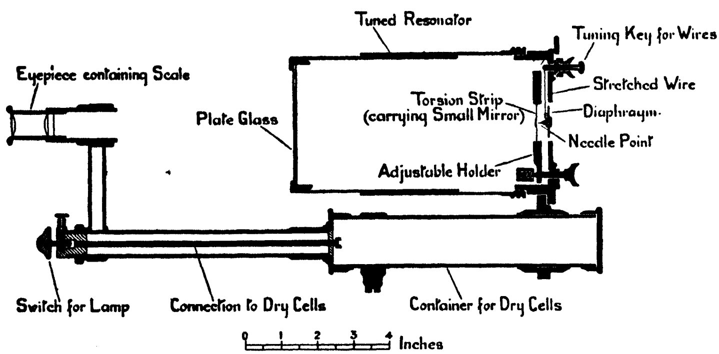


FIG. 35.—Webster's portable selective phonometer
(Davis and Kaye's *Acoustics of Buildings*)

a very small mirror on a torsion strip in such a way that the mirror is rotated when the diaphragm moves.

Light from a small lamp (lighted by a dry battery contained in the base of the instrument) passes through the glass back of the resonator, is reflected by the mirror, and observed by means of an eyepiece. From the swinging excursions of the image of the light, as observed on the scale of the eyepiece, the amplitude of vibration of the diaphragm may be calculated. Webster states that, by reading to 0·1 mm. on the graticule of the eyepiece, disc displacements of 0·00004 mm. can be detected. King * used the instrument in a survey at sea of the sounds of fog-signals.

Hot-wire Microphones. W. S. Tucker and E. T. Paris † have described a hot-wire microphone (fig. 36), consisting of an electrically heated grid of exceedingly fine platinum wire

* L. V. King, *Roy. Soc., Phil. Trans.*, 218, 211, 1919; *Frank. Inst. J.*, 183, 259, 1917.

† W. S. Tucker and E. T. Paris, *Roy. Soc., Phil. Trans.*, 221, 389, 1921.

0·0006 cm. in diameter. To obtain increased sensitiveness and selectivity they placed the grid in the neck of a tunable Helmholtz resonator. The cooling effect of air currents in the neck arising from resonance results in alteration of the electrical resistance of the hot wire. Their instrument actually measured the pressure variation in the acoustic field at the mouth of the resonator, for it is to this pressure variation that the air velocity fluctuations in the neck of the resonator are due.

The microphone is generally used with its axis vertical, the mouth of the resonator pointing upwards. Owing to the natural convection stream upwards, the periodic fluctuations of air in

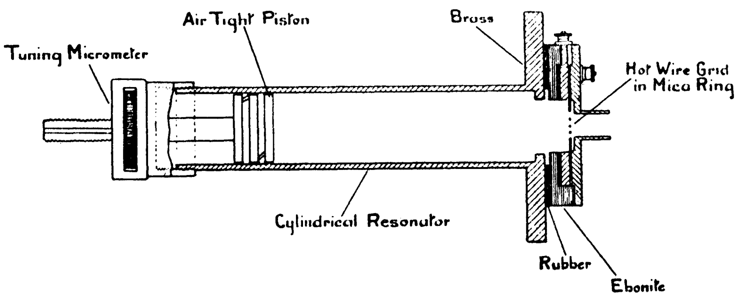


FIG. 36.—Tucker selective hot-wire microphone
(Davis and Kaye's *Acoustics of Buildings*)

the neck under the action of a sound may be somewhat complicated. Calibration may be effected by observing the cooling effect of steady air streams of known velocity U passing into the resonator. It is found that the resistance change is given by

$$R = R_0 + a(U - V_0)^2 + b(U - V_0)^4 \qquad (6)$$

Neglecting the fourth power term, and taking U to be alternating and equal to U sin pt, we find

$$R = R_0 + aV_0^2 + \tfrac{1}{2}aU^2 - 2aV_0U \sin pt - \tfrac{1}{2}aU^2 \cos 2pt \qquad (7)$$

Thus the sound gives rise to (i) a steady resistance change $\frac{1}{2}aU^2$ which is proportional to the intensity of the sound expressed in energy units, (ii) an oscillatory part which exhibits the fundamental frequency of the sound and also its octave, the latter being negligible when the sound amplitude is small. The fundamental oscillatory change in resistance is proportional to U, and is a measure of the amplitude of the sound.

There are thus two methods of using the instrument: (i) a

Wheatstone's bridge method depending upon the steady drop in average resistance of the heated grid which results from the average cooling effect of air currents set up by the sound, and (ii) a method in which the periodic resistance variations which accompany the fluctuations of air velocity are measured with the assistance of valve amplifiers.

The sensitivity of the instrument can be increased by increasing the heating current through the hot wire, and within limits is proportional to the temperature excess of the grid above its surroundings. The selectiveness of the tuned microphones may be illustrated by the fact that an instrument designed for frequencies of about 240 cycles per second gave deflections which fell off by 80 per cent. if the resonator was mistuned by 10 cycles per second.* The instrument is easily disturbed by draughts, and is therefore protected by a 'loofah' screen when used in the open. Owing to the thermal capacity of the wire, it has a practical upper limit of, say, 500 or 1000 cycles per second. Tucker † has given an interesting survey of its applications, and it may be mentioned that the instrument is specially sensitive for low-frequency sounds. When the instrument is used for sound-ranging in warfare, the resonator is excited impulsively by the sound pulse from a distant gun, and a record is obtained of the exact instant at which the sound arrives. By the use of several microphones situated widely apart it is possible to deduce the position of the gun from the various time intervals measured. When the hot-wire microphone is used in acoustical measurements in conjunction with a double resonator (p. 122) the selectivity is very sharp, and sounds barely perceptible to the ear may be measured.

A. v. Hippel ‡ has dealt with the theory of the unresonated hot-wire microphone, considering whether a faithful conversion of sound into electrical energy is possible by its means. He refers to three effects—the oscillatory temperature changes in the air at sound nodes, the oscillatory changes in air velocity between sound nodes, and these velocity changes in combination with convection currents both natural and forced: the first and

* E. T. Paris (*Phys. Soc., Proc.*, 43, 72, 1931) has discussed the acoustical characteristics of singly resonant hot-wire microphones, and of hot-wire microphones with double resonators (*Roy. Soc., Proc.*, 101, 391, 1922).

† W. S. Tucker, *Roy. Soc. Arts., J.*, 171, 121, 1923.

‡ A. v. Hippel, *Ann. d. Phys.*, 75, 521, 1924.

third of these are involved in the theory of the thermo-microphone. J. Freise and W. Waetzmann * criticised Hippel's calculations of the adiabatic thermal effect at the nodes, preferring their own, which was on the verge of publication and which is referred to elsewhere. Hippel † accepted the correction, but pointed out that the adiabatic thermal effect at nodes plays only a negligible part in the action of a thermo-microphone which depends mainly upon convection-current effects. In a further paper Hippel ‡ established the theory of the hot-wire microphone experimentally by varying current, wire thickness, frequency, amplitude, direction, and rapidity of the convection flow. E. G. Richardson § has employed an unresonated hot-wire microphone for exploring the amplitude of sound waves in pipes.

Resonators. As a selective element in the detection of sound of a definite pitch the Helmholtz resonator is of considerable value. It consists of a vessel which is completely closed except for one aperture. By sympathetic vibration of the contained air a resonator amplifies the effect of sounds produced in its neighbourhood. The pressure set up in the enclosure is greater than that set up by the incident sounds in the air outside, and the particle velocity of the air in the neck of the aperture is greater than the particle velocity in the incident sound wave. The resonators are very selective, since damping is very small.

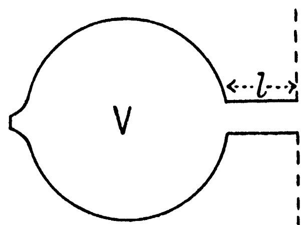


FIG. 37.—Helmholtz resonator

The theory of resonators was first given by Helmholtz, and later simplified by Rayleigh. The mathematical difficulties are reduced if the resonator neck is supposed to be fitted with an infinite flange. In this case an approximate theory may be set out succinctly as follows, by making use of impedance conceptions. A vessel (fig. 37) is supposed to be such that its diameter is small compared with the wave-length of the sound concerned. Under this condition the air in the neck will move practically as a whole, like a piston, and the pressure in the interior at any instant will be almost uniform. If ξ represents the small

* J. Friese and W. Waetzmann, *Ann. d. Phys.*, 76, 39, 1925.
† A. v. Hippel, *Zeits. f. Phys.*, 31, 716, 1925.
‡ A. v. Hippel, *Ann. d. Phys.*, 76, 590, 1925.
§ E. G. Richardson, *Roy. Soc., Proc.*, 112, 522, 1926.

displacement of the piston of air in the neck outwards from its mean position, the resulting force outwards acting on the area A of the neck is $-\gamma P\frac{\partial V}{V_0}A$ owing to adiabatic expansion of the air in the resonator, and may be written $-\rho c^2 A^2\xi/V_0$ since $c^2=\gamma P/\rho$, where ρ is the density of the air, P its undisturbed pressure, and c the velocity of sound. The mass of the piston is $\rho A l$ and, in the absence of impressed forces, the equation of motion of the system is $\rho A l\ddot{\xi}+\rho c^2 A^2\xi/V_0=0$. It is necessary, however, to take account of the reaction of the air outside the neck. The piston acts, indeed, as a source of sound as regards the semi-infinite medium into which it radiates, the specific impedance z_s of which is given on p. 61 as $\rho c\left(\frac{k^2a^2}{2}+i\frac{8ka}{3\pi}\right)$ for the case concerned, where a, the radius of the piston, is small compared with the wave-length $\lambda(=2\pi/k)$. The effect of this impedance upon the equation of motion of the piston (see p. 55) is to increase the effective mass and to supply a damping term. If the resonator is driven by an external force producing over the mouth a pressure represented by the real part of $pe^{i\omega t}$, the full equation of motion becomes

$$\rho A\{l+\alpha\}\ddot{\xi}+A\rho c\frac{k^2a^2}{2}\dot{\xi}+\frac{\rho c^2 A^2}{V_0}=Ape^{i\omega t} \qquad (8)$$

where $\alpha=8a/3\pi=0{\cdot}85a$. (*N.B.*—$A=\pi a^2$; $kc=\omega$.)

So far no allowance has been made for the motion of the air near the neck but inside the resonator. In this case the medium is confined and there is no radiation of sound, so that no addition to the damping term arises. However, assuming the added mass to be the same for this end of the neck as for the other, the added mass indicated above needs to be doubled. The final equation may then be written

$$\rho\left(\frac{l+2\alpha}{A}\right)\ddot{X}+\frac{\rho k\omega}{2\pi}\dot{X}+\frac{\rho c^2}{V_0}X=pe^{i\omega t} \qquad (9)$$

where X is written for $A\xi$, the volume displacement.

The quantity $A/(l+2\alpha)$ represents what is known as the conductivity (K) of the neck, as corrected for the motion of the air outside the ends. The correction $\alpha=0{\cdot}85a$ has been evaluated on the assumption that the air moves in the neck as a rigid piston.

Rayleigh * has shown, by hydrodynamical considerations of the air flow, that the correction α lies between $\pi a/4$ and $8a/3\pi$; that is, between $0{\cdot}785a$ and $0{\cdot}849a$. An empirical value is $0{\cdot}82a$.† If, as is usual in practice, the flange is omitted from the resonator, calculation is more difficult; the empirical value accepted in this case is usually $0{\cdot}6a$.

The equation (9) above is identical in form with that for the motion of a simple damped oscillating system, and its solution in complex form is

$$\dot{X} = iX\omega = \frac{pe^{i\omega t}}{\rho\left[\frac{k\omega}{2\pi} + i\left(\frac{\omega}{K} - \frac{c^2}{V_0\omega}\right)\right]} \tag{10}$$

The quantity in the denominator is equal to $pe^{i\omega t}/\dot{X}$, and is known as the acoustical impedance of the resonator.

The undamped resonance frequency is given by $\omega = c\sqrt{K/V_0}$ and differs but little from the damped resonance, the accurate expression for which is

$$\omega = c\sqrt{K/V_0}/\sqrt{1 + k^2c^2/8\pi^2} \tag{11}$$

The velocity amplification factor for the resonator is the ratio of the velocity in the neck to the velocity ($\xi = p/\rho c$, see p. 58) in the incoming plane wave. It is found to be equal to $4\pi c^2/A\omega^2$, *i.e.* to $\lambda^2/\pi A$. The pressure amplification factor is the ratio of the pressure set up in the bulb $(-\rho c^2 A^2 \xi_{max}/V_0)$ [p. 120] to the pressure $p(=\rho c\xi)$ in the incoming wave. It is found to be $cA/\omega V_0$ times the velocity amplification, and thus is equal to $4\pi c/\omega K$. It is usually less than the velocity amplification.‡

Helmholtz resonators have the advantages that the resonance is very sharp, that the overtones are all relatively high and are not in harmonic relation, and that the dimensions of the resonator need only be small compared with the wave-length of the sound concerned. Helmholtz § gave the theory of resonators with

* Rayleigh, *Roy. Soc., Proc.*, 19, 106, 1870.

† Calculation shows that the conductivity of an ellipse—even when the ellipse is so eccentric that the ratio of its axes is 2 : 1, is only 3 per cent. above that of a circle of equal area. It is thus useful to write a as $0{\cdot}46\sqrt{A}$. The conductivity $A/2a$ is thus $1{\cdot}1\sqrt{A} = 2a$.

‡ E. T. Paris (*Science Progress*, 20, 68, 1925) deals with the amplification of acoustic vibrations by resonators.

§ Helmholtz, *Crelle*, 57, 1–72, 1860.

cylindrical necks. Wertheim * showed that the effect of an open end could be represented by an addition α to the length independent, or nearly so, of the length and of the wave-length. Sondhauss † experimented with resonators with and without necks, and gave formulæ which are practically special cases of the formula given above.

Resonators are of value where it is desired to detect the presence of a note of definite pitch in the presence of other sounds. A series of resonators is therefore of value in the analysis of sound. Helmholtz used such a series in conjunction with the ear, by applying to the ear a small open pip at the base of the resonator. For detection, use has also been made of hot wires (p. 116) or Rayleigh discs in the neck (p. 123),‡ or of tuned mirror-carrying reeds in the mouth (Fournier d'Albe).

It may be mentioned here that when a source of sound such as a tuning-fork is placed near the mouth of a resonator, the intensity of the sound is enhanced. It must be clearly realised that the additional energy does not come from the resonator, but is merely the result of some interaction between it and the source, which facilitates emission of energy by the latter. The presence of the resonator increases the emission $1/k^2b^2$-fold in the case of a simple source, and $1/k^4b^4$-fold in the case of a double source, where k has its usual meaning $(2\pi/\lambda)$ and b (small) is the distance of the source from the aperture of the resonator.§ This ratio is greatest for the double source, *i.e.* for the source which is normally an inefficient radiator of sound (p. 59).

The effect of a resonator upon a sound field at a distance from a source to which it is tuned, is noteworthy. The resonator may absorb sound from a considerable area of the wave-front, diverting energy which at its maximum may extend over an area of λ^2/π, where λ, the wave-length of the sound concerned, is large compared with the dimensions of the resonator.§

Double Resonators.‖ When two resonators are connected in series the amplification is greater, and the combination has more

* Wertheim, *Ann. d. Chim.*, 31, 385, 1851.

† Sondhauss, *Pogg. Ann.*, 81, 235 and 347, 1850; 140, 53 and 219, 1872.

‡ See also Edwards, *Phys. Rev.*, 32, 23, 1911; Hewlett, *Phys. Rev.*, 35, 359, 1912.

§ Lamb, *Dynamical Theory of Sound*, p. 274, 1925.

‖ Rayleigh, *Sound*, 2, 189, 1896 ; *Phil. Mag.*, 36, 231, 1918.

than one resonant frequency. C. V. Boys * suggested this arrangement and pointed out its great sensitivity. Rayleigh has shown that if two reservoirs communicate with each other and with the external air by necks, and if the volumes and conductivities are as shown in fig. 38, then the formula for the natural pulsatances of the combination is

$$\omega^4 + \omega^2 a^2 \left\{ \frac{K_1 + K_2}{V} + \frac{K_3 + K_2}{V_1} \right\} + \frac{a^4}{VV_1} \{ K_1 K_3 + K_2 (K_1 + K_3) \} \quad (12)$$

If one end is closed K_3 is zero. If two ends are closed K_1 also is zero, and the formula gives the natural frequencies of two closed connected vessels. It is to be noticed that a double resonator closed at one end has two resonant frequencies, and that the inner resonator should be small compared with the outer for greatest amplification.

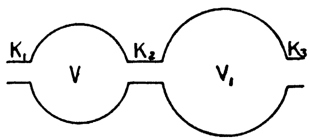


FIG. 38.—Double resonator

Paris † has extended the theory to cases in which a tube or a horn replaces one of the resonators.

Rayleigh Disc and Double Resonator. Where a very selective instrument is required in the measurement of sound intensity,

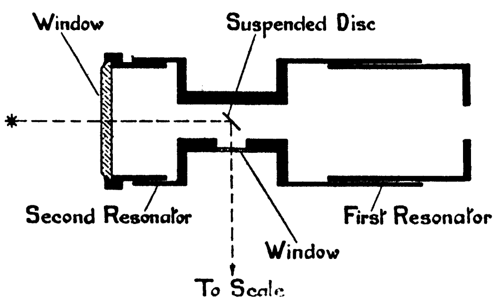


FIG. 39.—Rayleigh disc and double resonator
(Davis and Kaye's *Acoustics of Buildings*)

the Rayleigh disc may be used in conjunction with a double resonator (fig. 39). When the instrument is exposed to the influence of a note having the pitch to which it is tuned, the pronounced oscillatory excursions of the air set up in the neck are measured by means of a Rayleigh disc suspended there. Glass windows are provided in the neck and in the end of the

* C. V. Boys, *Nature*, 42, 604, 1890.
† E. T. Paris, *Roy. Soc., Proc.*, 101, 393, 1922; *Phil. Mag.*, 48, 769, 1924; 36, 231, 1926.

resonator so that the deflection of a beam of light reflected from the disc may be observed by suitable optical arrangements. The instrument is extremely sensitive and will measure sounds so feeble as to be barely perceptible by the ear. Its selectiveness is useful when it is desired to avoid the effects of any components of a sound other than that of chosen pitch.

Resistance Thermometer. In measuring the small adiabatic oscillations of temperature which occur at pressure loops in a stationary sound wave, resistance thermometers of fine wire or foil have been employed. K. Neuscheler * used a strip-resistance thermometer for measuring the adiabatic temperature changes just inside the stopped end of an organ pipe having a fundamental frequency of 33 vibrations per second. By employing an Einthoven galvanometer, he recorded wave-forms showing whether the pipe was speaking in a simple or in a compound tone. Temperature changes of 0·13° C. were measured, corresponding to amplitudes of vibration of 0·058 cm. K. Heindlhofer † gave a mathematical theory of a proposed method of determining the absolute intensity of a sound from its thermal effect. A gold leaf about 2 μ thick, in series with a coil and constant e.m.f., was proposed, the amplitude of the sound being deduced from the alternating current in a secondary coil. In a second paper Heindlhofer † corrected part of the earlier theory and verified the later theory experimentally, using a pistonphone arrangement for producing known pressure differences in the frequency range 5–32 vibrations per second.

J. Friese and E. Waetzmann ‡ explored the stationary sound waves in a tube for a frequency of 800 vibrations per second by relative temperature measurements at the nodes and loops by a resistance thermometer (of 4 μ wire 18 mm. long) in conjunction with a valve amplifier. Later (1925) allowance was made on theoretical grounds for the fact that the resistance thermometer does not follow the full temperature fluctuations of the air, particularly at higher frequencies, and absolute measurements were made. Simultaneously, the pressure amplitudes were measured by means of a membrane manometer, and the temperature amplitudes calculated from these proved to be in satisfactory agreement with those given by the resistance thermometer for

* K. Neuscheler, *Ann. d. Phys.*, 34, 131, 1911.

† K. Heindlhofer, *Ann. d. Phys.*, 37, 247, 1912; 45, 259, 1914.

‡ J. Friese and E. Waetzmann, *Zeits. f. Phys.*, 29, 110, 1924; 34, 131, 1925.

frequencies from 400–1000 cycles per second. In their theory of the resistance thermometer as used for absolute acoustical measurements J. Friese and E. Waetzmann * calculated that when a fine platinum wire of radius R is placed in air, of which the temperature varies with a frequency n, the ratio V of the mean temperature amplitude of the wire to that of the air is given by the following table:—

TABLE V

nR^2	V
0×10^{-6}	1·00
5	0·695
10	0·486
20	0·350
30	0·219
40	0·174
50	0·145

Their confirmation of this theory has been referred to above.

Radiometer. The positive radiation pressure exerted by sound waves upon an infinite wall normal to the waves has been calculated by Rayleigh † to be

$$P = (\gamma + 1)E \tag{13}$$

where E is the volume density of the incident wave-train, and γ is the ratio of the specific heats of the gaseous medium. In the case where the gas obeys an isothermal instead of an adiabatic law of expansion, the equation becomes $P = 2E$.

Rayleigh's proof involves general hydrodynamical equations, but Poynting ‡ and Larmor § derived the latter expression by a relatively simple treatment. Larmor's proof assumes the reflecting wall to be moving normally towards the oncoming waves with a small velocity. In consequence the waves are more crowded after reflection than before, or, in other words, the pitch of the sound is raised on reflection. Since the energy density of sound waves increases as the square of the frequency of the waves, the energy density of the sound is increased on reflection from the

* J. Friese and E. Waetzmann, *Zeits. f. Phys.*, 31, 50, 1925.
† Rayleigh, *Phil. Mag.*, 3, 338, 1902; 10, 364, 1905.
‡ J. H. Poynting, *Phil. Mag.*, 9, 393, 1905.
§ Larmor, *Ency. Brit.*, 11th ed., 22, 786, 1911.

moving wall. The resulting increase in wave-energy per unit time is equal to the work expended in moving the wall a distance v against the radiation pressure P, where v is the velocity of sound in the medium. When this equality is expressed in algebraical form it is found that $P=2E$. For further details of Larmor's

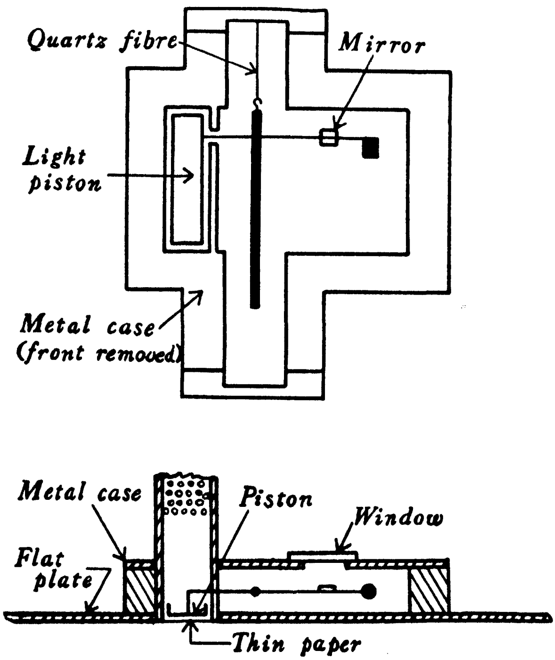


FIG. 40.—Acoustical radiation manometer

treatment and its relation to Rayleigh's calculation an article by Weaver * is of value.

Altberg † described an instrument for measuring the radiation pressure exerted by sound waves of high frequency and considerable amplitude, such as were produced by a mechanically excited Kundt's tube. The radiation pressure measured was about ¼ dyne per sq. cm. Later Altberg and Holtzmann ‡ used an instrument about 2000 times as sensitive; again, however, they confined themselves to sounds of very high pitch—5000 to 22,000 cycles per second. This later instrument is illustrated in fig. 40. It

* W. Weaver, *Phys. Rev.*, 15, 399, 1920.
† W. Altberg, *Ann. d. Phys.*, 11, 405, 1903.
‡ W. Altberg and M. Holtzmann, *Phys. Zeits.*, 26, 149, 1925.

consisted essentially of a large vertical reflecting plate having a rectangular hole through the centre. A light piston carried on the arm of a delicate torsion balance was mounted within this hole with a minimum clearance (½ mm.) consistent with free motion. The force exerted by the incident waves upon the piston was calculated from the deflection of the torsion balance as observed with the usual mirror arrangements. To prevent interference from thermal air currents the torsion system was entirely enclosed in a metal casing; a thin cover of cigarette paper over the hole in the reflecting plate protected the system from draughts without being thick enough to offer appreciable resistance to the passage of sound.

Altberg found the radiation pressure to be proportional to the square of the oscillatory pressure as measured by a vibration manometer. Zernov * compared the radiation pressure exerted at the closed end of a resonant pipe with that deduced from readings of a vibration manometer, and obtained agreement with Rayleigh's formula within 2 per cent., *i.e.* within the limits of error of the observations. His sound was of frequency 512 cycles per second, and the radiation pressure measured was about ½ dyne per sq. cm.

Clearly sound radiometers require very loud aerial sounds for their operation. They have been used in the form of a simple disc of metal upon a torsion balance, for the measurement of high-frequency radiation under water.†

* W. Zernov, *Ann. d. Phys.*, 21, 131, 1906.
† R. W. Boyle, *Roy. Soc. Canada, Proc.*, 3, 167, 1925.

CHAPTER VIII

MEASUREMENT OF SOUND INTENSITY

II. Electrical Microphones

Electromagnetic Microphones. An electrical microphone, combined with a device for measuring the alternating e.m.f. generated, is a most valuable instrument for acoustical measurement. There are, of course, many types of microphone. The carbon-granule type, as used in the transmitter of an ordinary telephone, is very inconstant and the e.m.f. generated is not strictly proportional to the displacement. High quality instruments are available, however, in which the diaphragm is so tightly stretched that it is free from resonances in the ordinary speech range, and in which the diaphragm carries two carbon buttons, one on each side—a device which practically eliminates non-linear distortion. Naturally in the double-button type only one side of the diaphragm is exposed to the sound. An ordinary electromagnetic telephone receiver (or loud-speaker unit) acts as a microphone under acoustic excitation, the moving diaphragm generating an alternating electromotive force in the windings around the magnet. It forms a more constant instrument for measurement purposes than a carbon microphone.

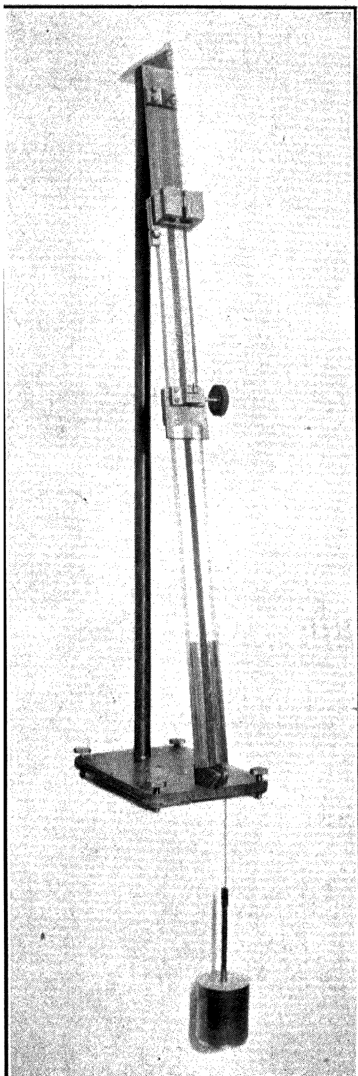
The theoretical equations for a telephone receiver when it is used as a microphone may be written

$$\mathrm{L}\frac{di}{dt}+\mathrm{R}i+\mathrm{M}\frac{du}{dt}=0 \qquad (1)$$

$$m\frac{d^2u}{dt^2}+r\frac{du}{dt}+su-\mathrm{M}i=\mathrm{F} \qquad (2)$$

where F is the applied force upon the diaphragm due to the incident sound, and the other symbols have the same meaning as those given in connection with the telephone receiver on p. 75.

PLATE IV



Wire Sonometer (Dye). See p. 162

The solutions of the equations are

$$\dot{i}=\frac{MF}{zZ}; \qquad \dot{u}=\frac{(R+j\omega L)F}{zZ} \tag{3}$$

and the microphone acts as a source of e.m.f. of magnitude MF/z, having internal impedance Z equal to the usual free impedance of the instrument.

If the microphone is in series with an external resistance, R must be taken to be the sum of the internal and external resistances ($=R_i+R_e$). The e.m.f. available externally is given by e.m.f.

$$\dot{i}R_e=\frac{MF}{z}\left(\frac{R_e}{Z+R_e}\right)=\frac{MF}{z} \text{ approx.} \tag{4}$$

if R_e is large compared with the impedance of the microphone, and

$$\dot{u}=\frac{F}{z}\left\{\frac{R_e+R_i+j\omega L}{R_e+R_i+j\omega L+M^2/z}\right\}=\frac{F}{z} \text{ approx.} \tag{5}$$

if the motional impedance is small. Since the motional impedance of the instrument is $Z_m=M^2/z$, we may write e.m.f. $=FZ_m/M$. The sensitivity of the microphone is therefore calculable if M is known—it is easily obtainable for moving coil instruments (see p. 78)—provided a measurement is made of the motional impedance Z_m of the receiver or of the mechanical impedance z of the diaphragm (see p. 74).

It must be commented that the moving system of an ideal telephone receiver should satisfy somewhat different requirements from that of an ideal microphone. For in use the receiver acts into a small enclosed cavity between the ear and the diaphragm, and the pressure developed within this enclosure per unit of current in the receiver coil should be independent of frequency if the coil impedance is constant. At low frequencies the pressure developed is very nearly proportional to the amplitude of vibration of the diaphragm. At high frequencies the acoustical impedance of the ear becomes important; this, however, varies greatly from ear to ear, and it appears that, on an average, constancy of the amplitude of motion of the diaphragm per unit current throughout the frequency range is still the best condition to attain in the design of a receiver. On the other hand, in the case of an electromagnetic microphone the voltage generated is proportional to the velocity of the diaphragm; consequently the diaphragm of a

uniformly sensitive microphone should have, at all frequencies, the same velocity for unit exciting pressure in the sound wave. Expressed in another way, the mechanical impedance (force per unit velocity) of a microphone diaphragm should be the same at all frequencies, whereas that of a receiver should be inversely proportional to the frequency.

Generally speaking, commercial types of microphones exhibit resonance within the range of acoustical frequencies. The methods of avoiding such resonance effects are to subject the diaphragm to either very high or low tension, so as to bring the resonant frequency well above or well below the speech or music range.

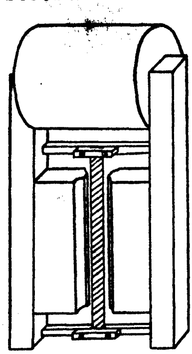


FIG. 41.—Ribbon microphone

Among the microphones with lightly controlled moving systems, the Marconi-Sykes microphone * may be mentioned. It is of the electrodynamic type. A thin annular coil of aluminium wire is loosely supported in the gap of a 'pot' electromagnet. The support is almost free from restraint—in one type of the instrument it is a trifilar suspension of silk thread, and in another is merely an annular pad of cotton-wool.

Another microphone of the same class is the 'strip' or 'ribbon' microphone of Gerlach and Schottky,† which is illustrated diagramatically in fig. 41. It consists essentially of a thin unstretched strip or corrugated ribbon (3 μ) of aluminium foil situated in the magnetic field of a strong electromagnet. The movement of the strip under the action of incident sound waves induces in it measurable electrical currents. The field of the electromagnet is so strong that the electromagnetic damping of the motion of the strip equals the damping by radiation—an optimum condition for efficiency of the receiver. Some microphones of the strip type are said to have a uniform characteristic over a wide frequency range—50–10,000 cycles per second. Gerlach made absolute measurements of sound intensity with this type of instrument by means of the compensation method, which he devised, and which has since been applied to other types of microphone. Gerlach passed through the strip an alternating current of suitable frequency, magnitude, and phase, so that the forces upon the

* H. J. Round, *Wireless World*, 15, 26, 1924.
† E. Gerlach and W. Schottky, *Phys. Zeits.*, 25, 672, 675, 1924.

strip due to the pressure of incident sound waves were exactly compensated, and the strip was retained at rest—a condition which was detected by an auxiliary device. The acoustical forces upon the strip were then deduced from the compensating electrical forces arising from the measured current in the known magnetic field.

H. F. Olson,* who gives the theory of the ribbon microphone, has described results obtained with a microphone of this type.

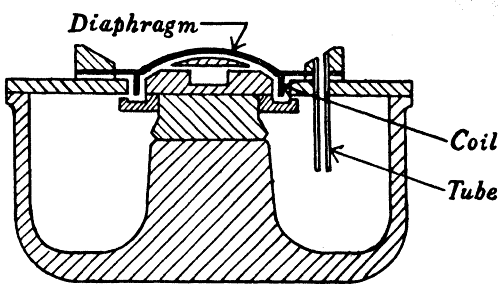


FIG. 42.—Moving-coil microphone

On calibration the microphone was found to have a response practically independent of frequency within the range covered (100–5000 cycles per second). The instrument may be used both with and without a baffle. When used without a baffle the acoustical air pressures have ready access to both sides of the microphone ribbon, and in consequence the microphone exhibits certain directional characteristics. To a reasonable degree of approximation the response in a direction inclined at an angle θ to the normal to the strip is proportional to cos θ. This relation is practically independent of frequency—except, of course, for frequencies such that the acoustic path from the front to the rear of the ribbon is comparable with the wave-length of the sound—so the directional characteristics of the microphone do not produce frequency distortion. In measuring the pressure difference between the two faces of the ribbon, the microphone measures the pressure gradient in the sound wave and thus the particle velocity. The instrument has advantages in outdoor tests of loud-speakers, in that its directional characteristics allow reflections from the ground to be ignored in the measurements.†

* H. F. Olson, *Acous. Soc. Am. J.*, 3, 56, 1931.
† I. Wolff and F. Massa, *Acous. Soc. Am. J.*, 4, 217, 1933.

Wente and Thuras * have described a high quality moving coil microphone (fig. 42), which is similar in general construction to the receiver described on p. 41, but has the cap omitted so that the diaphragm may be exposed to the incident sound waves. It has certain details designed to ensure fairly uniform response over the frequency range when the instrument is used as a microphone. In particular, a tube connecting the space at the rear of the diaphragm with that in front is intended to increase the low-frequency response to a desired degree. The original paper should be consulted for an explanation of the design.

Condenser Microphones. Among the microphones with a

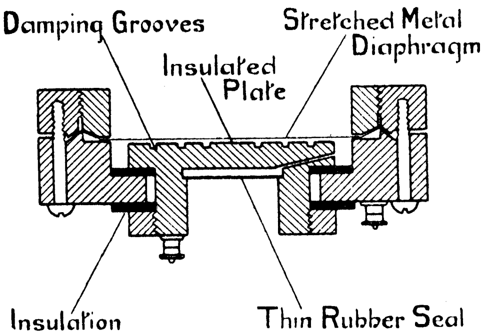


FIG. 43.—Diagram illustrating Wente's condenser microphone
(Davis and Kaye's *Acoustics of Buildings*)

stretched metal diaphragm is the electrostatic or condenser microphone described by Wente.† An early form of the instrument is illustrated in fig. 43. It consists essentially of a tightly stretched thin steel diaphragm separated from a metal back plate by about 1/1000-inch air-gap. The plate and the diaphragm form the two plates of an electrical condenser, which is charged by being permanently connected to a battery of about 200 volts through a high resistance. When the diaphragm vibrates under the action of sound, the capacity of the condenser varies, and an alternating electromotive force is set up. Annular grooves cut into the face of the back plate give the diaphragm the requisite degree of damping. Moisture is excluded from the space surrounding the back plate by means of a thin rubber sheet, which, being flexible, maintains the pressure within the air-gap of the instrument sub-

* E. C. Wente and A. L. Thuras, *Acous. Soc. Am. J.*, 3, 44, 1931.
† E. C. Wente, *Phys. Rev.*, 10, 39, 1917.

stantially atmospheric. Owing to the tight stretching of the metal diaphragm, its natural frequency is very high—some 10,000–20,000 vibrations per second, and so the microphone —although somewhat insensitive—is almost uniformly responsive to sounds over a wide range of acoustical frequencies. The microphone changes but little with time and atmospheric conditions, and is used with amplifiers which themselves, when properly designed, will maintain constancy for a long period.

The microphone is usually used in conjunction with a valve amplifier, and the electrical connections are shown in fig. 44. It may be mentioned that a condenser microphone having a

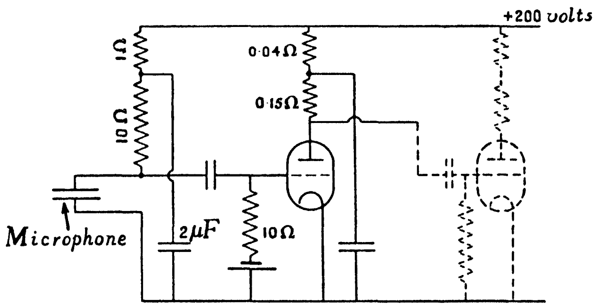


FIG. 44.—Condenser microphone circuit

capacity of, say, 300 micro-microfarads has an impedance of $6\frac{1}{2}$ megohms at a frequency of 50 cycles per second, and 0·064 megohm at 5000 cycles per second. Hence a resistance of the order of 10 megohms is necessary in the associated connector unit, to ensure that the input to the valve shall not fall off at low frequencies. Long leads from the microphone to the amplifier not only give trouble from the stray e.m.f.'s, which are very readily picked up, but also act as an appreciable shunt on the microphone: for it requires only a very few feet of ordinary leads to have a capacity greater than the capacity of the microphone. As a consequence the first stage of amplification is frequently incorporated in a housing as small as possible, attached to the back of the microphone.

Theory of the Condenser Microphone. When the diaphragm of a condenser microphone moves under the action of an incident sound the capacity C varies. Assuming the diaphragm to be replaced by an equivalent piston of area A at a distance $x_0 + x \sin \omega t$ from the back plate, where x_0 is the equilibrium distance, we

have, if K is the dielectric constant of air,

$$C=\frac{KA}{4\pi(x_0+x\sin\omega t)}=\frac{KA}{4\pi x_0}\left(1-\frac{x}{x_0}\sin\omega t\right)*\text{ approx.} \tag{6}$$

$$=C_0+C_1\sin\omega t\text{ say}$$

where C_1 is proportional to x if x/x_0 is small.

If the microphone is set up in series with a polarising potential E in series with a resistance R, this variation of capacity results in an alternating e.m.f. across R of

$$v=\frac{EC_1R}{C_0\sqrt{R^2+\left(\frac{1}{\omega C_0}\right)^2}}\sin(\omega t+\phi) \tag{7}$$

$$=\frac{Ex}{x_0}\sin(\omega t+\phi)$$

if R is sufficiently large and $C_1/C_0(=x/x_0)$ is small.

Equation (7) shows that the microphone acts as a generator having an internal impedance of $1/\omega C_0$ and giving an open circuit e.m.f. of $E(C_1/C_0)\sin(\omega t+\phi)$. The last equation shows that the amplitude of the alternating voltage across the resistance R is independent of frequency and proportional to the alternating diaphragm *displacement*. The e.m.f. thus accurately reproduces the wave-form of the diaphragm motion, and, so far as the diaphragm reproduces the pressures of the sound wave, reproduces the wave-form of the sound itself. The conditions are that x/x_0 be small—otherwise octaves are introduced—and that R shall be large compared with the impedance of the microphone.

As regards the dynamical theory of the condenser microphone, it is to be noted that the presence of the back plate has an important effect on the apparent stiffness and damping owing to two kinds of motion which occur in the air film between it and the diaphragm. There is simple compression—which adds to the apparent stiffness of the diaphragm—and a lateral escape of air towards the edge. This latter takes place in confined conditions, and appreciable damping occurs owing to viscosity of the air. At low frequencies the air has ample time to escape, and the

* Further terms in $\sin^2\omega t$ lead to the production of the octave of the fundamental in the e.m.f. at the terminals when large displacements of the microphone diaphragm are involved.

stiffness effect is small. At high frequencies very little air can escape laterally, and the chief effect is an increased stiffness. Fig. 45 shows resistance and total stiffness factors calculated for a condenser microphone used by E. C. Wente. The high resistance at low frequencies and high stiffness at high frequencies are clearly shown.

It is desirable in a condenser microphone to adjust the resistance suitably. This can be done by altering the separation

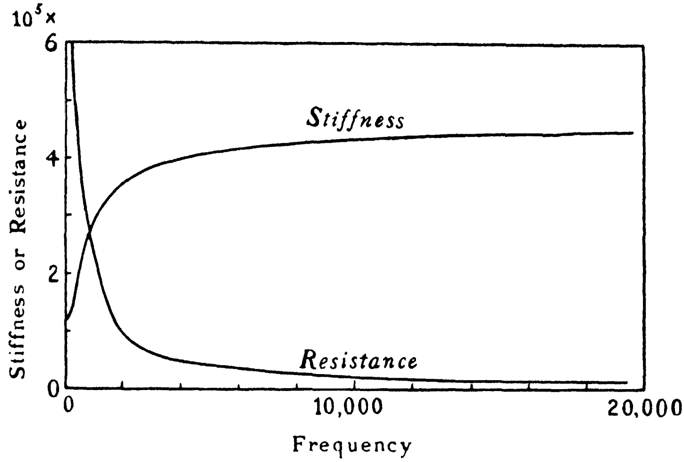


FIG. 45.—Resistance and stiffness factors of Wente's condenser microphone

between the plate and the diaphragm. Unfortunately increasing this separation reduces sensitiveness. Consequently in practice a design is employed which makes the most of the close adjustment, but facilitates the escape of air from the film by perforating the back plate or cutting deep grooves in it so that the proper combination of resistance and added stiffness is obtained. The addition of the grooves, by its effect on the stiffness, decreases the frequency of diaphragm resonance and increases particularly the response at low frequencies, where the stiffness due to the air film is often the greater part of the total stiffness.

Types of Condenser Microphone. In Wente's microphone and in other types the diaphragm is often situated at the bottom of a shallow cavity, and resonance of this cavity leads to an increase of pressure on the diaphragm. A. J. Aldridge * pointed out the importance of the cavity when the microphone is used in free

* A. J. Aldridge, *P.O. Elec. Eng., J.*, 21, 223, 1928.

air. West * and Ballantine † have calculated the effect. It may be mentioned that a cavity ½ in. deep in front of a diaphragm of 1·6 in. diameter leads to a resonance at about 3000 cycles per second, the response being some 2–3 times as great as for a sound of frequency 1000 cycles per second. Doubling the length of the cavity lowers the resonance point to 2000, but doubles the height of the amplitude resonance peak.

Oliver ‡ has designed a microphone of the Wente type in which

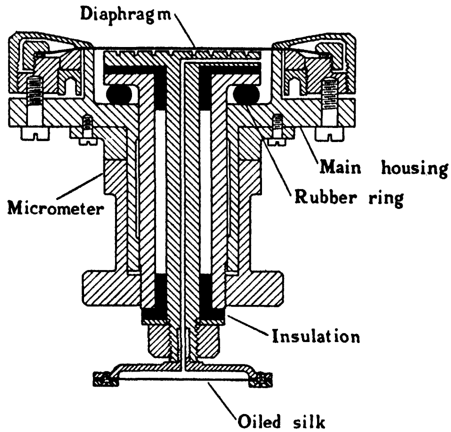


FIG. 46.—Improved condenser microphone (Oliver)

the diaphragm is only very slightly recessed, and is practically flush with the case (fig. 46). The instrument is used in an oval shield which fits over the whole of the back of the instrument.

To minimise the distortion of the sound field which arises when a large microphone is introduced, W. West § used a microphone of 1¼ in. diameter, and E. Meyer ‖ also employed a condenser microphone of very small dimensions. H. C. Harrison and P. B. Flanders ¶ have described a laboratory model of a Wente type of condenser microphone so small (about ½ in. diameter) that reflection from it is negligible (p. 138), and the pressure is

* W. West, *I.E.E., Proc.*, 5, 145, 1930.

† S. Ballantine, *Inst. Radio Eng., Proc.*, 18, 1206, 1930; see also H. C. Harrison and P. B. Flanders, *Acous. Soc. Am. J.*, 4, 451, 1932.

‡ D. A. Oliver, *J. Sci. Insts.*, 7, 113, 1930.

§ W. West, *I.E.E., J.*, 67, 1137, 1929.

‖ E. Meyer, *E.N.T.*, 4, 203, 1927.

¶ H. C. Harrison and P. B. Flanders, *loc. cit.*; see also W. M. Hall, *Acous. Soc. Am. J.*, 4, 83, 1932.

substantially the same (*i.e.* to 1 or 2 db.) at all points on the diaphragm even when the sound is incident upon it at grazing incidence. For the latter purpose the diameter needs to be less than half the wave-length of the highest note concerned, and the microphone they have described is said to respond uniformly up to a frequency of 10,000 cycles per second. The impedance of the microphone is specially high, and short leads to the first stage of amplification are essential. The first stage is therefore contained in a tube about 7½ in. long attached to the back of the microphone, the diameter of the tube (0·8 in.) being very little greater than that of the microphone. The whole therefore resembles a rod about 7½ in. long and of 0·8 in. diameter, one end of which constitutes the microphone diaphragm. A shielded cable of special construction connects the unit to the second stage of amplification.

The Riegger condenser microphone (described by Trendelenburg *) is of a different construction from the Wente type, particularly as regards the method of supporting the diaphragm. The vibrating system consists of thinnest metal foil, and is sandwiched between two silk membranes. The metal foil with its silk membranes is situated between two fixed plates, of which that in front is slotted to admit sound. The damping due to the two air films is very great. The natural frequency, determined entirely by the cushion action of the enclosed air space, is very high since the mass of the system is extremely small. In use the metal foil and the metal plate form a condenser which varies in capacity as the diaphragm moves under the action of sound waves. The Siemens condenser microphone, of the Riegger type, is claimed to cover the range 50–8000 cycles per second. In use it is not polarised, and unlike the Wente type is not used as the source of small audio-frequency e.m.f.'s corresponding electrically to fluctuations in pressure in the incident sound waves. Indeed the microphone is used as a tuning condenser in a high-frequency circuit. Audio-frequency variations in its capacity result in audio-frequency variations in the high-frequency oscillations, *i.e.* in a modulated high-frequency current.

The complete arrangement is as follows: Oscillations from a separate high-frequency oscillating valve circuit are passed, *via* the resonant microphone circuit, to the grid of a rectifying valve.

* F. Trendelenburg, *Zeits. f. tech. Phys.*, 5, 236, 1924; see also Backhaus and F. Trendelenburg, 7, 630, 1926.

The circuit is arranged so that the condenser microphone circuit is tuned (nearly, but not quite) to the frequency of the high frequency oscillator. Then, when the tuning of the resonant microphone circuit fluctuates consequent upon the arrival of sound waves at the microphone, considerable fluctuations are applied to the grid of the rectifier valve, and corresponding audio-frequency fluctuations are observed in the rectified current.

The procedure adopted in mis-tuning the frequencies of the oscillator and microphone circuits to the desired degree, is to keep constant the tuning of one circuit whilst the other is varied. The reading of an ammeter in the rectifier circuit then varies in the manner indicated in fig. 47 as the tuning of the circuit passes through the resonance with the other. The value of the ammeter reading at resonance (A) is noted, and the tuning then altered until it falls to half this value, corresponding to a point P on the curve. At this point small fluctuations in the frequency of the microphone circuit cause proportional variation in the anode current.

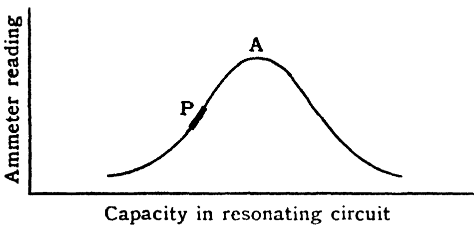


FIG. 47.—Adjustment of high-frequency circuit

Absolute Calibration of Microphones. A high quality microphone, when calibrated in absolute acoustical units, is a most convenient and flexible instrument for the absolute measurement of sound intensity. It should be clearly recognised, however, that such instruments are often about 2 or 3 in. in diameter, and are large enough to act as appreciable reflectors of high-frequency sounds. Now when sound is reflected normally the pressure at the reflecting surface is twice what it would be if the reflecting surface were removed. Consequently the pressure on the diaphragm of a condenser microphone placed normally in a high-frequency sound field tends to be equal to twice the pressure which existed at the point before the microphone was introduced. At very low frequencies, however, the microphone, being small compared with the wave-length of the sound, does not act as an appreciable reflector, and the pressure upon the diaphragm is the same as that which existed in the sound field before the microphone was introduced. At intermediate frequencies the

relation is often incalculable, because microphones do not have simple shapes.*

The calibration usually required is that in terms of the oscillatory pressure existing in the field of sound before the introduction of the microphone, and this takes account of any effect which the size and shape of the instrument may have on the sound pressures actually imposed upon the diaphragm.

It is not necessary to be able to calculate the difference because the e.m.f. developed by microphones can be calibrated as desired, either in terms of the pressure on the diaphragm or in terms of

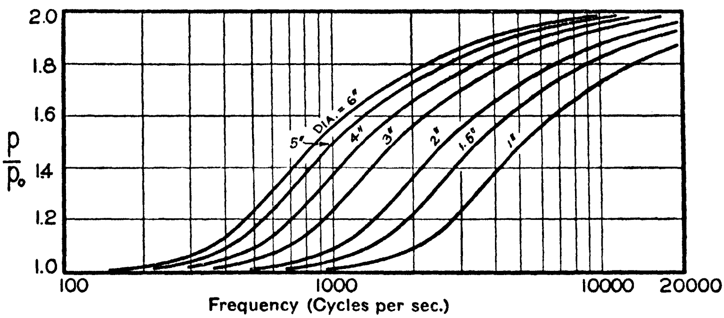


FIG. 48.—Increase of pressure due to diffraction for a plane wave and spherical microphone housings of various diameters (Ballantine)

the pressure previously existing in the sound field. It is convenient to note, however, that Bostwick † inferred from some indirect experiments that there was a twofold difference between the two calibrations for frequencies above 2100 cycles per second in the case of a microphone of $2\frac{7}{8}$ in. diameter, and equality for frequencies below 1100 cycles per second. Ballantine ‡ attempted to evaluate the correction by employing a spherical mounting of which the diaphragm occupied a small area about the pole. The ratio of increase in pressure for this mounting is calculable theoretically on lines developed by Rayleigh, and is determined by the ratio that the diameter of the sphere bears to the wavelength of the sound. It is indicated in fig. 48 as a function of frequency for the particular cases of spheres of from 1 in. to 6 in. diameter. Reasonable agreement with the theoretical curves has been indicated by experimental calibrations.§ Experiments with

* G. v. Békésy (*Ann. d. Phys.*, 14, 51, 1932) has studied the effect of the head and ear passages upon sound fields.

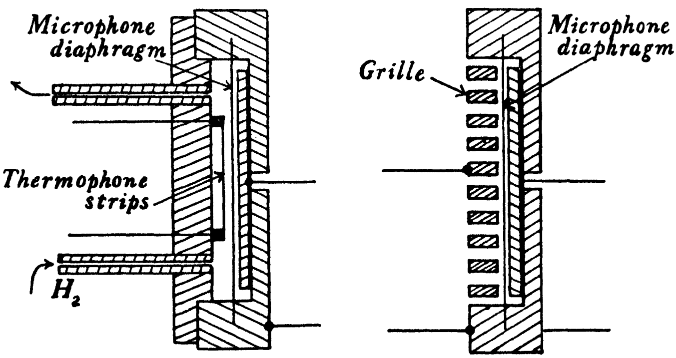
† L. G. Bostwick, *Bell Sys. Tech. J.*, 8, 135, 1929.

‡ S. Ballantine, *Phys. Rev.*, 32, 988, 1928.

§ S. Ballantine, *Acous. Soc. Am. J.*, 3, 319, 1932.

Oliver's * 4-in. microphone of 4 in. diameter have shown that the pressure ratio increased from unity to 2 in the region between 400 and 1800 cycles per second.

'Actual Pressure' Calibrations. Various methods have been employed in determining the calibration of microphones in terms of the pressure actually exerted on the diaphragm. At the lowest frequencies a pistonphone (p. 32) may be used to produce known pressure variations in a small enclosure closed at one end by the diaphragm of the microphone. At frequencies above about



(*a*) By thermophone (*b*) By electrostatic grille

FIG. 49.—Pressure calibration of microphones

200–300 cycles per second the pistonphone usually becomes unreliable. At frequencies above this value therefore a thermophone is sometimes used to replace the piston as a generator of known pressures in an enclosure (p. 34), and may be used up to frequencies for which the wave-length of the sound is comparable with the size of the enclosure. Usually the largest dimension of the enclosure is its diameter, and if this is 1½ in. the thermophone can be relied upon up to, say, 8000 cycles per second. By using hydrogen as the gas in the enclosure the wave-length is increased about fourfold, and the thermophone can then be employed up to the highest audible frequencies (fig. 49). A recent discussion of the theory of the thermophone and detailed description of its use is that by Ballantine (p. 35).† The disadvantage of the thermophone is the complicated formula involved, which includes a dozen or more physical and electrical quantities, some of which (*e.g.* thermal conductivity of the gas and thermal capacity of the

* D. A. Oliver, *J. Sci. Insts.*, 7, 113, 1930.
† S. Ballantine, *Acous. Soc. Am. J.*, 3, 319, 1932.

foil) are not known with satisfactory accuracy. In another method of applying a known mechanical force to the diaphragm of a microphone membrane, a grille-shaped electrode is mounted in front of the diaphragm at a small distance d of, say, 0·1 cm. (fig. 49 (b)). An electrostatic force is then applied between the diaphragm and the grille. Owing to the perforations in the grille and the separation employed the acoustical reaction on the diaphragm is negligible. The force exerted by the grille can be calculated from its physical dimensions and separation, and incidently is independent of the frequency—a real convenience. The usual excitation consists of an alternating voltage $E \sin \omega t$ superimposed upon a large direct voltage E_0. If p is the alternating pressure thus applied to the diaphragm we have

$$p = 8{\cdot}85 \times 10^{-7} E_0 E K / d^2 \text{ (dynes, volts, cm.)}$$

where K depends upon the geometrical details of the grille. Agreement with a thermophone calibration has been obtained within about 5 per cent. above frequencies of 200 cycles per second, and within 18 per cent. down to 20 cycles per second.*

Meyer † used electrostatic forces in making absolute measurements of pure tones with a condenser microphone, and adopted a compensation principle which had previously been used by Gerlach.‡ Meyer compensated the sound forces acting upon the diaphragm of the microphone by oppositely directed electrostatic forces. The natural frequency of the diaphragm plays no part in the measurement, the method being a null one. The reduction to zero of the vibration of the diaphragm was detected by means of a high-frequency circuit (say 3 million cycles per second) very similar to that described in connection with the normal method of operation of the Riegger condenser microphone. A steady and an alternating voltage applied to the microphone suffice for the compensation of the sound forces. It can be obtained conveniently from a Larsen a.c. potentiometer deriving its current from the same oscillator as the electrically driven source of sound under study, and may be conveyed to the microphone through a high-frequency choke circuit. The audio-frequency compensating voltage must be prevented from reaching the high-frequency detecting circuit by means of a high-frequency transformer.

* S. Ballantine, *loc. cit.*
† E. Meyer, *E.N.T.*, 4, 86, 1927.
‡ E. Gerlach, *Wiss. Veröff. a.d. Siemens Konzern*, 3, 139, 1923.

The compensation principle is only applicable to pure tones, but an absolute calibration is possible, the forces set up by the electrostatic voltages being measured as follows. A steady negative pressure is applied by suction to the microphone diaphragm, and is measured by means of a suitable manometer. A change in anode current of the high-frequency detector valve results from the displacement of the diaphragm. By applying a direct voltage to the microphone the diaphragm is brought back to its original position. If p is the applied pressure and V the applied voltage the calibration constant of the condenser microphone is given by $k = p/V^2$. When the instrument is used for measuring sound the effective sound-pressure amplitude is given by the equation $p = 2kVv$ where V and v are the compensating direct and alternating voltages, V being large compared with v.

A calibration of a microphone in terms of the pressure on the diaphragm may also be carried out by a method which involves observation of stationary sound waves set up in a pipe.* The microphone diaphragm closes one end of the pipe, and a source of sound giving pure notes faces the other. A Rayleigh disc is suspended at a distance of a quarter of a wave-length from the closed end and the particle velocity $\dot{\xi}$ of the air at this point—which is a velocity anti-node—is thereby measured. From the measurement the magnitude of the oscillatory pressure p at the end closed by the diaphragm is inferred from the relation $p/\dot{\xi} = \rho c$ (p. 58). The e.m.f. set up by the microphone under the action of this pressure is of course noted. If the Rayleigh disc is permanently suspended at the middle of the pipe it may be viewed through a small window of glass or celluloid, but it is at a velocity anti-node only for a definite series of frequencies for which the total length of the tube is an odd number of half wave-lengths of the sound. If any particular frequencies not included in this series are required, it is necessary to employ a tube in which arrangements have been made for moving a Rayleigh disc along its length, and for observing the disc in any position to which it may be moved. At very low frequencies the pipe method is somewhat unwieldy on account of the length of pipe necessary to accommodate one-quarter of a wave-length of the sound, and it fails at high frequencies because the diameter of the pipe sets an upper limit to the pitch of the note which may be employed without exciting resonant radial vibrations of the contained air.

* E. Meyer, *loc. cit.*; W. West, *P.O. Elec. Eng., J.*, 20, 127, 1927.

The wave-lengths of the frequencies of radial* resonance of sound in a pipe of radius r are $1{\cdot}64r$, $0{\cdot}90r$, $0{\cdot}62r$ . . .; the fundamental corresponds to a frequency of about 8000 cycles per second for a pipe of 2 in. diameter.

Undoubtedly the pressure upon the diaphragm of the microphone at the end of a pipe could be inferred from measurements of the radiation pressure exerted upon a suitable acoustic radiometer placed in substitution for the microphone. Indeed, such an experiment was performed by Zernov.

When a microphone, which has been calibrated in the above manner, is available, other microphones may be calibrated by comparison. The calibrated and uncalibrated microphones are used to close the two ends of a small (cylindrical) chamber to which the sound is supplied through a small hole in the centre of the curved side. The chamber is so small that the pressures set up within it are substantially uniform. C. A. Hartmann described the method and has given a brief outline in English.†

'Free Air' Calibrations. The calibration of microphones, in terms of the pressure p existing in a plane wave before the introduction of the microphone, may be performed by measuring the particle velocity $\dot{\xi}$ at a point by means of a Rayleigh disc, which is too small to distort the sound field, and then substituting the microphone. In a progressive plane sound wave $p=\rho c\dot{\xi}$, where ρc, the product of density of the medium and velocity of sound, is $42{\cdot}6$ for air at 0° C. In practice the comparison must be performed in an enclosure, since draughts which occur in open air would disturb a Rayleigh disc. For frequencies above, say, 500 or 300 cycles per second it is possible to work in a room or chamber the walls of which are generously lagged with 6 in. or 1 ft. or more of highly absorbent material such as cotton-wool or cotton-waste. This lagging is sufficient to suppress the reflection of sounds of the higher frequencies from the walls to such an extent that the inverse square law ‡ holds for a reasonable working distance from a point source. Pl. III, p. 112, shows a photograph of a lagged chamber, some 6 ft. cube, lagged with 6 in. of cotton-waste behind wire-netting. One side of the

* Other transverse resonances (side to side) occur for wave-lengths $3{\cdot}4r$, $1{\cdot}18r$, $0{\cdot}73r$ . . . (Lamb, *Sound*, p. 263).

† C. A. Hartmann, *E.N.T.*, 3, 458, 1926; *I.E.E., J.*, 64, 1058, 1926.

‡ The pressure follows an inverse law, but the particle velocity is given by equations (p. 58) which depart from an inverse law near the source.

chamber has been removed for the purpose of taking the photograph.

The fact that such chambers are not suitable for work at frequencies below about 500 cycles per second is not of very great importance in the calibration of microphones, for at lower frequencies the 'free air' and 'actual pressure' calibrations are practically identical for instruments of ordinary dimensions. The rest of the calibration may therefore be obtained by one of the methods enumerated above.

Measurements in a lagged chamber are often taken with the microphone relatively near to the source so that the effect of reflection from the walls shall be so minimised as to be negligible. In consequence the wave is spherical rather than plane. Calculations of the diffraction of a spherical wave around a microphone show that the pressure-doubling tends to be more complete as the microphone approaches the source. At distances of the order of 1 ft., however, it is possible that there is very little difference between a spherical and a plane wave as regards the pressure-doubling effect.

When a microphone diaphragm is of a size which is comparable with the wave-length of the sound concerned, the pressure exerted on the diaphragm by an incident sound is uniform only if the microphone is directly facing the sound waves. The response depends upon the angle between the wave normal and the normal to the diaphragm. The directivity curves for a $1\frac{1}{2}$-in. microphone diaphragm forming substantially part of a spherical surface of 5 in. diameter are given in fig. 50 from results by Ballantine.* The dotted curve for 1660 cycles represents values computed from an integration of calculations made by Rayleigh † for the pressure rise at *points* in the surface of a sphere; the experimental curve is in fair agreement. The smaller the microphone, the less are its directivity effects; and a microphone of about 0·1 in. diameter would be practically free of such effects ‡ for all frequencies up to 15,000 cycles per second. Directivity measurements involve only the comparison of the readings of the microphone when its orientation to the oncoming waves is varied, and can be carried out in open air.

Fig. 51, compiled as far as possible from the sources where the instruments have been first described, gives calibration data for

* S. Ballantine, *Acous. Soc. Am. J.*, 3, 319, 1932.

† Rayleigh, *Scientific Papers*, 5, 151, 1904.

‡ H. C. Harrison and P. B. Flanders, *Acous. Soc. Am. J.*, 4, 451, 1932.

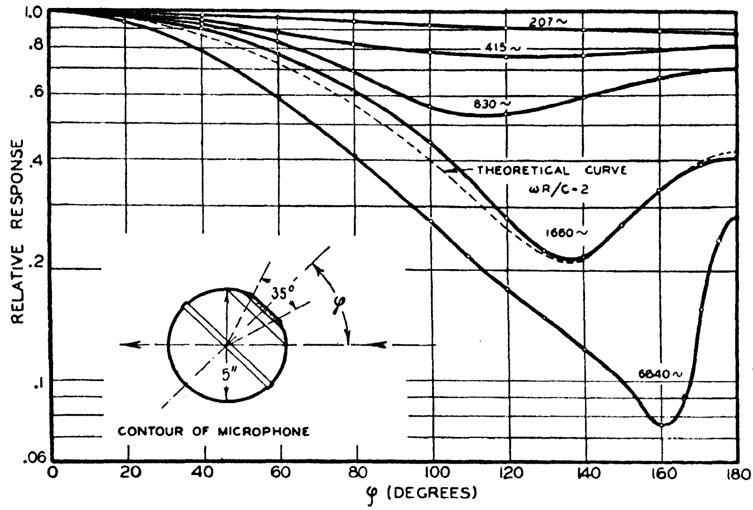


FIG. 50.—Directivity characteristic (plane waves) of spherical condenser microphone (Ballantine)

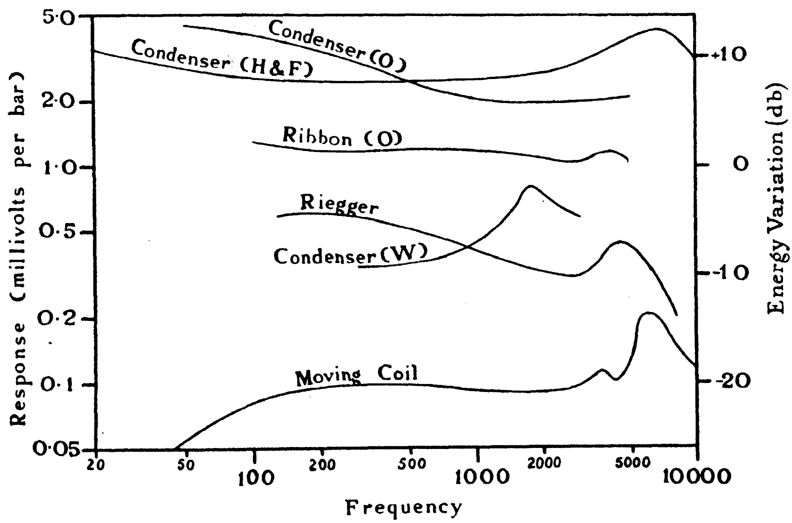


FIG. 51.—Some 'free-air' microphone calibration curves

Condenser microphone (Oliver)
Condenser microphone (Harrison and Flanders)
Ribbon microphone (Olson)
Moving-coil microphone (Wente and Thuras)
Condenser microphone (Wente, early type). See Aldridge, *P.O.O.E., J.*, **21**, 223, 1928
Riegger condenser microphone in high-frequency circuit (zero arbitrary)

microphones of various types. It must not be assumed that all instruments of the same type will have similar characteristics, for the effects of variations of diaphragm resonance or damping and other causes may be appreciable. Moreover, commercial instruments are not always as free from subsidiary resonances as these curves suggest.

The Measurement of Telephonic e.m.f.'s. The e.m.f.'s set up by several types of microphone are so small that, in order to measure them, valve amplifiers are used in conjunction with thermal milliameters or with rectifiers and galvanometers. It is well to adopt an arrangement which is convenient for the frequent calibration of the amplifier. For this purpose it is useful to incorporate a small resistance of, say, 1 ohm, in series with the leads from the microphone to the amplifier. If a known current of the same frequency as that of the microphone current is passed through this resistance by means of an external oscillator, a known e.m.f. is thereby injected into the microphone circuit. If now the magnitude of the current is adjusted so that the deflection of the recording instrument is the same whether the excitation is due to a current through this resistance or to the e.m.f. set up by the action of the sound upon the microphone, then the value of this latter is known. For the known and unknown e.m.f.'s give the same deflections, and they are both injected into the same amplifier circuit without alteration of the input impedance. The procedure is specially convenient when the source of sound used is an electrical loud-speaker. Fig. 52 illustrates diagrammatically the circuits in such a case. The loud-speaker is actuated by means of an oscillator unit, and the calibrating voltage is obtained through an attenuator in a parallel circuit. A key is so arranged that in one position it supplies current to the loud-speaker for the acoustical experiment, and in the other supplies the calibrating current, through an attenuator and an ammeter, to the resistance. Potentiometer methods may also be employed.

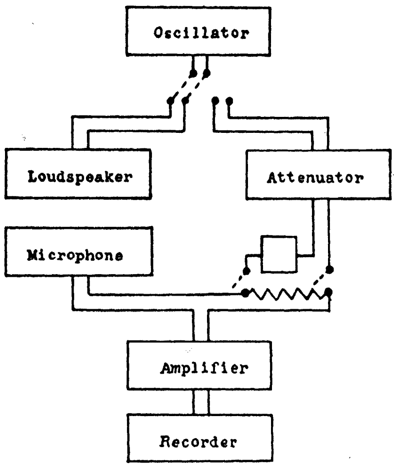


FIG. 52.—Diagram of convenient circuit for acoustical calibrations

CHAPTER IX

REVERBERATION METHODS OF MEASUREMENT

In the chapters relating to the measurement of sound intensity, reference has been made to the fact that, owing to the reflection and interference of sound from the walls, floor, and ceiling, pronounced unevenness of sound intensity occurs in ordinary rooms unless the bounding surfaces are rendered very absorbent by the generous application of sound-absorbing materials. Reference has been made to the desirability of conducting certain types of experiment in deadened chambers in which reflections are suppressed.

In some circumstances, however, an alternative method of treatment may be adopted. In this case the walls of the chamber are kept highly reflecting, the sound is mixed as adequately as possible by means to be referred to later, and an average measurement of the intensity of sound is made by means of a moving microphone or by some corresponding device. In practice, measurement of this 'reverberant' sound is most readily conducted by means of experiments in which the interference system is changing rapidly, as is the case when the sound is dying away after the source has been stopped. The rate of decay depends upon the absorbing power of the room, and the duration of audibility on the rate of emission of acoustical energy by the source. The method thus demands calibrated rooms. It is of obvious value in determining the absorbing power of introduced materials—the use to which it was first applied—but should have important applications in measuring the total emission in all directions from sources, and in the study of the overall sensitivity of receivers to randomly directed sound. The potential importance of the method lies in the fact that in everyday life sound in rooms is mainly reverberant, so that measurements of reverberant sound, and of the behaviour of sources, receivers, absorbents, and other acoustical devices in reverberant rooms, bear a close relation to practical requirements.

Theoretical Equations. If a source of sound which has been sounding within a closed room is suddenly stopped, the sound heard in the room does not cease at once, but dies away gradually. This is due to the fact that it is being reflected to and fro in the room, and indeed would not die away at all if it were not absorbed to some extent by the surfaces of the room at each reflection. The subject of reverberation deals with this repeated reflection of sound to and fro within a closed space. Reverberation is the general effect arising from a multitude of random reflections following each other in rapid succession, and differs from echo in which the reflected sound is a detectable repetition of the original.

Sabine's Formula. The first experimental work on reverberation appears to have been carried out by W. C. Sabine,* and the earlier equations were his. The theory has been expressed since in rather more comprehensive forms by Jaeger,† Buckingham,‡ and others. In all cases it is recognised that a sound suffers some two or three hundred reflections before it finally dies away, and, in consequence, it is assumed that through random arrangement the sound energy rapidly becomes uniformly distributed throughout the enclosure. Interference phenomena give rise to a complicated distribution of maxima and minima of sound throughout the space and make it difficult to assess the average acoustical energy, and the shifting of the interference system during the decay of the sound causes it to appear to die away in waves. The ear, however, appears to ignore these fluctuations in decay and to concentrate upon averages over short-time intervals, so that the interference phenomena are not of primary importance. They are ignored in reverberation theory, so results are only valid to the extent to which this is justifiable.

Jaeger's analysis notes that sound waves from a simple source of musical sound proceed outwards and suffer reflection, diffraction, and absorption at the various enclosing surfaces. If for a ray of sound the mean number of incidences upon the surfaces is n per second, and if the mean absorption coefficient (or ratio of absorbed to incident energy) of the surfaces is a, and if the mean intensity (energy per unit volume) in the room at any

* W. C. Sabine, *Collected Papers on Acoustics*, 1922.
† G. Jaeger, *Akad. Wiss. Wien. Sitz. Ber.*, 120, 613, 1911.
‡ E. Buckingham, *Bur. Stds. Sci. Papers*, No. 506, 1925.

instant is I, then the change in intensity by absorption at the surfaces in a time dt is $dI = -I \, . \, a \, . \, n \, . \, dt$.*

By a statistical method it is shown that for a ray of sound in a closed room the mean free path is $4V/S$, so that the mean number of reflections per second is given by $n = cS/4V$, where c is the velocity of sound and V and S are respectively the total volume and the total surface area of the room. If E denotes the rate of emission of energy from the source, the total rate of change in unit volume of the room is given by

$$dI/dt = E/V - caSI/4V \tag{1}$$

Solutions of this fundamental differential equation for various special cases are:

(i) Uniform sound emission beginning at zero time

$$I = \frac{4E}{vaS}(1 - e^{-caSt/4V}) \tag{2}$$

(ii) Steady state reached by uniform emission

$$I_{max} = \frac{4E}{vaS} \tag{3}$$

(iii) Emission stopped in steady state

$$I = I_{max} e^{-caSt/4V} \tag{4}$$

The standard period of reverberation of a room is the time which elapses after the finish of a sustained note before the intensity of the reverberant sound falls to one millionth of its initial value. For this case $10^{-6} = e^{-catS/4V}$, whence putting

* It is necessary to comment here that this equation is only an approximation, for it assumes that the amount of sound absorbed during the small interval of time is proportional to the number of reflections. This is only true if the absorption per reflection is relatively small so that the intensity of the incident sound is not materially altered by a limited number of reflections. It is more satisfactory to suppose that the intensity is reduced at each reflection to a definite fraction of its incident magnitude, so that the formula for repeated reflection should be $I_t = I_0(1 - a)^{nt}$. This yields $dI = I.n.(-\log_e \overline{1 - a}).dt$. Thus in Jaeger's analysis $-\log_e(1 - a)$ should be substituted for a: when a is less than 0·2 there is no appreciable difference. If the substitution is made in formula (5) of p. 150 the modified formula becomes identical with that proposed by Schuster and Waetzmann and also by Eyring for the case of diffuse reflection within a room (p. 151).

T for this standard period we have

$$T = \frac{55{\cdot}2V}{caS} \tag{5}$$

The total absorbing power aS is, of course, equal to the sum of the absorbing powers of the component surfaces S_1, S_2, S_3, etc., and is given by

$$aS = a_1S_1 + a_2S_2 + a_3S_3 \tag{6}$$

W. C. Sabine found that the duration of audibility of reverberant sound in a room was largely independent of the positions of the source of sound, of the observer, and of the absorbent surfaces. Moreover, the standard reverberation period agreed satisfactorily with the above formula in the case of ordinary rooms and halls. V. O. Knudsen * has made measurements of the mean free path of sound in three dimensional models of eleven typical auditoriums such as rectangular rooms with and without balconies, fan-shaped theatres, churches, and rooms with gabled, barrelled, or domed ceilings. He found the mean free path to vary from 3·8V/S in a large rectangular room with a low ceiling to 4·3V/S in a cruciform church. Corresponding to this result, the formula for the reverberation period would depend slightly upon the form of the auditorium, and the constant 55·2 in equations (5), (9), and (11) would vary between 52 and 59.

Later Reverberation Formulæ. Since W. C. Sabine's time the formula has been found to be defective when applied to very large halls, or to rooms which are unusually absorbent, such as highly lagged acoustical studios. Schuster and Waetzmann † first pointed out that Sabine's equation applies essentially to 'live' rooms, and indicated that the reverberation time may depend to some extent upon the shape of the room. They derived a new formula. C. F. Eyring ‡ derived the same formula by considering the sound reflected from the walls as arising from a sequence of image sources which all come into action at the moment when the source starts. More recently G. Millington § has proposed another formula, which differs from Eyring's mainly in the manner in which the average absorbing power of a number of different surfaces in a room is reached. Millington's

* V. O. Knudsen, *Acous. Soc. Am. J.*, 4, 20, 1932.
† K. Schuster and E. Waetzmann, *Ann. d. Phys.*, 5, 671, 1929.
‡ C. F. Eyring, *Acous. Soc. Am. J.*, 1, 217, 1930.
§ G. Millington, *Acous. Soc. Am. J.*, 4, 69, 1932.

analysis may be regarded as following the lines of Jaeger's up to a point.

He notes that in a time t any particular ray of sound will make nt reflections, where as before $n = cS/4V$. He states that the simple ray theory can be applied provided each of the areas S_1, S_2, etc., is large enough to act as a reflector of the sound, and that the edge effects are negligible. Provided that none of the ratios S_1/S, S_2/S, etc., is very small, then of the nt reflections a fraction S_1/S will take place on the surface S_1, a fraction S_2/S will take place on the surface S_2, and so on. At each reflection from S_1 the energy reflected is $(1 - a_1)$ times the incident energy, and is $(1 - a_2)$ for the surface S_2, and so on. If the initial energy is I_0 then the energy remaining after t seconds is

$$I_t = I_0(1 - a_1)^{(S_1/S)(nt)}(1 - a_2)^{(S_2/S)(nt)} \ldots (1 - a_n)^{(S_n/S)(nt)} \qquad (7)$$

and it does not matter in what order the reflections have occurred. This may be rewritten

$$I_t = I_0 e^{-kt} \quad \text{where} \quad k = -\frac{v}{4V}\Sigma S_1 \log_e (1 - a_1) \qquad (8)$$

which yields for the standard reverberation period

$$T = -\frac{55{\cdot}2V}{c\Sigma S_1 \log_e (1 - a_1)} \qquad (9)$$

Eyring's formula differs from this only in the method of averaging the absorption contributions of different elements, and contains the term $S \log_e (1 - a)$ instead of $\Sigma S_1 \log_e (1 - a_1)$. In Eyring's formula $aS = \Sigma a_1 S_1 +$. Eyring stresses the point, however, that in rooms of special shape special methods of averaging may be necessary. Millington regards the sound as reflected from various different surfaces in turn, and Eyring considers it to suffer an average absorption on each reflection.

Comparison of Reverberation Formulæ. In effect Eyring takes an arithmetical mean for the average absorption coefficient of the room, Millington the geometrical mean. At the present time the difficulty of selecting the correct formula has not been overcome. For specified values of absorption coefficients Eyring's formula tends to give higher values of the calculated period of reverberation than Millington's; and in the converse process in which a reverberation room is used to find the absorption coefficient of a given sample, Eyring's gives the highest value

for the coefficient calculated. Indeed, sometimes in practice it comes embarrassingly high. On the other hand, Millington's formula fails when an area of the chamber is completely absorbent.

A possible line of research would be to ascertain the true absorption coefficients of a material by making measurements when the material was used to cover the whole surface of the chamber—for which case Eyring's and Millington's equations agree—and then to experiment again with only a limited area (say 100 sq. ft.) of the absorbent in the chamber. Again a comparison of absorption coefficients obtained by decay methods and by steady-state measurements might be of assistance.

It should always be borne in mind that, in so far as the various reverberation formulæ depend upon statistical averages, they presuppose a complete mixing of the sound in the chamber. In very absorbent rooms the sound dies away in a few reflections, and the statistical basis of the formulæ is weakened. Similarly, in a very large hall, as the sound cannot cross the room many times during an observed reverberation period of a few seconds, the validity of the formulæ is affected. Sabine's formula has the defect that from it a finite reverberation period can be calculated for a hall of which all the surfaces are completely absorbent, a defect from which Eyring's and Millington's formulæ are free. Millington's formula fails, however, when an isolated area of the chamber is completely absorbent.

A comparison of Sabine's and Millington's formulæ is interesting. The only difference between them is that Millington's contains $-\log_e(1-a_1)$, etc., where Sabine's contains a_1, etc. Thus, if in Millington's we write $a_1{}^1$ for $-\log_e(1-a_1)$, etc., his equation is identical in form with Sabine's.

This fact has an interesting practical consequence in connection with auditorium acoustics. It is usually desired to ascertain what period of reverberation a hall would have if certain areas of specified absorbents were introduced. Experiments are therefore necessary in a reverberation chamber to determine the absorbing powers of the materials proposed. On Sabine's formula certain absorption coefficients a_1, etc., are obtained which, equal to $a_1{}^1$ on Millington's formula, are to be regarded on Millington's basis as a logarithmic function of the absorption coefficient rather than the absorption coefficient itself. To calculate the reverberation period of the hall concerned, it is

necessary to introduce the values determined (a_1 in the Sabine case and the identical $a_1{}^1$ in the Millington case) into the same formula, and identical values are therefore obtained for the reverberation period of the hall.

Summarising, therefore, Sabine's equation is applicable only to 'live' rooms, and gives incorrect relations between reverberation and *true* absorbing power in 'dead' rooms. Both Eyring's and Millington's are based upon a more accurate statement of the law of decay, but differ in the manner in which the average absorbing power of surfaces of a room are obtained. Where absorbing powers used in calculations have been obtained by reverberation experiments, differences between the three formulæ are not great if the same formula is used consistently for the experiments and for the calculations. If, however, the true absorbing power is known from theoretical or other considerations (for instance, 1 for a large open window) Eyring's or Millington's formulæ unquestionably give a nearer approximation to the true period of a 'dead' hall than Sabine's.

Damped Free Vibrations of Air in an Enclosure. M. J. O. Strutt * and also K. Schuster and E. Waetzmann † have considered reverberation by regarding it as a case of free damped vibration of the volume of the air enclosed in the room. The analysis involves the general wave equations, with suitable boundary conditions imposed. They regard as unsatisfactory theories which deal with the paths of separate sound rays. The various 'eigentones' or modes of resonant vibration of the air columns in the room appear in the analysis. In Strutt's investigation Sabine's law is revealed as an asymptotic property to which the decay of reverberation tends as the frequency of the (forcing) sound becomes infinitely great compared with the lowest free frequency of the air itself; or, in other words, when the dimensions of the room become infinitely great compared with the wave-length of the sounds concerned. He finds that during the decay of reverberation the frequency of the sound remains substantially constant, although not identically so, particularly if the absorbing power of the surfaces of the room is not markedly dependent upon frequency. The rate of decay is proportional to the volume divided by the total absorption, and, as Sabine found, is independent of the shape of the room and of the positions

* M. J. O. Strutt, *Phil. Mag.*, 8, 236, 1929.

† K. Schuster and E. Waetzmann, *Ann. d. Phys.*, 5, 671, 1929.

of the source and observer. His equations show that on an average the sound decays in an exponential manner with time, as in Sabine's law, but the equations also contain certain terms which imply that oscillatory fluctuations are imposed upon the instantaneous rate of decay. These fluctuations correspond, of course, to interference systems floating through space and causing the sound to die away in waves when heard by the ear. Strutt states that it is always possible to dispose absorbent material in a room in a manner such that Sabine's law is not followed, namely, by placing absorbent to coincide with the loops or nodes of a free mode of oscillation in the neighbourhood of the frequency of the test sound. One special mode of oscillation then decays faster than the other modes. In general, however, the high free modes of oscillation all die out at exactly the same rate.

V. O. Knudsen * has observed, by oscillographic methods, the form of decay of sound in rectangular rooms, which were not large compared with the wave-length of the sounds concerned. The observed 'eigentones' were shown to agree with the theoretical frequencies n for the vibrating volume of air in a rectangular parallelopiped, namely,

$$n=\frac{c}{2}\sqrt{\frac{p^2}{l_1^2}+\frac{q^2}{l_2^2}+\frac{r^2}{l_3^2}} \tag{10}$$

where c is the velocity of sound, l_1, l_2, and l_3 the dimensions of the room, and p, q, and r are integers. One or more of these integers vanish when one or more surfaces in the room are covered with absorptive material. When a tone, which is nearly of the same frequency as the frequency of one of the 'eigentones,' is sounded and then stopped in the room, the decay, especially during the latter stages, reveals the frequency of the 'eigentone' rather than that of the original sound. Thus in a room in which the fundamental frequency was calculated to be 70 cycles per second, oscillograms revealed that sounds of frequencies of 65, 68, 70, 72, or 75 cycles per second all decayed at the fundamental frequency of 70 cycles. The reverberation was about 50 per cent. longer for the resonant frequency (70 cycles) than it was for frequencies only 5 cycles lower or higher. Similar effects were observed in the vicinity of several of the overtones for the room. The results show that volume resonance must be considered in

* V. O. Knudsen, *Acous. Soc. Am. J.*, 4, 20, 1932.

measurements by the reverberation method, and suggest that frequencies in the region of resonant frequencies should be avoided.

The Effect upon Reverberation of Absorption of Sound in the Air. The above analyses assume that no absorption of sound takes place in the air of the reverberation chamber itself, but that all occurs at the reflecting surfaces. However, P. E. Sabine * and E. Meyer † found experimental evidence that for frequencies above about 2000 cycles per second, the reverberation period of a room depended upon the humidity of the air of the room. It is true that this might have been due indirectly to the effect of humidity upon the absorption of sound by exposed surfaces in the room. However, by conducting experiments in two rooms of different sizes but with surfaces of the same absorbing power (varnished painted concrete in each case) Knudsen ‡ was able to disentangle the two effects of surface absorption and absorption by the air. As a result he found that at high frequencies absorption does take place in the air, and he has also shown how the necessary allowance may be made in the theoretical equations. When absorption occurs the intensity in a plane wave which has travelled a distance x in a medium is, if displacements are not too large, $I_0 e^{-sx}$, where I_0 is the intensity at the position $x=0$, and s is the attenuation coefficient for the wave in the medium (p. 223). We have seen that the rate of decay of sound in a room, if all the absorption takes place at the boundaries, is given by the equation, $I=I_0 e^{-kt}$, where k has various values according as Sabine's, Eyring's, or Millington's expressions are accepted. If the effect of air absorption is introduced into this equation it becomes $I=I_0 e^{-kt} e^{-sx}=I_0 e^{-(k+sc)t}$, since $x=ct$. As a result the Eyring form of the formula for the reverberation period of a room becomes

$$T=\frac{55 \cdot 2V}{c[-S \log_e (1-a)+4sV]} \qquad (11)$$

Knudsen found in his experiments that s had the values given in fig. 53. From them it appears that absorption of sound by slightly humid air is important when frequencies above about 1000 cycles per second are concerned.§ Knudsen noted also that

* P. E. Sabine, *Frank. Inst., J.*, 207, 341, 1929.
† E. Meyer, *Zeits. f. tech. Phys.*, 11, 253, 1930.
‡ V. O. Knudsen, *Acous. Soc. Am. J.*, 3, 126, 1931.
§ When $s=0 \cdot 002$ ft.$^{-1}$, the attenuation is about 46 db. per mile or about 9·8 db. per second.

for humidities up to 80 per cent. the absorption coefficients of the painted concrete itself were practically independent of the humidity and of the frequency of the test note. Above that limit, however, an increased surface absorption at saturation was probably attributable to condensation upon the walls of the chamber. V. L. Chrisler and C. E. Miller * have recently indicated that for large rooms having a volume of, say, one million cubic feet, aerial absorption can be measured at frequencies as

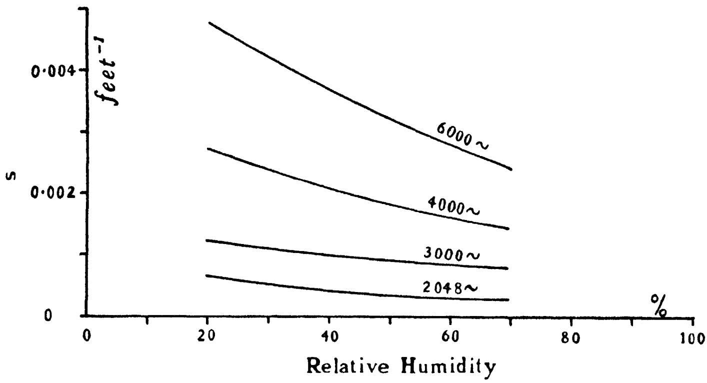


FIG. 53.—Dependence of *s* upon humidity for tones of frequency 2048–6000 cycles per second (Knudsen)
(Temperature of air 21° to 22° C.)

low as 512 cycles per second, and that the humidity effect depends upon temperature and barometric pressure as well as upon the relative humidity of the air.

Knudsen,† experimenting at various temperatures in two small steel reverberation chambers (one a 2-ft. cube and the other a 6-ft. cube), has generally confirmed his earlier results on sound absorption in air, and has extended them. He finds maximum absorption in the region of low humidities—a result of theoretical interest (p. 224).

Experimental Measurements of Reverberation and of Absorbing Power. The fundamental experiment in reverberation measurement is the determination of the rate of decay of sound in the room, and thus of the standard reverberation period. By virtue of the theory of reverberation, the total absorbing power of the surfaces of the room is then known. Sabine's technique employed the human ear to note the duration of audibility t_1 of a

* V. L. Chrisler and C. E. Miller, *Bur. Stds. J. Rch.*, 9, 175, 1932.
† V. O. Knudsen, *Acous. Soc. Am. J.*, 5, 112, 1933.

certain source, and also the duration t_m of another source of m times the activity. The difference $t_m - t_1$ represents the time taken for an m-fold decay in the intensity of the sound in the room, and from this the standard period of reverberation can be calculated. For best results the same source should be used in both cases, the activity being varied in the m-fold proportion.*

Nowadays instrumental methods have been substituted for aural methods of observing the decaying sound. The source is sounded and then stopped; the act of stopping the source sets a chronograph in operation. The decaying sound affects a microphone-amplifier-rectifier system which operates a relay and stops the chronograph when the sound has decayed to some pre-determined value.† The experiment may be repeated with an m-fold change in the activity of the source, or alternatively with the gain of the amplifier altered in an m-fold ratio. Thyratron relays are suitable for the work (p. 96). In practice the experiment is carried out for several values of m, and the results are plotted. If the ordinary reverberation law is being obeyed a linear law should be found between the duration of reverberation and log m.

In another instrumental method of studying reverberation oscillograms of the decay of the sound are taken, but the chronograph technique is the more rapid.

Considerable accuracy is necessary when measurements are being made of the *changes* in the absorbing power of a room resulting from the introduction of a test absorbent or other experimental variable, and the chronograph is satisfactory. In cases where it is merely desired to ascertain the reverberation period of an auditorium or of a studio less accuracy is demanded, and various reverberation meters and bridges may be employed.‡ In some a balance is obtained between the decay of electric current due to sound picked up by a microphone and rectified, and the decay of potential of a condenser discharging through

* A. H. Davis, *Phil. Mag.*, 2, 543, 1926; and A. H. Davis and N. Fleming, *Phil. Mag.*, 2, 51, 1926.

† M. J. O. Strutt, *E.N.T.*, 7, 280, 1930; E. Meyer, *Zeits. f. tech. Phys.*, 11, 253, 1930; *N.P.L. Annual Report*, p. 85, 1930; E. C. Wente and E. H. Bedell, *Acous. Soc. Am. J.*, 1, 422, 1930; W. F. Snyder, *Bur. Stds. J. Rch.*, 9, 47, 1932 (bibliography); R. F. Norris and C. A. Andree, *Acous. Soc. Am. J.*, 1, 366, 1930.

‡ H. E. Holmann and T. Schultes, *E.N.T.*, 8, 387, 1931; 8, 539, 1931; H. F. Olson and B. Kreuzer, *Acous. Soc. Am. J.*, 2, 78, 1930.

an adjustable resistance. In another the sound is picked up by a microphone, and when the intensity falls to a pre-determined level the source is automatically switched on again. It remains on until the level of intensity rises to another definite value, when it is automatically switched off and the operation repeats itself. The frequency with which the controlling relays operate is a measure of the reverberation period of the room.

If the reverberation period of a room is known, it is possible to calculate the energy emission from a source by measuring the average intensity of sound ultimately set up in the room by continuous sounding of the source. Also changes in the absorbing power of the room may be observed by noting the change in the average intensity set up by a given source. These measurements of the steady state are difficult, however, on account of the variations of intensity which occur from point to point in the room owing to interference effects. Warble notes are therefore necessary, and it is desirable to employ several microphones situated in different parts of the room.* In many ways the steady-state measurement must be inferior to the decay form of measurement, because in the latter case the interference system moves continually as the sound dies away, and the process of averaging is thus much simplified.

Conditions Desirable in Acoustical Measurements by Reverberation Methods. It is well to recapitulate some of the requirements of reverberation methods of measurement, particularly those where accuracy is required, as in the measurement of the absorbing power of test absorbents.† In the first place, interference effects—which are ignored in the theory—should be minimised. To this end Sabine ‡ employed large rotating reflectors in the experimental room, and thus kept the interference system constantly on the move. Later experimenters have employed moving, warbling (p. 93), or intermittent § sources. A large room is desirable so that its natural resonances shall be as low as possible. A large room is also advantageous in order that its area may be large compared with the area of any test absorbent, and uniformity of mixing of the sound thereby ensured. Resonance frequencies of the air of the room should be avoided when

* V. O. Knudsen, *Phil. Mag.*, 5, 1240, 1928.
† A. H. Davis, *Phil. Mag.*, 2, 543, 1926.
‡ P. E. Sabine, *Phys. Rev.*, 19, 402, 1922.
§ M. J. O. Strutt, *Rev. d'Acoustique*, 2, 1, 1933.

measurements are made, partly because of the special rates of decay which may occur near resonances, but also in some instances because of the special reaction the room may have upon the source. Clearly a room with parallel sides is liable to have several series of pronounced resonances, and for that reason the reverberation chamber at the National Physical Laboratory (fig. 54) has been designed to have five walls, no two of which are parallel, and a ceiling which is not parallel to the floor. Special precautions have been taken to exclude noise (pp. 29, 292).

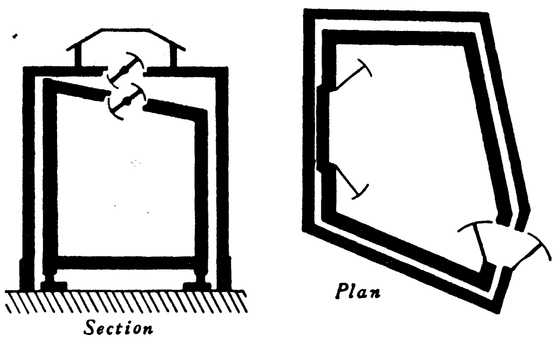


FIG. 54.—Chamber for reverberation experiments (N.P.L.)
(To exclude noise the room is double-walled, the inner chamber being supported upon insulated piers)

It is desirable to work at a constant temperature and humidity of the air, particularly for tests at frequencies above 1000 cycles per second, where the effect of humidity is possibly appreciable. The importance of absorption in the air of the chamber can be minimised by making the surface of the reverberation chamber moderately absorbent, but the remedy reduces the reverberation and increases the accuracy necessary in measurements. To average out any lack of uniformity in the distribution of sound within the room, the positions of source, absorbents, and observer should be varied as far as possible: satisfactory rooms should show little dependence upon these variations. Sources should preferably be of a type which are not greatly affected by the reaction of the room upon them.

When the absorbing power of test absorbents is being measured, the sample should not be small compared with the wave-length of the sound concerned, and a standard size—say 10 ft. by 10 ft.—should be employed. For owing to diffraction effects, the absorption per unit area of small samples is liable to depend

noticeably upon the size and shape of the sample. Also variations which arise from humidity effects, etc., make it advisable to conduct the experiments with the chamber empty, on the same day as the test is carried out with the absorbent present. It is convenient therefore to have heavy steel shutters (fig. 54) for covering on such occasions the wall recess which accommodates the absorbent.

CHAPTER X

MEASUREMENT OF FREQUENCY

In modern acoustics electrical methods are freely employed in measurements of frequency—as in the measurements of intensity—on account of their general convenience. In their highest development they are capable of an accuracy approaching even that with which time itself may be measured. When the frequency of the note of an electrical source of sound is required, it is merely necessary to ascertain the frequency of the exciting electrical current. In cases where the source is not electrical it is often possible to tune an electrical source to exact unison with it and then to measure the frequency of the electrical source. On the other hand, the sound could be converted into electrical form by microphone and amplifier apparatus, and then measured. The actual procedure depends upon circumstances, and it is sufficient to outline the frequency-measuring devices which may be used.

Reed Frequency Meter. In approximate measurements at frequencies of the order of, say, 20–150 cycles per second, it is often convenient to employ an instrument containing a graduated series of reeds, each clamped at one end and tuned to a definite frequency. A convenient arrangement is to have approximately a 1 per cent. frequency interval between successive reeds. The electrical current of which the frequency is desired is passed through an electromagnetic system which attracts all the reeds, but it is only the reed which is tuned exactly to the electromagnetic forces which is set in appreciable vibration. This reed may be observed visually and thus the frequency is known. Two frequency ranges are possible, one double the other, according as the unknown a.c. is applied alone to the electromagnet coils, or is superimposed upon a moderately large direct current (*cf.* electromagnetic telephone diaphragm, p. 38). Kempf-Hartmann *

* R. Kempf-Hartmann, *E.T.Z.*, 22, 9, 1901; *Phys. Zeits.*, 11, 1183, 1910.

constructed reed meters covering ranges from 2 cycles per second to 1500 cycles per second. Incidentally, if such a meter is merely set upon a vibrating platform or machine the appropriate reed will be set into vibration by mechanical action alone, so that the frequency of vibration is measurable in this manner if it is within the range of the meter.

Deflectional Frequency Meter. In addition to the vibrating-reed type of frequency meter, instruments may be obtained in which a pointer indicates on a scale the instantaneous value of the frequency of the applied e.m.f. In the instrument, which is of the 'moving iron' type, the current passes through two operating windings which act differentially upon the pivoted moving iron which is attached to the pointer. If the frequency of the current changes, the current through one winding increases, and that through the other decreases, thus altering the forces upon the pointer and causing it to move to a new position of equilibrium. The controlling and actuating forces are entirely electrical, and the reading of the instrument is practically unaffected by, say, 30 per cent. of variations in the applied voltage. The instrument is not sensitive to temperature variations. To render the indications independent of wave-form, inductances are inserted in both windings to filter out higher harmonics.

Deflectional meters are used for observing the frequency of ordinary a.c. electric supply, and different instruments are necessary for different frequency ranges. Each instrument can cover a range of the order of an octave. Types are made for ranges from, say, 20–60 cycles per second to, say, 550–700 cycles per second. By reducing the range of a given instrument it is possible to obtain a more open scale and greater precision.

Wire Sonometer. For higher frequencies a wire sonometer may be constructed, on the resonance principle, capable of giving an accuracy of about 1 in 1000 over a considerable range. The meter was first used by L. V. King * for the range 360–1600 cycles per second. Kennelly and Mannebach † developed a form suitable for the range 250–2500 cycles per second.

An instrument of the same type designed by Dye ‡ is shown in Pl. IV, p. 129. A phosphor-bronze wire, 0·3 mm. diameter, held in a terminal at its upper end, has a weight hung on its

* L. V. King, *Frank. Inst. J.*, 187, 611, 1919.
† Kennelly and Mannebach, *Frank. Inst. J.*, 192, 349, 1921.
‡ D. W. Dye, *N.P.L. Annual Report for 1924*, p. 83.

lower end. It is tightly strained by this between a fixed nodal point in the form of a V groove between two small steel balls in contact, and a lower groove formed by a small wheel on a light steel spindle. This lower nodal point is on a sliding carriage which can be raised or lowered by screw motion, so that the free length of the wire—and thus its natural frequency—can be varied between wide limits. The carriage carries pointers which indicate the natural frequency on direct reading scales. A current of, say, 100 milliamps. from the source of unknown frequency is passed through the wire, a portion of the free length of which lies between the poles of a permanent magnet. When, through adjustments of the position of the frequency-controlling carriage, the natural frequency of the free length of the wire equals that of the source, a large resonant vibration occurs, and may be noted visually or aurally. The frequency corresponding to this position of the carriage is noted. Since the wire may be made to vibrate in 1, 2, 5, or any number of loops (provided the magnet is moved so that it is not situated at a node), the instrument may be fitted with a number of scales covering a long range. Actually in one instrument the range is from 200 to 400 when the wire is vibrating as a single loop, from 400 to 800 when it is vibrating in two loops, and so on; the top range with ten loops is from 2000–10,000 cycles per second. The range for one loop serves for two loops when multiplied by 2, but the other ranges have separate scales because the stiffness of the wire causes them to depart slightly from exact multiples of the unit scale.* Over practically the whole range a frequency change of 1 part in 1000 corresponds to a length of not less than about $\frac{1}{2}$ mm. Frequencies between 100 and 200 are obtained by using a weight of a quarter of that used for other ranges. The power taken by the apparatus is only about 1 milliwatt.

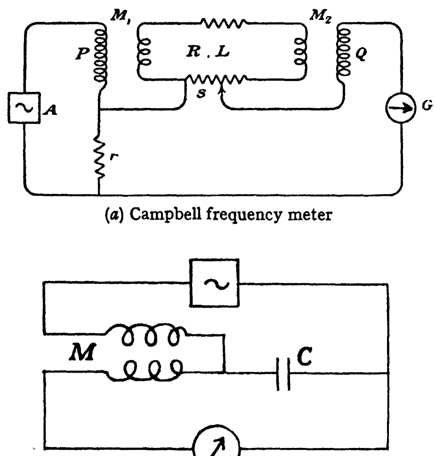
Campbell Frequency Meter. A different type of instrument applicable to electrical frequency measurement is the Campbell † frequency meter, a diagram of which is given in fig. 55 (*a*). Two mutual inductances, M_1 and M_2, are connected with an alternating source A and a detector G (*e.g.* a vibration galvanometer or telephone) as shown, their secondary circuits forming a loop having total self-inductance L and resistance R, of which a portion s is tapped off into the detector circuit. r is a

* See also G. E. Allan, *Phil. Mag.*, 4, 1324, 1927.
† A. Campbell, *Phys. Soc., Proc.*, 37, 97, 1925.

non-inductive resistance. If n is the frequency of the source the bridge will be balanced and G will show no current when (putting $\omega = 2\pi n$)

$$\omega^2 M_1 M_2 = Rr \quad \text{and} \quad M_1 s = Lr \tag{1}$$

The latter condition is satisfied once for all by fixing M_1, r, L, and s. Then for any given frequency the first condition can be satisfied by adjusting only M_2. Then $\omega^2 = Rr/M_1M_2$, or $n \propto aR/\sqrt{M_2}$ where a is a constant. In the actual instrument the scale of the variable M_2 is marked to read n directly for a given value of R, and a number of different ranges are obtained by altering R in suitable proportions to give simple scale multipliers. An instrument with five ranges covers the range 180–4000 cycles per second, the accuracy over the middle portion of the range (600–2500 cycles per second) being generally within 1 per cent. The ranges of the instrument are practically free from any temperature coefficient.



(a) Campbell frequency meter

(b) An earlier frequency circuit

FIG. 55

Campbell * had earlier described an electrical method of measuring large capacities which is applicable to measuring frequency; the circuit is illustrated in fig. 55 (b). The bridge is balanced when $MC\omega^2 = 1$, where M is measured in henries and C in farads. The method is sensitive to small changes in frequency of the order of, say, 1 part in a million. It provides a simple and sensitive method of testing the purity of the applied electrical current, for if a harmonic of only one ten-thousandth of the amplitude of the fundamental were present, it would be detected. Even the best condensers show, however, a considerable variation of capacity with temperature, and the frequency scale of the circuit thus has a considerable temperature coefficient.

Phonic Wheel. Absolute frequency measurements resolve

* A. Campbell, *Phys. Soc., Proc.*, **21**, 69, 1908.

themselves into counting directly or by inference the number of vibrations which occur in a measured time. A useful instrument for counting electrical oscillations is a phonic wheel, or synchronous motor. Invented independently by La Cour and Rayleigh,* it consists essentially of two electromagnets and a rotating wheel-armature having an even number of iron teeth spaced equally on its circumference. The phonic wheel designed by Dye † consisted of an aluminium wheel, about 11 cm. in diameter, on which was forced a steel rim about 4 cm. thick,

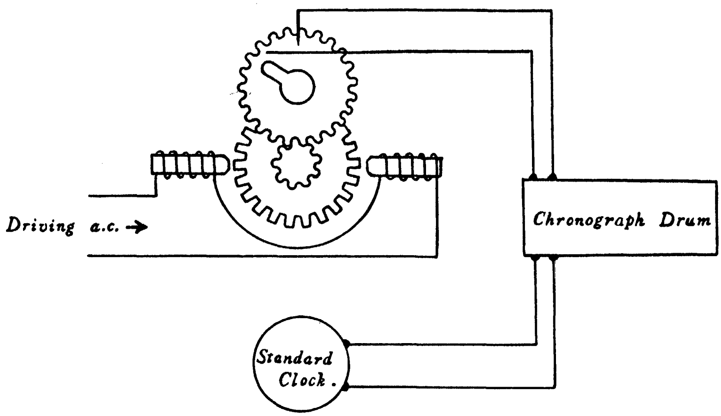


FIG. 56.—Phonic wheel and auxiliary apparatus

and having 20 teeth and spaces about 1·5 mm. deep cut evenly on its circumference. It was mounted on a hardened steel spindle running on jewelled bearings. A pair of telephone receivers were used as electromagnets, and were so adjusted that diametrically opposite teeth were simultaneously passing between the polarised magnets of the respective receivers. As a synchronous motor a phonic wheel, after it has been run up to speed by an Eccles valve-driven motor (Dye) or by a blast of air directed against the teeth of the wheel,‡ continues to run for hours, if it is actuated by steady alternating current. The condition for continuance is that the passage of teeth past the electromagnets shall correspond exactly to the fluctuations of the exciting current. To ascertain exactly the frequency of the steady

* Rayleigh, *Nature*, 18, 111, 1878; P. La Cour, *Comptes Rendus*, 87, 499, 1878.

† D. W. Dye, *Roy. Soc., Proc.*, 103, 240, 1923.

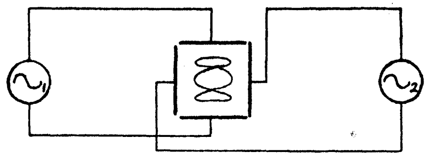
‡ E. Klein and G. F. Rouse, *Op. Soc. Am., J.*, 14, 263, 1927.

current driving the motor, it is thus only necessary to count the number of teeth on the rim which pass the magnets per second. By suitable gear, as shown diagrammatically in fig. 56, it is possible for the motor itself to record upon a chronograph at the end of, say, every 50 revolutions, *i.e.* every 1000 alternations of the current. If the apparatus is run steadily for several hours in conjunction with a standard clock the number of oscillations in the electric current in that period may be estimated with great accuracy. The instrument finds its most accurate use when it is employed for calibrating high precision standards of frequency, such as valve-maintained tuning-forks, which will run steadily for long periods of time. Clearly the frequency of any other type of source of sound could be determined by arranging an electric oscillator and loud-speaker to give a note exactly in tune with the unknown source, and then counting the oscillations in the electric circuit by means of the phonic motor.

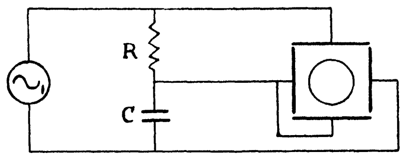
Cathode-ray Oscillograph. Frequency calibrations can often be conveniently carried out, particularly where the frequency concerned is exactly or approximately the harmonic of a standard frequency, by means of a cathode-ray oscillograph. For instance, if an e.m.f. of a known standard frequency is applied to two plates of the oscillograph so as to give a deflection of the spot in one direction in the field of view, and the e.m.f. of unknown frequency is applied (amplified if necessary) to the other plates to give movement to the spot in the direction at right angles, then the moving spot will trace out Lissajous' figures in the field of view (fig. 57 (*a*)). The figures will be stationary if the known and unknown frequencies are in exact harmonic relation, and will move slowly at a rate which can be counted if the relation is only approximately integral.

A more easily interpreted arrangement, however, is to apply an e.m.f. of the standard frequency and pure wave-form to both sets of plates, but with a phase difference of 90°, so that, in the absence of the unknown frequency, a circle appears on the screen of the oscillograph. A suitable circuit is shown in fig. 57 (*b*), where the current of standard frequency is applied to a resistance R and a variable capacity C in series. The potential difference across the resistance is applied to two plates of the oscillograph, whilst the potential difference across the capacity is applied to the other two plates. If the resistance and capacity are suitably adjusted in magnitude, the two e.m.f.'s applied to the plates are

equal in magnitude, but differ by 90° in phase; the oscillograph spot thus traces out a circle upon the screen. The size of this circle depends upon the anode voltage applied to the cathode-ray oscillograph. If therefore the unknown frequency is applied in series with the anode battery of the oscillograph, the circle will tend continually to increase and decrease in diameter. The trace on the oscillograph will thus resemble the circumference of a toothed wheel, and the number of teeth in the wheel will be equal to the ratio which the unknown frequency bears to the standard frequency. If the ratio is not exactly integral, the wheel will revolve one way or the other at a rate which may be counted and which may be allowed for in computing the unknown frequency. This method is very useful for setting an oscillator to give a series of definitely known high frequencies (up to, say, 15,000 cycles per second), such as might be of value in calibrating Galton whistles.



(*a*) Two sources giving Lissajous' figures on oscillograph screen



(*b*) Single source giving circular figure on oscillograph screen

FIG. 57.—Cathode-ray oscillograph circuits for use in comparing integrally related frequencies

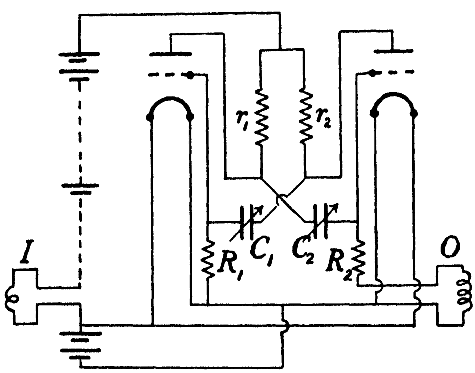


FIG. 58.—Abraham-Bloch multivibrator

Multivibrator. An extremely useful equipment for obtaining frequencies which are an exact multiple of a standard (without recourse to frequency-counting devices such as a cathode-ray oscillograph) is the multivibrator, originally described by Abraham and Bloch.* It is essentially a form of controllable thermionic valve oscillator which gives a note which is exceptionally rich in

* H. Abraham and E. Bloch, *Ann. de Physique*, 12, 237, 1919.

harmonics. A circuit is indicated in fig. 58. With the resistance and capacities as shown, the fundamental frequency of the oscillator is roughly, but not accurately, equal to $1/(C_1R_1 + C_2R_2)$.* Suitable values for a frequency of about 1000 cycles per second are $r_1 = r_2 = 50{,}000$ ohms, $R_1 = R_2 = 75{,}000$ ohms, $C_1 = C_2 = 0{\cdot}008$ μF. By putting at I, in series with the anode battery, a small voltage from a valve-maintained fork having a frequency approximately equal to that of the multivibrator, the frequency of the multivibrator may be controlled and its fundamental frequency thus maintained at a steady standard value. The output from the oscillator may be obtained from the coil connected at O in series with R_2. As stated the output is rich in harmonics. Any required harmonic may be selected from the constituents by

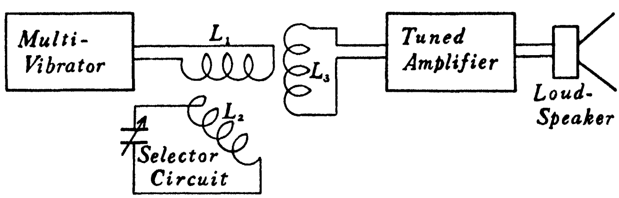


FIG. 59.—Circuit arrangement for selecting known high frequencies from the output of a multivibrator

means of the general arrangement shown in fig. 59. Here the output from the multivibrator passes through the coil L_1. A harmonic selector consisting of a variable condenser and a coil L_2 very loosely coupled to L_1 may be set to resonate at any desired harmonic of the multivibrator note. A pick-up coil, L_3, is so placed that it picks up nothing directly from the multivibrator coil L_1, but picks up from L_2 an e.m.f. having the resonant frequency of the selector. This e.m.f. is passed to an amplifier (preferably regenerative and tunable to the desired harmonic) and thus to a loud-speaker or telephone, from which a note of the required high frequency will be emitted. Such an apparatus is useful in calibrating whistles, etc., at high frequencies.

The principle of the controlled multivibrator may be extended and a second instrument, tuned to, say, 20,000 cycles per second, can be controlled by the first. Thus Dye attained frequencies up to $1{\cdot}5 \times 10^6$ per second in two stages, using as standard a 1000-cycle maintained tuning-fork.

It may be stated that the advantage of the multivibrator and

* C and R are measured in farads and ohms respectively.

cathode-ray oscillograph devices lies in the fact that the direct phonic-wheel calibration of the frequency of electrical currents cannot conveniently be carried out at frequencies above 1000 a second. Thus, whilst highest accuracies are possible at frequencies below this value, great accuracy at higher frequencies can only be attained by ascertaining that the high frequency is an exact multiple of an accurately known frequency within the range of the phonic wheel. Incidentally, if a single-standard frequency of, say, 1000 cycles per second is available, sub-multiple frequencies of 100, 200, etc., can be accurately compared with it by reversing the process and using the unknown frequency to give the fundamental circle on the oscillograph, or the fundamental frequency of the multivibrator. It should be mentioned, however, that the multivibrator indicated in fig. 58 can be controlled at frequencies which are exact sub-multiples of the frequency injected at I by the standard fork; also, to a lesser extent, at a number of subsidiary frequencies which are fractions $p/(p+q)$ of the fork frequency, where p and q are simple integers. Thus with a multivibrator and a controlling fork of frequency 1000 cycles per second, a series of standard frequencies may be obtained in the range 50 to 20,000 cycles per second.

Electrically-maintained Tuning-forks. It is well known that tuning-forks may be maintained in vibration by electrical means. For forks of low frequency, below 100 cycles per second, the principle of the electric bell may be employed, the vibrations of the prong making and breaking an electrical contact through which the exciting electromagnet is controlled. For precision various features of design are desirable,* and these have been incorporated in certain forks which are commercially obtainable. The fork should have a massive stiff base, prongs parallel to the base, and contacts designed to give exact timing of the 'make' and 'break' of the current as the prongs move to and fro. A fairly large inductance is necessary in the circuit to ensure efficient running, and it is therefore desirable to include a condenser in parallel with the fork contacts to reduce the liability of arcing at the break. The frequency of the fork varies slightly with the setting of the contacts operated by the prongs, and with the change in the electrical constants of the electromagnetic circuit.

* H. M. Dadourian, *Phys. Rev.*, 13, 337, 1919; A. B. Wood, *J. Sci. Insts.*, 1, 330, 1924; A. B. Wood and J. M. Ford, *ibid.*, 1, 161, 1924.

Eccles * has described a method of driving a tuning-fork by means of a thermionic valve in appropriate circuits, which suitably excite electromagnets which act upon the prongs. Mechanical contacts with the moving prongs are avoided, and the drive, which is one of considerable precision, is applicable to forks of higher frequency. A 1000-cycle fork used by Dye † in studying the value of a fork as a precision standard, is illustrated in fig. 60. Eckhardt, Karcher, and Keiser ‡ have described

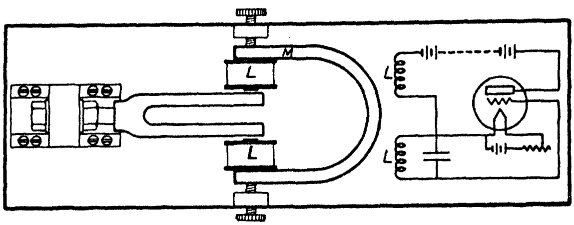


FIG. 60.—Valve-maintained tuning-fork (Dye)

a modified form of drive which has some advantages, and Horton, Ricker, and Marrison § adopted a variation of the latter for maintaining a 100-cycle fork. In Dye's apparatus two electromagnetic coils L, situated respectively in the grid and plate circuits of the thermionic valve, are mounted upon a laminated projection of a polarising magnet M. The fork is self-starting—a convenience, since for constancy such forks are often kept in a constant temperature enclosure where they are not readily accessible.

Variation in the temperature of forks is the chief cause of the variations of frequency. Such variations with steel forks are usually of the order of 0·01 per cent. decrease in frequency per degree rise in temperature, and arise mainly from changes in elastic properties. To obtain constancy to 1 part in a million, it would be necessary to keep the temperature constant to 0·01° C., and Dye pointed out that 'elinvar' (a nickel steel) would give a tenfold improvement in this respect. No important frequency changes arise from ordinary variations in the electrical voltages

* W. H. Eccles, *Phys. Soc.*, *Proc.*, 31, 269, 1920.

† D. W. Dye, *Roy. Soc.*, *Proc.*, 103, 240, 1923.

‡ E. A. Eckhardt, J. C. Karcher, and M. Keiser, *Op. Soc. Am.*, *J.*, 6, 949, 1922.

§ J. W. Horton, N. H. Ricker, and W. A. Marrison, *A.I.E.E.*, *Trans.*, 42, 730, 1923.

applied to the valves, nor from changes in the electrical constants of circuits.

Butterworth,* in a mathematical treatment of the problem, pointed out the utility of connecting condensers in parallel with the plate and grid coils. Hodgkinson,† considering circuits without condensers, found it advantageous to employ transformers between the valve and the magnet coils, in the case of forks having a frequency of about 50 cycles per second.

* S. Butterworth, *Phys. Soc., Proc.*, 33, 345, 1920.
† T. G. Hodgkinson, *Phys. Soc., Proc.*, 103, 240, 1923.

CHAPTER XI

ANALYSIS AND FILTRATION OF SOUND

Analysis with Selective Microphone Equipment. Where it is desired to identify the frequencies and intensities of component tones of a complex sound, the analysis may be performed in various ways. On the one hand, instruments like the Helmholtz resonator —in conjunction with suitable hot wire, Rayleigh disc, or membrane detectors—may be used for measuring the intensity of sounds of a definite pitch. Consequently a continuously variable resonator or a series of such resonant instruments would enable the whole audible range of pitch to be explored. In practice it is not possible to cover the whole audible range with one instrument, and a series is required. This is a disadvantage. A more elastic method of analysis is to employ a high-quality microphone to reproduce the acoustical wave in electrical form, and to analyse the electrical e.m.f. generated. The method has the advantage that continuous tuning can be adopted, and control is possible at a distance from the point where the microphone is situated.

A simple form of analyser consists of a tunable electrical circuit introduced into the electrical amplifier associated with a microphone. This tuned circuit may be located at the input of the amplifier, at the output, or at an intermediate stage.

Crandall and MacKenzie * analysed speech by means of electrically tuned output circuits. After preliminary amplification the wave-form to be analysed was transmitted to twin single-stage amplifiers in parallel. Coupled to the output of one of these amplifiers was a resonant circuit containing a hot-wire microammeter, the deflections of which gave a measure of the amount of energy which lay within the limits of transmission of the tuned circuits. Coupled to the other was an untuned circuit and microammeter which dealt with all frequencies equally, and

* I. Crandall and D. MacKenzie, *Phys. Rev.*, 19, 221, 1922.

thus gave the total energy of the syllable uttered. The syllable was necessarily repeated many times, and the untuned circuit gave a check upon the total loudness on any occasion. The frequency range 75–5000 cycles per second was covered in 23 steps. The damping * (Δ varied from 75–250) was such that the response fell off by about 50 per cent. if the analyser were mistuned by about 12–40 cycles per second.

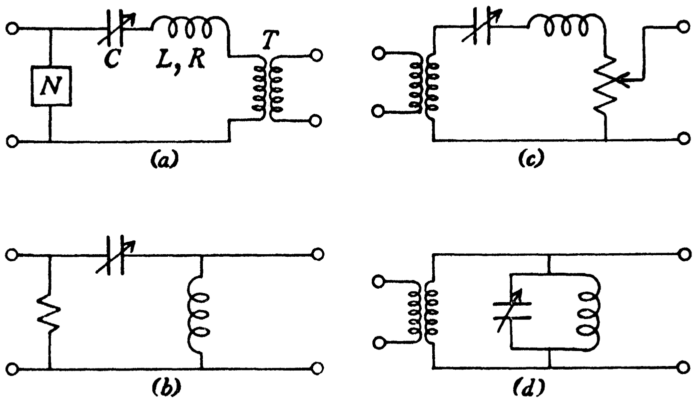


FIG. 61.—Single selector circuits for analysis
(In cases (*b*) and (*d*) the sensitivity varies considerably with frequency for a given sound pressure)

Wegel and Moore † described an analyser in which the variable-tuned circuit was located at the input of the amplifier. The instrument, which was extremely elaborate, was direct recording and covered a frequency range of 20–5000 cycles per second. Fig. 61 (*a*) is a diagram of the essential elements of the arrangement. The wave to be analysed was introduced to an input-shunting network N and to the variable-tuned circuit. The input network (actually a condenser in parallel with an inductive resistance) had a low impedance varying from 0·4 ohm for direct current to 10 ohms at 5000 cycles per second. A low impedance in parallel with the input was essential to the performance of the analyser, but the variation of this impedance

* If an analyser circuit has a damping factor Δ, two equally strong components, of pulsatance ω_1 and ω_2 respectively, will need to be separated by an amount of about $\omega_1 - \omega_2 = 2\Delta$ before they can be separately detected, and by an amount $\omega_1 - \omega_2 = 10\Delta$ before they can be separately measured without more than, say, 25 per cent. error in amplitude.

† R. L. Wegel and C. R. Moore, *A.I.E.E., J.*, 43, 798, 1924.

with frequency represents an empirical design found necessary to compensate for certain high-frequency losses in the amplifier rectifier system, and thus to make the arrangement equally sensitive to all frequencies in the range 80–5000 cycles per second.

The tuned circuit consisted of a variable condenser of capacity C and inductance L of ohmic resistance R. The value of the capacity C was varied from 0·05–200 microfarads in small steps by an automatic device. The inductance L consisted of four identical windings (100 millihenries and resistance 3–16 ohms) on a toroidal core which, by means of a switch, could be thrown in series or in parallel, thereby changing the value of the inductance in the ratio 16 : 1. With the same range of capacity the instrument then had two frequency ranges 20–1250 cycles and 80–5000 cycles per second. By means of the high ratio transformer T of negligible impedance the response of the circuit was applied to an amplifier-rectifier unit, and registered by means of a recording meter.

The impedance of the source of the complex wave was in practice maintained high at all frequencies in comparison with that of the input network, so that the input wave-form was independent of the small changes in impedance of the analyser due to varying the condenser C. For each capacitance value there was a frequency ($f_r = 1/2\pi\sqrt{\mathrm{LC}}$) for which the impedance Z of the tuned circuit was R. For other frequencies Z was much greater, being actually $\mathrm{R} + jw\mathrm{L} + 1/jw\mathrm{C}$. An incoming current of frequency f_r was therefore largely shunted through the tuned circuit, while current of any other frequency passed through the input network. In this way, as the capacitance C was varied gradually, the tuned circuit shunted selectively from the input network the successive components of the complex wave. The apparatus measured component frequencies as close together as 15 cycles per second at the lower end and 200 at the upper end of the range, and detected even closer components. Sharper tuning arrangements would be advantageous by affording better resolution in this latter region. Pl. V, p. 225, shows an analysis of a buzzer note made with the instrument. Firestone * described a portable arrangement which was designed closely on the above general principle, but which was not automatic in operation. The analysing circuit (fig. 61 (*b*)) was included in

* F. A. Firestone, *Soc. Auto. Eng., J.*, 19, 461, 1926.

an intervalve coupling, and a simple 3-ohm resistance was used instead of an equalising network in parallel with the tuned circuit, and the output from the analyser was obtained from the potential difference across the inductance L instead of from a transformer. This arrangement is most sensitive to high frequencies. Spooner and Foltz * have also described analysers (fig. 61 (*c*) and (*d*)) with two types of intervalve coupling, one of which (*c*) resembles that of Wegel and Moore, except that a resistance replaces the transformer T.

The disadvantage of a single-tuned circuit is that no very great sharpness of resonance is obtainable, particularly at low frequencies. Unless coils of prohibitive weight are employed, their resistance is such that blunt tuning only is attained. Improvement may be sought by using a second tuned circuit to follow the first in the same apparatus, and may be achieved in various ways. Selectivity may be afforded by means of two tuned circuits inserted at two successive intervalve stages of an amplifier. This double tuning of the amplifier gives better resolving power between notes of neighbouring frequency, without too great a sensitivity to small variations in the frequency of the component being measured—*i.e.* without too great a sharpness in the immediate neighbourhood of the tuned frequency. It discriminates against extraneous harmonic components near the tuned frequency in proportion to the square of the differences between their frequencies and that to which the circuits are tuned. The response curve is therefore approximately parabolic in form —flat near the tuned frequency, but falling off sharply at frequencies substantially removed. The two tuned circuits may be inserted at two successive stages of an amplifier, or may be coupled together at one intervalve stage. Churcher and King † used two tuned circuits loosely coupled together as an output coupling. For high frequencies the coupled coils had an air core, but at low frequencies coils with an iron-nickel core were employed so that requisite values of resistance and inductance could be obtained in a small space at reasonable cost. Each core had an air-gap which alone controlled the inductance of the coils, so that there was no appreciable variation of inductance with current.

The coupling between the iron-core coils was varied by having

* T. Spooner and J. P. Foltz, *A.I.E.E., J.*, 48, 199, 1929.
† B. A. G. Churcher and A. J. King, *I.E.E., J.*, 69, 97, 1930.

a variable number of turns on the second coil in series with the first coil.

Analysers Involving Heterodyne Arrangements. Just as it proves convenient in generating audio-frequency currents to avoid unwieldy condensers by heterodyning two high-frequency oscillations, it has proved possible to get increased convenience and great selectivity * in analysers by adopting heterodyne arrangements. In this method of analysis (fig. 62) the constituent tones of complex sound, after being converted into the

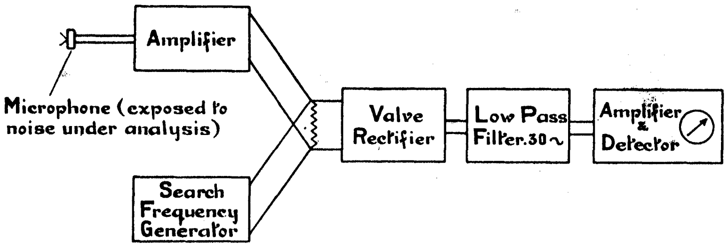


FIG. 62.—Analysis of complex sounds using a search frequency (Grützmacher) diagram of apparatus

corresponding electrical audio-frequency variations by a high-quality microphone, are heterodyned in turn by an electrical exploring voltage of pure wave-form and constant amplitude, but variable frequency. The mixture is then rectified and passed on to an amplifying and detecting unit. The rectifier unit generates—and passes on with the general noise and search frequencies—the combination tones between the search tone and components of the noise. That is, if the search tone has a frequency s, and if a component of the noise has a frequency n, the rectifier passes also the difference tone of frequency $s-n$ (or $n-s$) and the summation tone of frequency $s+n$. Actually tones of frequency $2s$ and $2n$ are also generated.

In Grützmacher's † form of analyser a filter now suppresses all frequencies greater than 30 cycles per second. When, as is usually the case, the fundamental of the noise and the search tone are both higher than 30 cycles per second, the indicator is affected only when $s-n$ is less than 30, *i.e.* when the search tone —of which the frequency is known—lies within 30 cycles per

* This selectivity is usually welcome. Where, however, a noise is not of fixed pitch, but is varying slightly, too great a sharpness renders the analyser inconvenient in operation.

† M. Grützmacher, *E.N.T.*, 4, 533, 1927.

second of a component of the noise. For the other frequencies s, n, $2s$, $2n$ and $s+n$ are all rejected by the filter. Thus the frequencies of constituent tones of the noise may be detected as the frequency of the search tone is gradually varied over the audible range. For a search tone of constant magnitude the amplitudes of the components of the noise are respectively proportional to the amplitudes of the difference tones they set up, and may therefore be measured.

One inconvenience of Grützmacher's form of analyser is that each component of the noise gives two indications upon the indicator. The first of these arises when the search tone is 0–30 cycles per second below the frequency of the component of the noise, and the other when it is 0–30 cycles above. When the search tone and the noise component are identical in frequency the difference tone is of zero frequency (*i.e.* is not alternating), and nothing is passed by the amplifier to the recorder.

The analyser of Moore and Curtis * avoided double indications. It employed as search tone the output from a variable high-frequency oscillator ranging from 11,000 to 16,000 cycles per second. For detecting the combination tone $s-n$ a highly selective resonator was used instead of the filter shown in fig. 71. Then, as the search frequency was varied continuously, an indication was obtained on the detector instrument whenever $(s-n)$ became equal to the natural frequency of the resonator. The resonating element was tuned to 11,000 cycles per second. As the search frequency was always above this, no summation frequencies affected the resonator. The instrument was not used for sounds above 5000 cycles per second, so that the octave term in $2n$ lay below the resonant frequency of the selector. Indeed, the frequency ranges had been carefully chosen to exclude all undesired terms which could not be made negligibly small.

It may be mentioned that if the resonating element had a frequency of 16,000 cycles per second, whilst the search tone varied as before between 11,000 and 16,000 cycles per second, the instrument would operate on the summation tone $s+n$.

Naturally, an extremely sharply tuned resonant circuit was necessary, and Moore and Curtis employed a mechanical one. They made use of the longitudinal vibrations of a steel bar clamped in the middle, and having a natural frequency of 11,000 cycles per second. It was excited magnetically at the driving

* C. R. Moore and A. S. Curtis, *Bell Sys. Tech. J.*, 6, 217, 1927.

end by means of the magnetic element of a telephone receiver, deriving its current from the rectifier of the analyser. The response at the other end of the bar was detected by another magnetic element which, in turn, communicated with the indicator. The bar had a sharpness of resonance (ω/Δ) of about 30,000. Corresponding to this a departure of 10 cycles per second from the resonant frequency gave a loss of over thirtyfold in voltage, so that, even at frequencies as low as 50 cycles per second, the frequency discrimination was quite satisfactory.

Some further details of heterodyne analysers may be noted. The indicator should be quick acting and well damped. By means of a recording drum rigidly coupled to the frequency-changing condenser of the heterodyne oscillator, Grützmacher (who analysed the sounds of combinations of pure notes, of the violin, of organ pipes, and of German vowels) was able to record photographically (Pl. V, p. 225) the deflections corresponding to the separate partial tones of a sound. Complete records over the frequency range 80–10,000 cycles per second were taken in a few seconds. There are limits to the speed at which the frequency range can be traversed, because oscillations will not be fully set up in the filter circuit or resonator unless a certain amount of time is allowed for exciting them. Grützmacher claimed that a speed giving in each second a frequency change of 300 cycles per second could be combined with a sharpness of 30 cycles. He indicated also that, even if only impulsive excitation is applied to the filter circuit, a quantitative evaluation of the analysis is possible. Salinger * has dealt with the theory of analysis by means of exploring tones.

It is desirable to note that mixing the sound frequencies and the search frequency in a common resistance, as shown in fig. 62 and in fig. 63 (*a*), has disadvantages. In the case of a noise, or of a complex sound having several components close together, the measuring instrument may give a deflection entirely independent of the frequency of the search tone, and which persists even when the search tone is removed. This is due to difference tones—produced by the rectifier—between components of the sound which are so close together that the difference tone is lower than 30 cycles per second. Grützmacher's solution of the difficulty † is to adopt a push-pull form of rectifier, using accurately

* H. Salinger, *E.N.T.*, 6, 294, 1929.

† M. Grützmacher, *Zeits. f. tech. Phys.*, 10, 570, 1929.

matched valves, and to apply the sound and search frequencies as shown in fig. 63 (*b*). With proper winding of the centre-tapped transformer the sound frequencies themselves are not passed onwards, as the two valves give equal and opposite effects in the output. Combination sounds between the search tone and the sound frequencies are, however, passed by the rectifier to the filter as before.

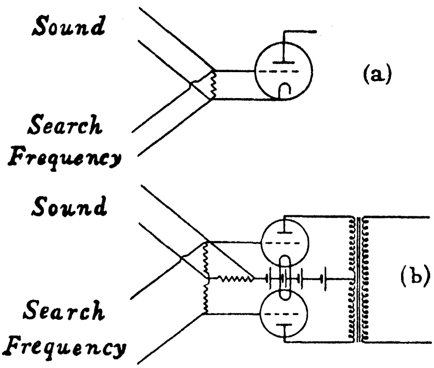


FIG. 63.—Rectifier units for heterodyne analysers

In the rectifiers use is made of the quadratic portion of the valve characteristic, since no difference tones of higher order are then produced. Calculation for quadratic rectification gives for the anode current I_A:

$$\begin{aligned} I_A &= K(a_0 + a_1 \sin pt + a_2 \sin qt)^2 \\ &= K\{a_0^2 + 2a_0a_1 \sin pt + 2a_0a_2 \sin qt + \tfrac{1}{2}a_1^2 \cos 2pt \\ &\qquad + \tfrac{1}{2}a_2^2 \cos 2qt - a_1a_2 \cos (p+q)t - a_1a_2 \cos (p-q)t\} \qquad (1) \end{aligned}$$

p and q being respectively the frequencies of a component tone of the complex voltage, and of the search tone. Only the difference tone passes the filter. The amplitude of the difference tone is seen to be proportional to the producing voltages a_1 and a_2, so that, when the amplitude a_2 of the search voltage is constant over the frequency range, the amplitude of the difference tone is proportional to the amplitude a_1 of the component of the sound.

It is important that the characteristic of the rectifier valve shall not be of a higher order than the second, for difference tones of higher order such as $2p-q$ and $2q-p$ would be produced, and result in false partial tones in the analysis. Care is therefore necessary in selecting the rectifier valve, its working point, and its loading. By 'analysing' a known pure tone, the functioning of the analyser may be tested and any tendency to produce false partials detected. Also by analysing a mixture of two pure tones one may detect any defect in the apparatus which would give rise to a difference tone between the two pure tones.

The disadvantage of heterodyne analysers of the above types is the necessity for employing only an accurately quadratic portion of the characteristic of a rectifying valve. The elimination of false components due to failure in this respect is a matter of some difficulty. T. Theodorsen * employed a method of search-tone analysis in which this difficulty was obviated. He availed himself of the heating effect of audio-frequency currents—a property which depends essentially upon the square of the current concerned, and thus is accurately quadratic. His method may conveniently be regarded as based upon the fact that the ohmic loss in an electrical resistance is equal to the sum of the losses of the harmonic components of a complex wave except for the case in which two components are nearly equal in frequency and equal (or opposite) in phase. In the latter case the ohmic loss is then increased (or decreased) by a definite amount. When the frequencies are nearly equal, the resistance wire thus exhibits a slow periodic temperature variation at a frequency equal to the difference between the frequencies of the two components. The heat capacity of the resistance ensures that although it follows these slow fluctuations it does not follow the higher-frequency fluctuations. The selectivity of the device thus depends upon the heat capacity of the wire and it is not dependent upon frequency.

In practice the ohmic resistance employed is the filament of a valve, and the complex current and search tone are both passed through it. The filament is chosen to have the correct dimensions to obtain suitable selectivity, and the temperature fluctuations are easily made manifest by changes of anode voltage due to variations in the electron emission from the filament. For small fluctuations of filament temperature it was considered sufficiently accurate to assume proportional changes in anode voltage. The device is followed by an amplifier, and, through a transformer if necessary, by a recording milliammeter having a natural frequency of about three vibrations per second.

Provided the search-tone current is constant in magnitude, the indications so obtained are proportional in magnitude to the amplitudes of the constituents of the complex wave.

A diagram of the actual arrangement is shown in fig. 64. Two valves, V_1, V_2, are used, and the complex wave and search tone are supplied to their filaments so that if the phase-angle

* T. Theodorsen, *U.S. Nat. Adv. Cttee. Aero., Report*, No. 395, 1931.

difference is α in one filament it is $\pi + \alpha$ in the other. This push-pull method of introduction of the voltages, combined with accurate matching of the valve circuits, ensures that the output voltage taken from across the anode resistances R_1 and R_2 shall not contain fluctuations due to the combination of two constituents of the sound which may happen to be close in frequency, but only those due to the combination of the search tone with separate constituents of the noise. Certain high-resistance potentiometers which permit adjustment of the grid bias and

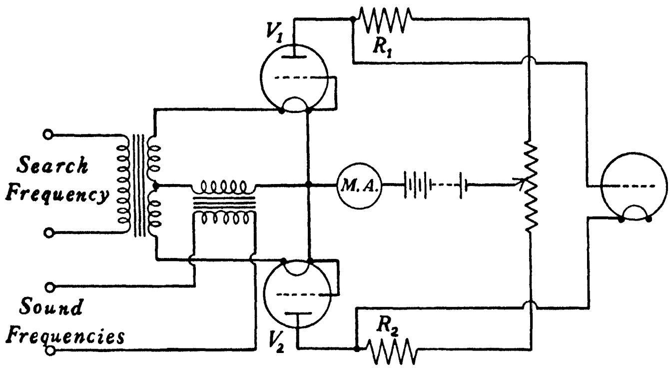


FIG. 64.—Essential features of heterodyne analyser due to Theodorsen

thus of the valves have been omitted from the diagram. The output is led to an amplifying valve (shown to the right of the figure) and thence to a recorder. When the apparatus is adapted for automatic recording about 5 minutes are required to obtain a record (Pl. V, p. 225).

Meyer * has devised a very simple method of sound analysis which requires a minimum of apparatus and which he claims to be sufficiently good for many purposes. The principle is that an *alternating* current $J \sin 2\pi st$ is passed through a carbon microphone instead of the usual direct current, and if at the same time the microphone is subject to a pure tone of frequency p, then the voltage across the microphone is given by

$$E = (R + dR \sin 2\pi pt) J \sin 2\pi st \qquad (2)$$

where R is the constant and dR the variable part of the microphone resistance. This gives

$$E = JR \sin 2\pi st + \tfrac{1}{2}J \cdot dR \cos 2\pi(s-p)t - \tfrac{1}{2}J \cdot dR \cos 2\pi(s+p)t \qquad (3)$$

* E. Meyer, *E.N.T.*, 5, 398, 1928.

In order to carry out an analysis the search frequency s is varied at constant current over the desired range, and an a.c. instrument tuned to a low frequency is employed to select the difference frequency $s-p$ from the voltage E. This difference frequency is proportional to dR and thus, with a distortionless microphone, to the amplitude of the partial tone in the sound under analysis. The summation tone $s+p$ does not affect a low-frequency instrument. The search frequency may do so, however, at low frequencies, and is eliminated by the bridge circuit shown in fig. 65. A capacity C in parallel with a low-resistance choke serves to shunt the higher frequencies—especially the frequency $p-s$ when it reaches high values. With a Reiss microphone at M and a sensitive galvanometer, measurements down to 0·1 dyne/cm.^{2} can be made without amplifying arrangements.

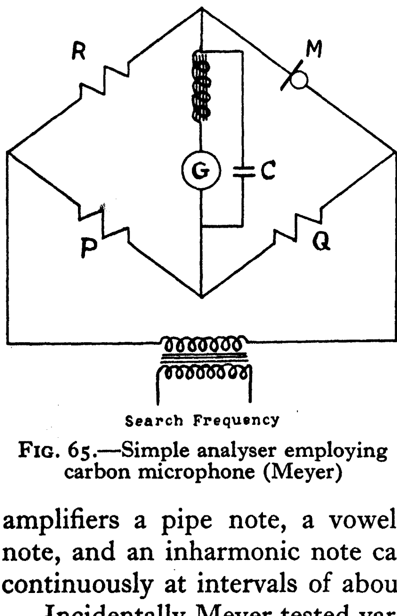


FIG. 65.—Simple analyser employing carbon microphone (Meyer)

Meyer analysed without amplifiers a pipe note, a vowel sung for 15 seconds, a violin note, and an inharmonic note caused by striking an electric bell continuously at intervals of about 1 second.

Incidentally Meyer tested various types of carbon microphone for non-linear distortion, by 'analysing' a mixture of two pure sounds with the aid of the test microphone. When non-linear distortion set in the microphone gave rise to combination tones in the e.m.f., and these were detected in the analysis. It was found that an ordinary carbon microphone gave considerable non-linear distortion. A Reiss microphone gave only small distortion, practically negligible up to intensities of about 6 dynes per sq. cm.—a considerable intensity. Distortion was found to be practically negligible in a double microphone, consisting of a highly stretched diaphragm of duralumin carrying a small carbon button on each side. In such a microphone the sound acts only on one side of the diaphragm. The arrangement of the microphone in the analyser bridge is such that the second carbon button occupies the place of the resistance P.

Analysis by Means of Oscillograph Records. Regular notes consisting of a single harmonic series of tones may be analysed by obtaining an oscillograph record of the wave-form, and analysing this into its Fourier components by mathematical processes. Inspection of the oscillograph record may also give useful information where (as in fig. 9 (*b*), p. 30) a fundamental note is accompanied by an overtone which is inharmonic. With sounds or noises in which several unrelated tones occur, the oscillograph record is not an accurately repeating pattern, and analysis into Fourier components is not possible. Consequently, whilst for complete measurement of a complicated sound an oscillograph record is valuable, there are limitations to its use in analysis, particularly where irregular noises are concerned. On the other hand, the electrical analysers described above will detect components irrespective of whether they are harmonically related or not. Sacia * has analysed vowel sounds of short duration by obtaining an oscillograph record in the form of a black profile on a transparent strip, then reproducing the wave-form as a fluctuating beam of light controlled by the passage of the strip in front of a lamp at a steady rate, then converting the wave-form of the fluctuating light into electrical form by means of a light sensitive cell, and finally analysing the wave-form by a resonant electrical analyser of the type described above. The oscillograph records of vowels were naturally of short duration, so the ends of the strip were joined to form an endless belt, and continuous repetition was possible.

Electrical Filters. We have seen that the incorporation of a single resonant element in a circuit leads to the selection of a particular frequency from a complex electrical current, and the practical suppression of frequencies appreciably outside the resonant region. G. A. Campbell has shown that it is possible to devise circuits such that selected *ranges* of frequency are transmitted without appreciable distortion or attenuation, whereas other frequencies are suppressed entirely. In general such electrical filters, as they are called, consist of a chain of similar networks, each composed of specially arranged capacities and inductances. Ideally the chain consists of an infinite number of sections; alternatively it may be closed at any section by an impedance corresponding to that of the rest of the infinite line (*i.e.* to the impedance of the infinite line itself). This impedance

* C. F. Sacia, *Op. Soc. Am.*, *J.*, 9, 487, 1924.

with which the end of the line can be replaced without affecting the currents in previous sections is known as the characteristic impedance of the filter. The theory is outlined on p. 329.

There are two general types of filter section. Each is shown in two forms in fig. 66 according as the section is arranged in a T circuit or in a Π circuit. Assuming that the impedances of the limbs of the filters are represented by Z_1 and Z_2 as indicated in the figures, and that the inductances are resistanceless, Type I will transmit two bands of frequency without attenuation and suppress all other frequencies as shown below the circuit diagram for Type I. The four critical angular velocities ω_1, ω_2, ω_3, and ω_4 have the same values for the T and the Π circuit, and are given by the solution of the equations

$$\frac{Z_1}{Z_2}=0 \qquad \frac{Z_1}{Z_2}+2=0 \tag{4}$$

The values are tabulated later for important special cases, but further details may be obtained from textbooks, or from a paper by A. B. Morice * which has been used in selecting the present material. For reference it may be stated that the characteristic impedances Z_0 of the two circuits are

$$\text{T circuit:—}Z_0=\sqrt{Z_1{}^2+2Z_1Z_2}\,;\quad \Pi\text{ circuit:—}Z_0=\sqrt{\frac{Z_1Z_2{}^2}{Z_1+2Z_2}} \tag{5}$$

and the four critical frequencies are related as follows :—

$$\omega_1/\omega_2=\omega_3/\omega_4$$

Filter Type I is a double 'band-pass' wave filter. There are several ways of reducing it to a single band-pass filter, including the following :—

(*a*) If $L_1C_1=L_2C_2$, then $\omega_2=\omega_3$, and the two bands become confluent.

(*b*) If $L_1=C_2=0$, all frequencies above a specified value are transmitted (high-pass filter).

(*c*) If $L_2=C_1=\infty$ (*i.e.* L_2 disconnected and C_1 short-circuited) all frequencies below a specified value are transmitted (low-pass filter).

Filter Type II is illustrated in the T and Π forms to the right of fig. 66. It will transmit all frequencies except a band between ω_2 and ω_3. As before, the critical frequencies are given by the solution of equations (4), and the characteristic impedances

* A. B. Morice, *Phil. Mag.*, 3, 801, 1927.

by equations (5). This type of filter has only one critical frequency

(*a*) if $C_1 = 0$ or $L_2 = 0$ (low-pass filter); or
(*b*) if $C_2 = \infty$ or $L_1 = \infty$ (high-pass filter).

It should be reiterated that the properties of these two types of filter only apply rigorously when an infinite number of sections,

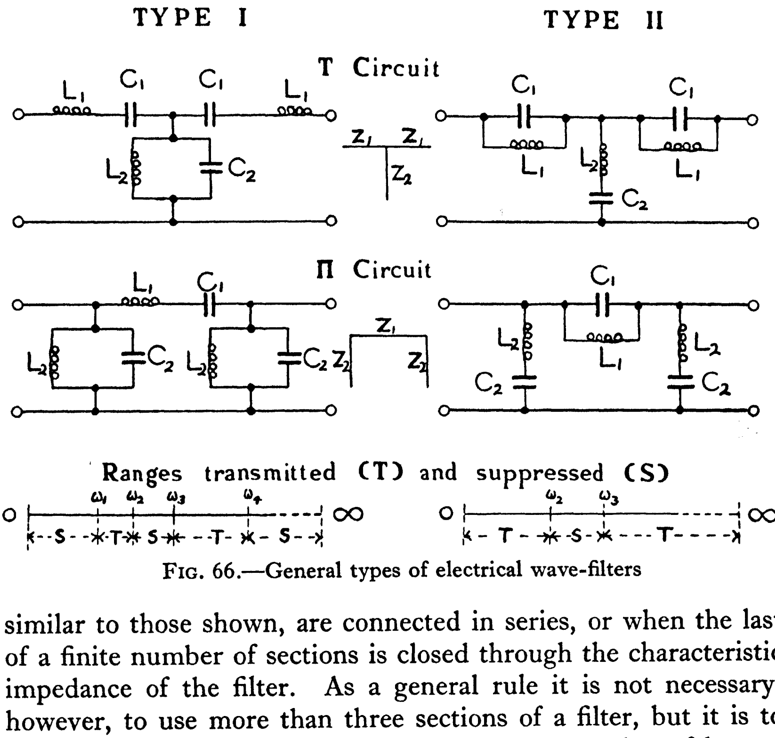


FIG. 66.—General types of electrical wave-filters

similar to those shown, are connected in series, or when the last of a finite number of sections is closed through the characteristic impedance of the filter. As a general rule it is not necessary, however, to use more than three sections of a filter, but it is to be noted that a multi-section filter will have a number of humps in the pass section of its transmission curve equal to the number of sections used. Further, the greater the number of sections the sharper the cut-off. The effect of resistance in the coils and of leakance in the condensers is to make the action of the filter more or less imperfect: the critical frequencies are, however, the same as those of an ideal filter.

Important particular types of the above filters are shown as three-section filters * in fig. 67. The corresponding critical

* Note that two capacities C in series, one from each T section, have a resultant capacity of C/2.

frequencies and characteristic impedances of infinite filters of these types are set out in Table VI. Ultimately, in the pass region, the characteristic impedances all become of the form $k\sqrt{L/C}$, where k is a numeric of value depending upon which circuit is

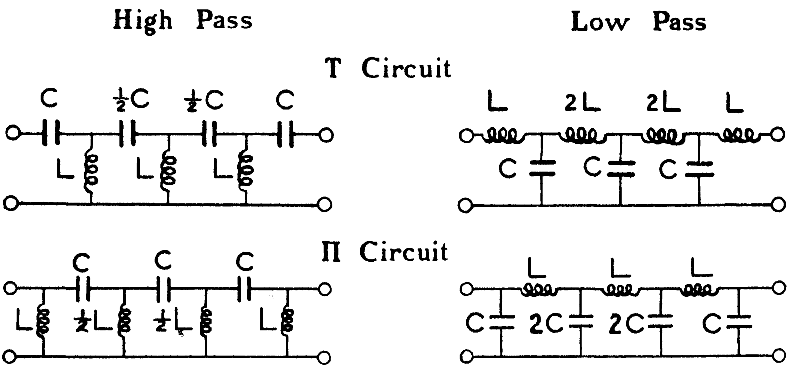


FIG. 67.—High-pass and low-pass filters (3 sections)

involved. This impedance is of the nature of a pure constant resistance, so that often a filter circuit may be appropriately terminated by a suitable output resistance, as far as the greater part of the transmission band is concerned.

TABLE VI

Characteristics of High-pass and Low-pass Filters

Type	Critical Frequency	Characteristic Impedance
	$\omega_c =$	$Z_0 =$
High-pass—T type . .	$\sqrt{\frac{1}{2LC}}$	$\frac{\sqrt{2LC\omega^2 - 1}}{\omega C}$
Π type . .	$\sqrt{\frac{1}{2LC}}$	$\frac{\omega L}{\sqrt{2LC\omega^2 - 1}}$
Low-pass—T type . .	$\sqrt{\frac{2}{LC}}$	$\sqrt{\frac{L(2 - LC\omega^2)}{C}}$
Π type . .	$\sqrt{\frac{2}{LC}}$	$\sqrt{\frac{L}{C(2 - LC\omega^2)}}$

C. A. Beer and G. J. S. Little * give useful information concerning the design of filters. They mention that the

* C. A. Beer and G. J. S. Little, *P.O.E.E., J.*, 17, 298, 1925.

terminating impedance of a finite filter cannot be made equal to the characteristic impedance at all frequencies, but that it can be at all frequencies which lie well in the transmission band. A good empirical rule with a low-pass filter is to make the equality at about 0·8 times the critical frequency. Details of the construction of various filter ranges are given in the paper, together

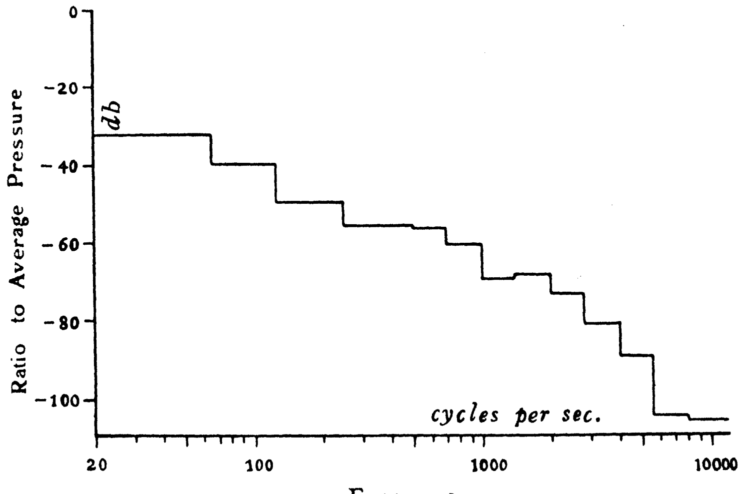


FIG. 68.—Distribution of sound energy in street noise (Sivian)
(Average pressures per frequency interval of 1 cycle per second)

with information on the stalloy stampings suitable for producing the iron-core inductances (with air-gap) that make suitable inductances for filters.

Analysis with Band-pass Filters. L. J. Sivian * described a method of analysis of sound in which exact analysis is not achieved, but in which measurement is made of the sound energy which occurs in each of a series frequency bands, each about half an octave wide, which together cover the audible scale from 30–10,000 cycles per second. For this purpose band-pass filters are adapted for use with the microphone-amplifier equipment employed in sound measurement, and arranged to pass into it, one after the other, selected frequency bands—say, 30–100 cycles per second, then 100–150 cycles per second, and so on. Fig. 68 shows an analysis of this kind for street noise, due chiefly to street traffic and an elevated railway. It should be noted

* L. J. Sivian, *Bell Sys. Tech. J.*, 8, 646, 1929.

that the pressures recorded are regarded as equally distributed within the bands, and the results are therefore plotted in the form of "average pressures per interval of 1 cycle per second." The chief present limitation of the method is that the separation of successive bands selected by the electric filters is not as sharp as might be desired.

Stewart's Acoustical Filters. Following the development of electrical filters, G. W. Stewart * has proposed certain acoustical filters which are applicable to transmission of sound through tubes, and which consist of repeated resonator sections. The impedance theory upon which Stewart worked is analogous to the theory of electrical filters. It involves the 'lumping' of acoustical elements in the manner which was adopted (p. 119) in the theory of Helmholtz resonators, where a short column of air was regarded primarily as a lumped *mass* of air, and an enclosed volume was regarded as a *capacity*. In order that this procedure may be justifiable in the theory of acoustical filters, the length of each section and the dimension of each branch must be small compared with the wave-length of the sound, so that no change in phase may occur in any element.

In the theory of acoustical filters a circuit of tubes and containers is analogous to an electrical circuit, and the direct analogue to electrical current in the electrical circuit is the volume flow of air in the mechanical network. Thus for filter circuits the corresponding equations of p. 21

$$\mathrm{L}\ddot{q}+\mathrm{R}\dot{q}+\frac{q}{\mathrm{C}}=\mathrm{E}_0\cos\omega t \tag{6}$$

$$m\ddot{\xi}+r\dot{\xi}+s\xi=\mathrm{F}_0\cos\omega t$$

become most strictly analogous when we deal with the acoustical volume changes $\mathrm{X}(=\mathrm{S}\xi)$ and acoustical pressures per unit area $f_0=(\mathrm{F}_0/\mathrm{S})$, where S is the area of cross-section of the acoustical conduit at the point concerned: that is, when the mechanical equation is written in the form

$$\frac{m}{\mathrm{S}^2}\ddot{\mathrm{X}}+\frac{r}{\mathrm{S}^2}\dot{\mathrm{X}}+\frac{s}{\mathrm{S}^2}\mathrm{X}=f_0\cos\omega t \tag{7}$$

Thus the true analogues to the electrical L and 1/C which occur

* G. W. Stewart, *Phys. Rev.*, 20, 528, 1922; G. W. Stewart and R. B. Lindsay, *Acoustics*, p. 161.

in the equations for electrical filters, are the inertance m/S^2 and the capacitance s/S^2 respectively.

Fig. 69 is a diagram of a low-pass acoustical filter. The total inertance—analogue of inductance—of each section of conduit of length l_1 is $\rho l_1/S_1$, where S_1 is the area of cross-section. If V_2 is the volume of the resonator chamber, its capacitance is $C_2 = V_2/\rho c^2$. Neglecting the inertance of the resonator neck which is strictly in series with the capacitance, the cut-off pulsatance ($\omega_c = 2\pi n_c$) is, by analogy with the formula for a low-pass electrical filter, $2c\sqrt{S_1/l_1V_2}$. Stewart shows that if allowance is made for the inertance ρ_0/K of the neck the equation becomes

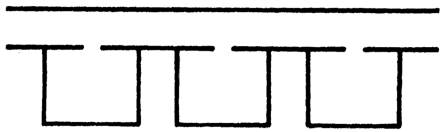


FIG. 69.—Low-pass acoustical filter

$$\omega_c = 2c\sqrt{\frac{S_1}{l_1V_2}\left(\frac{1}{1+\frac{4S_1}{l_1K}}\right)} \tag{8}$$

Here K, the conductivity of the neck, is given by $\frac{A}{l+2\alpha}$, where l is the length and A the sectional area of the neck (p. 120).

Stewart's acoustical analysis in this case may be set out briefly as follows. If the parts of the filter are small compared with the wave-length of the sound concerned, the mass of air in the conduit between successive junctions moves as a whole. Let the filter be infinite, let P_n, P_{n+1}, etc., represent the pressures at the junctions between the conduit and side branches, and let $\dot{X}_n$, $\dot{X}_{n+1}$, etc., represent respectively the volume currents in the sections of conduit following the junctions. The corresponding particle velocities may be denoted by $\dot{\xi}_n$ and $\dot{\xi}_{n+1}$, and the volume currents into the resonators at the junctions may be denoted $\dot{X}_n'$ and $\dot{X}_n'{}_{+1}$, etc.

From the motion in the air in the section between p_n and p_{n+1} we have

$$p_n - p_{n+1} = \rho l\ddot{\xi}_n = i\omega\rho l\dot{\xi}_n = i\omega\rho l\dot{X}_n/S_1 \tag{9}$$

From the continuity at the junction the volume flow into the resonator is equal to the difference between the flows in the conduit on the two sides of the junction. Consequently, if Z_2 is the

acoustical impedance of the resonator at its opening, we have

$$\left.\begin{aligned} \dot{X}_{n-1}-\dot{X}_n=\dot{X}_n{}'=p_n/Z_2, \quad i.e. \quad p_n=Z_2(\dot{X}_{n-1}-\dot{X}_n) \\ \text{Similarly} \qquad p_{n+1}=Z_2(\dot{X}_n-\dot{X}_{n+1}) \end{aligned}\right\} \tag{10}$$

Consequently, rewriting we have

$$Z_2(\dot{X}_{n-1}-\dot{X}_n)-Z_2(\dot{X}_n-\dot{X}_{n+1})=i\omega\rho l\dot{X}_n \tag{11}$$

Whence

$$\dot{X}_{n+1}-\left(2+\frac{i\omega\rho l}{Z_2S_1}\right)\dot{X}_n+\dot{X}_{n-1}=0 \tag{12}$$

It is readily evident that in an infinite filter the relation between adjacent sections will be independent of the position of those sections in the length of the filter, consequently we may write *

$$\frac{\dot{X}_{n+1}}{\dot{X}_n}=\frac{\dot{X}_n}{\dot{X}_{n-1}}=e^{-\lambda} \quad \text{say} \quad =\frac{p_{n+1}}{p_n}=\frac{p_n}{p_{n-1}} \tag{13}$$

The solution of (12) is then

$$\cosh\gamma=1+\tfrac{1}{2}\frac{i\omega\rho l}{Z_2S_1} \tag{14}$$

and it may be inferred (see p. 330) that the acoustical conduit acts as a filter, the limit of no attenuation being bounded by the limits

$$i\omega\rho l/Z_2S_1=0 \quad \text{and} \quad i\omega\rho l/Z_2S_1=-4 \tag{15}$$

Z_2 is given by the formula on p. 121, the term in $k\omega/2\pi$ having been neglected because damping due to radiation from the aperture into free air does not now occur. Thus

$$Z_2=\rho i\left(\frac{\omega}{K}-\frac{c^2}{V_2\omega}\right) \tag{16}$$

It follows on substitution in (15) that the region of no attenuation is bounded by the frequency given by (8) and by $\omega=0$.

Fig. 70 (*a*) gives a sketch of a Stewart low-pass filter calculated to have a cut-off at 3175 cycles per second, together with the experimental curve obtained.

A high-pass filter is indicated in fig. 70 (*b*). The branch lines consist merely of orifices. A somewhat empirical theory indi-

* A formal proof of this identity can be derived on the lines of that given for electrical filters on p. 329.

cates that the cut-off frequency for such filters is given by $\omega_c = c\sqrt{K/V_1}$, where V_1 is the volume of one section of the line, and K the conductivity of one of the branch lines.

Band-pass filters can be constructed. Such a filter due to Stewart is shown in fig. 70 (*c*). If Stewart's simple theory applied over the whole range only one band should be obtained, but the beginnings of an additional high-frequency band are noticeable in the experimental results. Physically the existence of additional transmission bands is due to the fact that for high frequencies the dimensions of the filter are no longer small compared with the wave-length. G. W. Stewart* has extended his theory to meet the case. W. P. Mason † has, however, abandoned the lumping of impedances, and studied the filters as regular combinations of acoustic tubes in each of which wave transmission occurs. In his treatment he ascertains the acoustical impedance of each tube or element, and incidentally is able to evaluate corrections for viscous and thermal dissipation. He also calculates the effect of inserting filters in any given acoustic

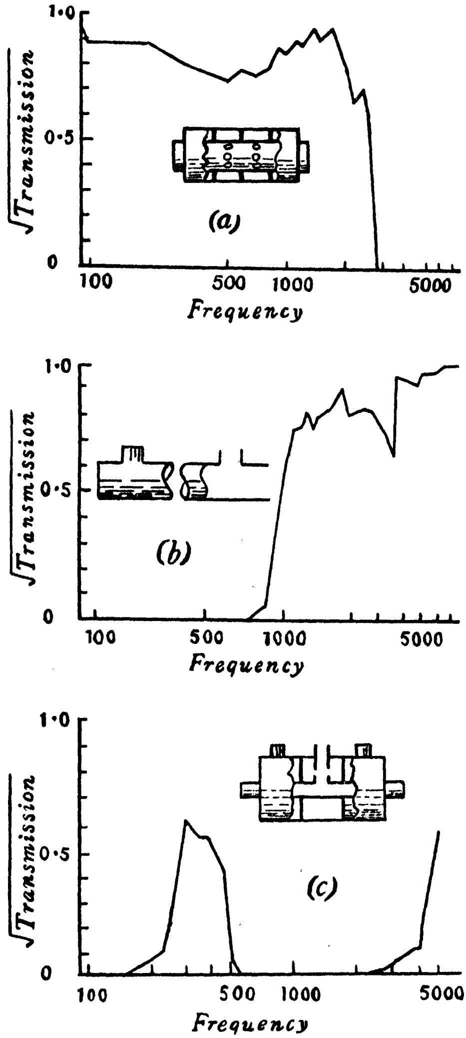


FIG. 70.—Acoustical filters (Stewart)
(*a*) Low pass, (*b*) high pass, (*c*) band pass, with corresponding experimental transmission curves

* G. W. Stewart, *Phys. Rev.*, 25, 90, 1925.
† W. P. Mason, *Bell Sys. Tech. J.*, 6, 258, 1927.

system. His theory reveals at once a number of transmission bands in the characteristics of the Stewart type of filter. Waetzmann and Noether have given measurements of transmission through filters.*

In conclusion it may be stated that, within limits, acoustical filters may be designed from Stewart's formulæ based upon lumping of the impedances. The formulæ fail at higher frequencies because it is no longer justifiable to assume no wave-motion in the elements, and acoustical filters differ from electrical filters with lumped impedances in that the former have an infinite number of transmission bands.

Quincke Acoustical Filter. When a resonator is fixed to the side of a sound conduit (fig. 71) it acts as a filter and suppresses

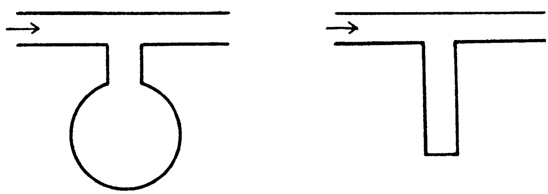


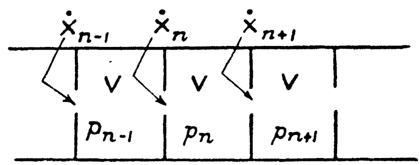
FIG. 71.—Quincke filters

certain frequencies in the sound transmitted through the conduit.† The frequencies suppressed are those natural to the resonator. A stopped pipe (of corrected length l) can of course be used as one form of resonator, and in that case the frequencies are those corresponding to wave-lengths, λ, where $l = n\lambda/4$, and n is an odd integer. The author has employed ripple photographs to illustrate the action of a Quincke acoustical filter.‡ Water waves were passed through a conduit fitted with a side tube of adjustable length.

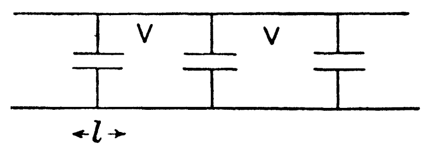
An interesting case of filtering action due to resonance was shown in some ripple experiments with a curved conduit representing a section of a loud-speaker horn.‡ Pronounced stationary waves occurred at a certain bend (the direction of vibration being transverse to the axis of the horn), and the bend acted as a filter over a moderate range of frequency. For frequencies above and below the resonant region, the waves emerged freely from the flare of the horn.

* E. Waetzmann and F. Noether, *Ann. d. Phys.*, 13, 212, 1932.
† G. Quincke, *Pogg. Ann.*, 128, 177, 1866; see Rayleigh, *Sound*, 2, 210.
‡ A. H. Davis, *Phys. Soc.*, *Proc.*, 14, 90, 1928.

Silencers for Engine Exhausts. So far as the writer is aware, no formal theory of the action of silencers of engine exhausts has been given. The following considerations show, however, that the ordinary silencer, which consists of a series of perforated baffle plates inserted in a conduit (fig. 72 (*a*)), is a form of acoustical filter. For simplicity the silencer is considered to be infinitely long and to have an infinite number of sections, and in the first instance it is assumed that each baffle is perforated by one hole only. The radius of the hole is denoted by '*a*,' the volume of the silencer between the baffles by V. In the figure the apertures in the baffle plates are all axial; the theory, however, would apply if they were not axial but were irregularly placed so that, as in practice, exhaust gases would necessarily take a somewhat tortuous path in moving from inlet to outlet.



(*a*) Baffle-plate silencer



(*b*) Modified baffle-plate silencer

FIG. 72.—Exhaust silencers

It is assumed, however, that the apertures in the baffle plates and the dimensions of the interbaffle space are small compared with the wave-length of sound. The pressures in any particular interspace at any moment are therefore uniform and calculable from the adiabatic volume changes which occur. Let the pressures at any moment in the various sections of the silencer be $p_1e^{i\omega t}$ . . . $p_ne^{i\omega t}$, etc., and let the volume velocities in the apertures be $\dot{X}_1e^{i\omega t}$, etc.; the volume displacements will then be $(\dot{X}_1/i\omega)e^{i\omega t}$, etc.

On expressing the excess pressure in the n^{th} section in terms of the excess of gas which has moved into it over that which has flowed out, we have

$$\frac{(\dot{X}_n - \dot{X}_{n+1})e^{i\omega t}}{i\omega V} = \frac{p_ne^{i\omega t}}{\gamma P} = \frac{p_ne^{i\omega t}}{c^2\rho}$$

where c = velocity of sound,
P = mean pressure of gas,
ρ = mean density,
γ = ratio of specific heats of gas,

whence

$$(\dot{X}_n - \dot{X}_{n+1}) = \frac{i\omega V}{c^2\rho} p_n \tag{17}$$

Since the apertures in the baffle plates are small compared with the wave-length of sound concerned, the pressures will be uniform and the gas in each aperture will oscillate to and fro almost as though it was incompressible. Let Z_1 be the acoustical impedance (ratio of pressure to volume flow) of the air in the orifice, an impedance which arises from the effective impedance of the air to pressure exerted upon it.

We then find from consideration of the motion of the air in the apertures

$$(p_{n-1} - p_n)e^{i\omega t} = \text{pressure on air in } n^{\text{th}} \text{ aperture} = Z_1\dot{X}_n e^{i\omega t}$$
$$(p_n - p_{n+1})e^{i\omega t} = Z_1\dot{X}_{n+1}e^{i\omega t} \tag{18}$$

Evaluating $(\dot{X}_n - \dot{X}_{n+1})$ from considerations (18) above and equating to the value given by (17) we find

$$(p_{n-1} - 2p_n + p_{n+1}) = Z_1(\dot{X}_n - \dot{X}_{n+1}) = \frac{Z_1 i\omega V}{c^2\rho} p_n$$

whence

$$p_{n-1} - \left(2 + \frac{i\omega V Z_1}{c^2\rho}\right) p_n + p_{n+1} = 0 \tag{19}$$

writing

$$\frac{p_{n-1}}{p_n} = \frac{p_n}{p_{n+1}}, \text{ etc., } e^{\lambda} \text{ say}$$

We may rewrite (3) in the form

$$e^{-\lambda} - \left(2 + \frac{i\omega V Z_1}{c^2\rho}\right) + e^{\lambda} = 0$$

whence

$$\cosh\lambda = \tfrac{1}{2}(e^{-\lambda} + e^{\lambda}) = 1 + \tfrac{1}{2}(i\omega V Z_1/c^2\rho) = 1 + \tfrac{1}{2}(Z_1/Z_2) \text{ say} \tag{20}$$

It follows from this (p. 320) that the region of no attenuation is bounded by the limits

$$\frac{Z_1}{Z_2} = 0 \qquad \frac{Z_1}{Z_2} = -4 \tag{21}$$

If the quantity Z_1 is taken to be due to the inertance * of an

* The term representing radiation into infinite space is omitted.

aperture of the same diameter in an infinite wall of zero thickness (see p. 200), we may write $Z_1 = i\rho\omega/2a$, where a, the radius of the aperture, is small compared with the baffle. The limits of the attenuation region then occur at frequencies given by

$$\frac{i\omega V}{c^2\rho} \cdot \frac{i\rho\omega}{2a} = 0 \quad \text{or} \quad -4$$

i.e. when $\omega = 0$ or when $\omega = 2c\sqrt{\dfrac{2a}{V}} = 2c\sqrt{\dfrac{c_0}{V}}$ (22)

where c_0 is the 'conductivity' of the aperture.

If, however, the aperture does not consist of a single hole, but of h holes each of radius a, the conductivity is increased. If they are widely separated the conductivity becomes hc_0. If they are close together it probably tends to be equal to that of a single aperture of h times the area of one, and thus of radius $a\sqrt{h}$. Thus for a silencer in which the baffle is perforated with h holes each of radius a the cut-off frequency is given by

$$\omega_c = 2c\sqrt{\frac{2am}{V}} \tag{23}$$

where

$$\sqrt{h} \ngtr m \ngtr h$$

For a few well-separated holes m tends towards h; for a number close together the tendency towards h is probably operative. Rewriting this in terms of the total area S of the h perforations we have

$$S = h \,.\, \pi a^2$$

and

$$\omega_c = 2c\sqrt{\frac{2m}{V}\sqrt{\frac{S}{h\pi}}} \tag{24}$$

If the apertures in the baffles of a silencer are of finite length, and possibly tubular (fig. 72 (*b*)), equation (20) still holds, but Z_1 takes the value $i\omega\rho\left\{\dfrac{l}{S} + \dfrac{1}{c_0}\right\}$ instead of $\dfrac{i\omega\rho}{c_0}$ where l is the length of the tube of cross-section S.

For the cut-off frequency to be lower by an octave or more than that for a simple perforated baffle, clearly $2l$ must be greater than $3\pi a$, *i.e.* $l > 5a$ approx. Thus there is but little difference between a simple perforated baffle and one in which tubes replace

the simple apertures, unless the tubes are fairly long. Tubes could be curved so as to enter the chamber obliquely, and thus impart to the gases a swirling motion which might assist to cool them and thus to reduce their pressure—a factor which is not considered in the above analysis.

When the sections of the filters are so large that they are comparable with the wave-length the above theory fails, and certain bands of high-pitched sounds are transmitted. For instance, transmission occurs at frequencies corresponding to longitudinal resonances in the silencer sections.

Full calculation of the behaviour of silencers is difficult, because they do not conform to ideal geometrical forms. Moreover, some allowance should presumably be made for the effect of the flow, turbulence, and progressive cooling of the exhaust gases as they pass through the conduit, and also for the fact that the large displacements which occur in an actual silencer may lie outside the limits to which pressures may be assumed to be proportional to displacements. In practice also, since an infinite series of sections is not used, it is important that the inlet and outlet impedances should be as nearly as possible the same as the characteristic impedance of the filter.

There are possibilities, for silencing purposes, in other types of conduit. For instance, if an infinite cylindrical exhaust pipe having a diameter d_1 has interposed in its length a short section of length l and diameter d_2, the ratio of the acoustical pressures before and after the expansion chamber so formed is given by

$$\frac{p_i}{p_t} = \sqrt{1 + \left\{\frac{1}{2}\left(m - \frac{1}{m}\right) \sin kl\right\}^2}$$

where $k = 2\pi/\lambda$ and $m = d_2{}^2/d_1{}^2$.

The attenuation, which is zero only when l is an even number of quarter wave-lengths, reaches a maximum value of $\frac{1}{2}\{m + (1/m)\}$ when l is equal to an odd number of quarter wave-lengths. An expansion chamber can thus be arranged to give attenuation in the region of a chosen low frequency such as that of the fundamental note of an engine exhaust. It will also reduce odd harmonics as well. Two chambers in series, suitably separated, would be more effective than one.

A low-pass Stewart filter (fig. 70 (*a*), p. 191) or a series of side-branch Quincke tubes with stopped ends (p. 202) also have

possibilities. Again, Schuster and Kipnis * have shown recently that high-pass acoustical filters of the side-branch type may be used as silencers.

Silencing by absorption (*cf.* p. 287) may be achieved if a section of an exhaust pipe is freely perforated with holes and surrounded with an absorbent material, such as glass wool, enclosed in an outer metal jacket: such a silencer attenuates high frequencies appreciably, although it has but little effect upon low.

It is desirable to mention a practical aspect of silencers which has not been referred to above. It has been assumed that the periodic discharge of exhaust gases into the silencer can be resolved into (*a*) a steady flow of gas, upon which is superimposed (*b*), an oscillatory flow of complex wave-form. Whilst preventing excessive oscillations of pressure from reaching the atmosphere, a silencer must at the same time release the exhaust gases steadily without exerting undue back-pressure upon the engine. Various arrangements of baffles for constricting and changing the direction of the exhaust gases give rise to appreciable back-pressure and accompanying loss of power. Many acoustical filters, however, present no serious obstacle to steady flow, and would appear to be advantageous in this important respect. Indeed, Kauffmann and Schmidt,† who recently designed certain motor-car silencers on acoustical principles, attained the degree of silencing usual with a "normal" car silencer by means of "filters" which gave considerably less back-pressure. Their filters consisted of absorbent sections of exhaust pipe, either alone, or in series with lengths of expansion chamber. The latter were tuned to reduce the low-pitched exhaust fundamental, and the former (which always followed expansion chambers to suppress turbulence noises, and sometimes preceded them as well) attenuated the high frequencies.

Whilst the present position appears to be that many commercial silencers have been arrived at by empirical methods, it is probable that design from acoustical principles would be of assistance, and there are signs of development on these lines.

* K. Schuster and M. Kipnis, *Ann. d. Phys.*, 14, 123, 1932.

† A. Kauffmann and U. Schmidt, *Schalldämpfer für Automobilmotoren* (M. Krayn), Berlin, 1932.

CHAPTER XII

ACOUSTICAL IMPEDANCE AND SOUND TRANSMISSION

Acoustical Impedance Defined. We have seen that the conception of mechanical impedance is of value in calculations of the behaviour of vibrating bodies, and that the quantity known as specific acoustic impedance is of convenience in dealing with the transmission of sound from a source to an infinite medium. It will be recalled that the mechanical impedance is the ratio which the total force on a body bears to the impressed velocity, and that the specific acoustical impedance of a medium is the ratio that the pressure at a point bears to the particle velocity; *i.e.* it is the ratio of the pressure to the volume flux measured over unit area. Often, as when dealing with the transmission of sound through tubes, it is more convenient to deal with the total volume flow of the air than to concentrate upon its velocity. For, particularly when the conduit branches, the total flow of air in the branches must be equal to the flow in the feeder tube. The name 'acoustical impedance' is given therefore to the complex quotient of the pressure (force per unit area) at a surface to the volume velocity (linear velocity multiplied by the area). The unit is the acoustic ohm.

To prevent misunderstanding it should be noted that if a piston of area S is vibrating with velocity $\dot{\xi}$, and a pressure p is developed in the medium at its surface, the acoustical impedance is equal to $p/S\dot{\xi}$. The specific acoustic impedance of the medium is equal to $p/\dot{\xi}$. When a piston of area S is acted upon by a pressure p and thus set in motion with a velocity $\dot{\xi}$, the mechanical impedance of the piston is $Sp/\dot{\xi}$.

Writing the acoustical impedance in the form $Z = R + iX$, the real component R of the acoustic impedance is known as the acoustic resistance; the imaginary part X is known as the acoustic reactance. Acoustic impedances are often written in the form

$Z = R + iX = R + i\left(M\omega - \frac{s}{\omega}\right)$ (*cf.* p. 56), which separates out the positive and negative parts of the reactance. In the positive part the quantity M is known as the 'inertance'; in the negative part the quantity $1/s$ is known as the 'capacitance.' M has the dimension $\frac{\text{mass}}{\text{area}^2}$ and s the dimensions $\frac{\text{stiffness}}{\text{area}^2}$ (*cf.* p. 189).

Some Values of Acoustical Impedance. The acoustic impedance of an infinite uniform tube of cross-section S is equal to $\rho c/S$ * where ρ is the density of the medium in the tube, and c the velocity of sound in the medium. It is a pure resistance. An ideal exponential horn having the same area S of cross-section at its throat should have the same acoustical impedance $\rho c/S$.

The acoustic impedances of the medium surrounding a pulsating sphere, and of the semi-infinite medium into which a piston radiates, may be inferred from the specific acoustic impedance z_s given on pp. 60, 61, by dividing z_s by the area of the driving surface.

The case of the semi-infinite medium driven by a piston in an infinite wall is of some importance. When the piston is small the acoustical impedance becomes

$$\frac{\rho c}{\pi a^2}\left[\frac{k^2a^2}{2} + i\frac{8ka}{3\pi}\right] \tag{1}$$

The real part is the acoustic resistance $(\rho c k^2/2\pi)$, and the acoustic reactance is positive corresponding to an inertance of value $8\rho/3\pi^2 a = 0{\cdot}27\rho/a$ (since $kc/\omega = 1$). This inertance is equal to that of a cyclindrical mass of air attached to surface of the piston and having a length $\alpha = 0{\cdot}27\pi a^2/a = 0{\cdot}85a$. Again from (1) the impedance of an aperture in an infinite wall may be deduced. For, assuming the air in the aperture moves to and fro as if it were a piston, the total impedance of the aperture will be due to motion on both sides of the wall, and will be twice the acoustic impedance of the semi-infinite air driven by the piston. In enclosed conditions where radiation of energy is prevented the radiation resistance term will vanish, and the total inertance of a small aperture will be $0{\cdot}54\rho/a$, its impedance being $i(0{\cdot}54\omega\rho/a)$.

* This follows, since in a plane wave $p = \rho c\dot{\xi}$, whence, for a plane wave proceeding through a tube of section S, $p/S\dot{\xi} = \rho c/S$. The effect of viscous dissipation in the tube is dealt with on p. 225.

Actually the air in the aperture does not move as a rigid piston, so these values are only approximations, but fairly close ones.

It is useful to note the relation which the inertance of an orifice bears to its conductivity c_0—a well-known quantity calculated by Rayleigh. If ϕ_1 and ϕ_2 are the velocity potentials on two sides of an aperture, and if $\dot{X}(=S\dot{\xi})$ is the volume flow through the aperture, then c_0 is defined by the relation $\dot{X}=c_0(\phi_1-\phi_2)$. The total kinetic energy of the moving medium is found to be $\frac{1}{2}\rho\dot{X}^2/c_0$, so that $\rho_0 S^2/c_0$ is the effective mass of moving fluid in the opening and ρ/c_0 its inertance. Rayleigh calculated the conductivity c_0 to be equal to $2a$, so that the inertance is $0{\cdot}5\rho/a$, which is in fair agreement with the calculation given above on the basis of the reaction upon a piston of air moving in a hole in an infinite wall.

With an open-flanged pipe the impedance of the air into which it opens can be represented by the formula on p. 61, on the assumption that the air in the end of the pipe moves as a piston. In the case of a pipe of small diameter (ka small) the inertance $0{\cdot}27\rho/a$ due to the open end is equivalent to that which would arise if a length $(0{\cdot}27/a)\pi a^2=0{\cdot}85a$ were added to the length on the pipe—the well-known 'end correction.' Rayleigh has shown, by hydrodynamical considerations, that the correction for a flanged end lies between $\pi a/4$ and $8a/3\pi$, *i.e.* between $0{\cdot}785a$ and $0{\cdot}85a$. An empirical value is $0{\cdot}82a$. If the flange is omitted the empirical value is usually $0{\cdot}6a$. It should be noted that the total correction for two ends is equal to S/c_0, where c_0 is the conductivity of an orifice of the same diameter in an infinite wall.

The acoustical impedance of a Helmholtz resonator is given on p. 121. If the resonator is not open to free air, but is, say, attached as a side branch to a tube, the expression for the radiation of energy disappears, and the acoustical impedance simplifies to the reactance $i\rho(\omega/K-c^2/V_0\omega)$.

Infinite Tube with a Side Branch. An infinite tube, of uniform cross-section, with a side branch of acoustical impedance Z_s (general in nature), is an interesting conduit (fig. 73) for which the sound transmission is calculable. Let the pressure in the incident plane wave measured at the junction be $p_i e^{i\omega t}$; then the associated volume change will be given by $X_i=p_i e^{i\omega t}/Z$, where Z is the acoustical impedance of an infinite tube, and is known to be $\rho c/S$. Let the pressures in the transmitted, reflected, and side-branched waves be p_t, p_r, and p_b respectively.

At the junction we have continuity of pressure so that

$$p_i + p_r = p_b = p_t \tag{2}$$

Moreover, if the diameter of the tube and of the branch near the junction are small compared with the wave-length of the sound, the pressure in the region of the junction at any moment will be uniform, and the air in the region will all be at the same density.

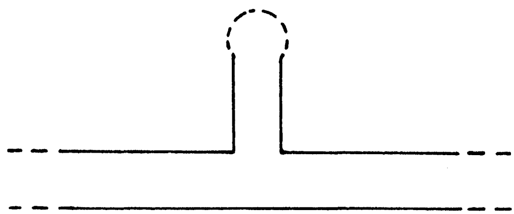


FIG. 73.—Acoustical conduit with side branch

Consequently there is continuity of the volume current at the junction, and we may write

$$\dot{X}_i = \dot{X}_b + \dot{X}_t + \dot{X}_r \tag{3}$$

where volume current amplitudes in the incident, side-branched, etc., waves are represented by $\dot{X}_i$, $\dot{X}_b$, . . . etc.

If the acoustical impedance of the side branch is Z_b, this becomes

$$\frac{p_i}{Z} = \frac{p_b}{Z_b} + \frac{p_t}{Z} + \frac{p_r}{Z} \tag{4}$$

Z being as before the impedance $\rho c/S$ of the tube.

From these equations p_b, p_r, and p_t can be found. For instance, to find p_t note that $p_b = p_t$ and then eliminate p_r, whence

$$\frac{p_t}{p_i} = \frac{1}{1 + \dfrac{Z}{2Z_b}} = \frac{1}{1 + \dfrac{\rho c}{2SZ_b}} \tag{5}$$

The acoustical impedance Z_b is in general complex, and may be due to several types of branch. It may be due to a Helmholtz resonator (p. 121)—a case in which G. W. Stewart * has compared theory with experiment with rather striking agreement. Although the resonator may have sharp selectivity when used in free air,

* G. W. Stewart, *Phys. Rev.*, 27, 487, 1926.

its influence as a side branch may extend so as to affect transmission seriously over more than an octave.

Another important form the side branch may take is that of a simple orifice radiating into outer air ($Z_b = \rho\omega k/2\pi + i\rho\omega/c_0$). The formula for this case too has been checked experimentally with generally satisfactory result. Orifices diminish transmission for high frequencies more than for low ; the effect of the radiation from them is small, and the reduced transmission is due primarily to reflection at the junction. When the side branch takes the form of a cylindrical tube, the arrangement constitutes a Quincke filter (p. 192). The appropriate impedance, Z_b, for this case is given on p. 207, except that a slight end correction should be added to l ($0{\cdot}82r$ where r is the radius of the orifice) to allow for the open end at the point of attachment to the main conduit.

Agreement with experiment is not quite so good in this case, possibly owing to uncertainty about the end correction and the neglect of damping in the Quincke filter. Nevertheless it is generally striking, and confirms the marked selectivity of a Quincke filter.

Filters of the Stewart type consist of a series of side branches equally spaced along the conduit ; they have been dealt with on p. 188. Papers by Mason * give attenuation, impedance, and design data for conduits having side-branch elements such as Quincke filters, and deal also with tapered filters and horns.

Infinite Tube, with sudden Change of Sectional Area. An infinite tube which suddenly changes its area from S_1 to S_2 at a point in its length is somewhat similar to a tube with a side branch. To determine the wave transmitted past the junction, and that reflected from it, the procedure may therefore be the same as in the previous section. Using the same notation for pressures and volume flows we have

$$p_i + p_r = p_t$$

$$\dot{X}_i = \dot{X}_t + \dot{X}_r$$

i.e.

$$\frac{p_i}{Z_{s_1}} = \frac{p_t}{Z_{s_2}} + \frac{p_r}{Z_{s_1}} \qquad (6)$$

Z_{s_1}, Z_{s_2} being the impedances $\rho c/S_1$ and $\rho c/S_2$ of tubes of cross-section S_1 and S_2.

* W. P. Mason, *Bell Sys. Tech. J.*, 6, 258, 1927; 9, 332, 1930.

From these we find

$$\frac{p_t}{p_i}=\frac{2}{1+\frac{Z_{s_1}}{Z_{s_2}}}=\frac{2}{1+\frac{S_2}{S_1}}=\frac{2}{1+m} \tag{7}$$

where m is the ratio S_2/S_1.

If the incident wave reaches a point where the cross-section diminishes, m is less than unity, and the pressure in the transmitted wave is greater than that in the incident wave. We also find

$$p_r=-p_i\left(\frac{m-1}{m+1}\right) \tag{8}$$

If $m<1$, p_r and p_i have the same sign, so that the incident and reflected waves are in the same phase. If $m>1$ they are in opposite phases. The fraction of the incident energy which is transmitted is $p_t^2S_2/p_i^2S_1$, which is found to be $4m/(1+m)^2$. The fraction reflected is $(m-1)^2/(m+1)^2$.

Acoustical Coupling Units. Gramophone Sound-boxes. The acoustical output of a diaphragm or piston source of sound may often be increased by coupling it, *via* a sound-box, with an orifice of which the cross-section S_2 is smaller than the cross-section S_1 of the diaphragm. Let fig. 74 represent the coupling unit, of thickness l. If the enclosure is small compared with the wave-length of the sound concerned, the pressure p and density ρ of the medium within it are substantially uniform. Considering a piston source for simplicity, we note that if the piston displacement is ξ_1, and the displacement of air in the orifice of the unit is ξ_2, the volume flows are respectively $X_1=S_1\xi$ and $X_2=S_2\xi_2$. At any instant when the piston displacement is ξ the volume of the enclosure is $S_1(l-\xi)$, and to a first approximation the mass of the enclosed air is $\rho_0S_1l-\rho_0S_2\xi_2$, for ρ_0S_1l represents the mass of air originally in the enclosure, and $\rho_0S_2\xi_2$ is the mass of air which has flowed out. Thus the density of air in the enclosure is

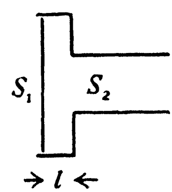


FIG. 74.—Sound-box

$$\rho_0\{1+(X_2/S_1l)\}\{1+(X_1/S_1l)\}=\rho_0\{1+(X_1-X_2)/S_1l\}$$

By definition (p. 52) the condensation s is given by

$$s=(X_1-X_2)/S_1l$$

Conse pressure in the enclosure is

$$p = \rho_0 c^2 s = \rho_0 c^2 \frac{X_1 - X_2}{S_1 l} \tag{9}$$

This relation may be expressed in terms of acoustical impedances by noting that if Z_1 is the impedance of the medium at the position of the piston, and Z_2 the impedance at the orifice, we have $p = Z_1\dot{X}_1$ and $p = Z_2\dot{X}_2$. By noting also that for a harmonic disturbance of frequency $\omega/2\pi$ we may write $\dot{X}_1 = i\omega X_1$ and $\dot{X}_2 = i\omega X_2$, we find

$$p = \frac{\rho_0 c^2}{i\omega S_1 l}\left(\frac{p}{Z_1} - \frac{p}{Z_2}\right) \tag{10}$$

whence

$$\frac{1}{Z_1} = \frac{i\omega S_1 l}{\rho_0 c^2} + \frac{1}{Z_2} \tag{11}$$

The acoustical impedance Z_1 at the piston may thus be inferred from that of the orifice or of the horn to which it is attached.*

The power radiated by the diaphragm is equal to $\frac{1}{2}p_{\max}\dot{X}_{\max}\cos\theta$, which is equivalent to $\frac{1}{2}\dot{X}^2_{\max} \times$ (real part of Z_1).

If the unit were replaced by a long tube of the same area as the piston, the acoustical impedance Z_1' would be $\rho c/S_1$. It follows that

$$\frac{Z_1'}{Z_1} = \frac{i\omega l}{c} + \frac{\rho c}{S_1 Z_2} \tag{12}$$

If the attachment were absent the impedance Z_1'' would be (p. 199) that for a semi-infinite medium driven by a piston of area S_1 in an infinite wall. Hence for a given motion $\dot{X}_{\max}$ of the piston the amplification factor of any coupling attachment is equal to the ratio that the real part of Z_1 bears to the real part of Z_1''.

Conduits of Variable Cross-section (Horns). To obtain the equation for the transmission of sound through a conduit of variable sections, the following procedure is satisfactory if the diameter of the conduit is everywhere small compared with the

* If l is not short, it may be shown that

$$Z_1 = \frac{i\omega Z_2 \cos kl - \dfrac{\rho k c^2}{S_1}\sin kl}{-\dfrac{\omega^2 S_1}{\rho k c^2} Z_2 \sin kl + i\omega\cos kl} \tag{11a}$$

where $k = 2\pi/\lambda = \omega/c$.

wave-length of sound concerned, and if the particle velocity $\dot{\xi}$ and the condensation s are small.

Consider (fig. 75) a thin lamina of the medium of thickness dx transverse to the conduit at a point (distant x from one end of the conduit) where the cross-section area is S. If the mean density of the medium is ρ_0 the mass of the lamina is $\rho_0 \mathrm{S}dx$. If the pressures on the two faces of the lamina are p and $p+(\partial p/\partial x)dx$ respectively, the total force on the lamina in the x direction is $-\mathrm{S}(\partial p/\partial x)dx$. Its equation of motion is thus

$$\rho_0 \mathrm{S}\ddot{\xi}dx = -\mathrm{S}(\partial p/\partial x)dx \qquad (13)$$

if ξ represents the displacement in the x direction.

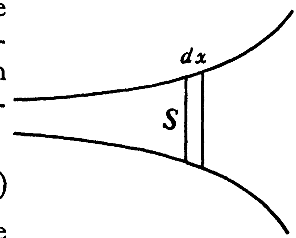


FIG. 75.—Horn

The difference between the rate of flow into the lamina and that out of it is

$$-\frac{\partial}{\partial x}(\rho \mathrm{S}\dot{\xi})dx$$

a quantity which is equal to the rate ($\dot{\rho}\mathrm{S}dx$) of increase of mass of the fluid in the lamina. Consequently

$$-\frac{\partial}{\partial x}(\rho \mathrm{S}\dot{\xi}) = \mathrm{S}\dot{\rho} = \frac{\mathrm{S}}{c^2}\dot{p} \qquad (14)$$

for $\dot{\rho}$ may be shown to be equal to $\rho_0\dot{s} = \dot{p}/c^2$, since $p = \rho_0 c^2 s$ (p. 52). To a first approximation this may be written

$$-\rho_0\frac{\partial}{\partial x}(\mathrm{S}\dot{\xi}) = \frac{\mathrm{S}}{c^2}\dot{p}$$

or

$$-\rho_0\frac{\partial}{\partial x}(\mathrm{S}\ddot{\xi}) = \frac{\mathrm{S}}{c^2}\ddot{p} \qquad (15)$$

as the variation of ρ with x gives only a second order effect if the condensation s (defined by $\rho = \rho_0\overline{1+s}$) is small. Hence, combining (13) and (15), we find

$$\frac{\ddot{p}}{c^2} = \frac{\partial^2 p}{\partial x^2} + \frac{1}{\mathrm{S}}\frac{\partial \mathrm{S}}{\partial x}\cdot\frac{\partial p}{\partial x} = \frac{\partial^2 p}{\partial x^2} + \frac{\partial(\log \mathrm{S})}{\partial x}\cdot\frac{\partial p}{\partial x} \qquad (16)$$

If p varies harmonically with frequency $\omega/2\pi(=kc/2\pi)$, this becomes

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial(\log \mathrm{S})}{\partial x}\frac{\partial x}{\partial p} + k^2 p = 0 \qquad (17)$$

the fundamental differential equation first used by A. G. Webster.*

The solution of (17) shows that the pressures (p_1, p_2) and volume displacements (X_1, X_2) at the two ends of the horn are related by the equations

$$p_2 = ap_1 + bX_1$$
$$X_2 = fp_1 + gX_2 \qquad (18)$$

where a, b, f, and g depend upon the geometrical form of the horn. Their values in certain cases are given in Table VII.

TABLE VII

Transmission of Sound through Conduits

Values of a, b, f, and g

	Tubular Conduit	Conical Conduit, $S = S_0x^2$	Exponential Horn, $S = S_0e^{mx}$
a	$\cos kl$	$\dfrac{x_1}{x_2}\dfrac{\sin k(l+\epsilon_1)}{\sin k\epsilon_1}$	$e^{-\left(\frac{ml}{2}\right)}\left[\dfrac{m}{2\gamma}\sin\gamma l + \cos\gamma l\right]$
b	$\dfrac{\beta}{S}\sin kl$	$\dfrac{\beta}{S_1}\dfrac{x_1}{x_2}\sin kl$	$\dfrac{\beta k}{S_1\gamma}e^{-\left(\frac{ml}{2}\right)}\cdot\sin kl$
f	$-\dfrac{S}{\beta}\sin kl$	$-\dfrac{S_2x_1}{\beta x_2}\cdot\dfrac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 . \sin k\epsilon_2}$	$-\dfrac{S_2k}{\beta\gamma}e^{-\left(\frac{ml}{2}\right)}\cdot\sin\gamma l$
g	$\cos kl$	$-\dfrac{S_2x_1}{S_1x_2}\cdot\dfrac{\sin k(l-\epsilon_2)}{\sin k\epsilon_2}$	$-\dfrac{S_2}{S_1}e^{-\left(\frac{ml}{2}\right)}\left[-\dfrac{m}{2\gamma}\sin\gamma l + \cos\gamma l\right]$

Where l is the length of the conduit $= x_1 - x_2$, and where $\beta = kc^2\rho_0$, $kx_1 = \tan k\epsilon_1$, $kx_2 = \tan k\epsilon_2$, and $\gamma^2 = k^2 - m^2/4$.

(Suffix $_1$ relates to one end of conduit and suffix $_2$ to the other.)

Since the terminal impedances Z_1 and Z_2 are respectively equal to $p_1/\dot{X}_1$ and $p_2/\dot{X}_2$, *i.e.* to $p_1/i\omega X_1$ and $p_2/i\omega X_2$, we find

$$Z_1 = \frac{i\omega g Z_2 - b}{i\omega a + \omega^2 f Z_2} \qquad (19)$$

and the impedance at one end of the horn can be calculated from that at the other.

Thus for a horn excited at its small end the pressure p_2 and volume displacement X_2 at the large end can be calculated. From these latter the energy radiation from the large end can be inferred

* A. G. Webster, *Nat. Acad. Sci., Proc.*, 5, 275, 1919; 6, 316, 1920.

if it is regarded as similar to that from a piston of equal area in an infinite flange.

Closed Tube. In view of various uses to which closed tubes may be put in acoustics, a consideration of their properties is of value. Let the end D of a closed tube of sectional area S (fig. 76) be closed by a perfectly reflecting piston moving with velocity $\xi_0 e^{i\omega t}$. Suppose absorption in the tube is negligible, but that the end A, distant l from D, is closed by a partially absorbing plug or other form of acoustical impedance Z. Let

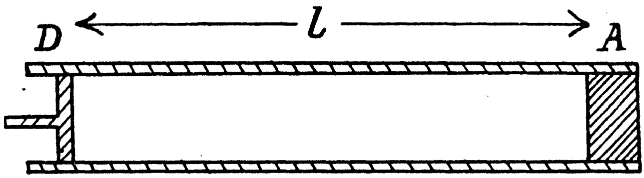


FIG. 76.—Tube closed by absorbing plug A

the reflection factor of the impedance for pressures or velocities be g, a complex quantity. It is possible to infer the magnitude of Z (or of g, which is related to it) from various measurements of the stationary sound waves in the tube.

The acoustical impedance at the driving piston may be calculated as follows :—*

The air velocity at D is equal to the piston velocity $\xi_0 e^{i\omega t}$.

The pressure at D is made up of the first outgoing wave ($p_1 = R\xi_0 e^{i\omega t}$) plus the first reflected wave from A ($p_2 = gR\xi_0 e^{i(\omega t - 2kl)}$) plus the wave again reflected from D ($p_3 = gR\xi_0 e^{i(\omega t - 2kl)}$), and so on, where $R = \rho c$. Thus the total pressure at D is

$$p = R\xi_0 e^{i\omega t}[1 + 2ge^{-2kl} + 2g^2 e^{-4kl} + \quad +] = R\xi_0 e^{i\omega t}\left[\frac{1 + ge^{-2ikl}}{1 - ge^{-2ikl}}\right] \tag{20}$$

Hence the acoustical impedance at D($=p/S\xi_0$) is given by

$$Z_D = \frac{R}{S}\left[\frac{1 - g^2}{1 + g^2 - 2g\cos 2kl} - i\frac{2g\sin 2kl}{1 + g^2 - 2g\cos 2kl}\right]\dagger \tag{21}$$

---

* W. West, *P.O.E.E., J.*, 21, 293, 1929.

† *If the piston has a reflection coefficient* g and the other end of the tube is occupied by a perfect reflector, we find the impedance at D is given by

$$Z = \frac{R}{S}\left(\frac{1 + e^{2ikl}}{1 - e^{2ikl}}\right),$$

a result independent of g.

The impedance Z of the reflector is related to the reflection coefficient g by the relation

$$Z=\frac{R}{S}\left(\frac{1+g}{1-g}\right) \qquad g=\frac{Z-R/S}{Z+R/S} \tag{22}$$

For as the driving piston is brought close to A, the distance l approaches zero, and Z_D approaches the value $R(1+g)/S(1-g)$.

The pressure at any point in the tube distant x *from the piston* may be shown to be,* if $\dot{X}_0=S\dot{\xi}_0$

$$p=\frac{R}{S}\dot{X}_0e^{i\omega t}\left[\frac{Z\cos kl+i\frac{R}{S}\sin kl}{\frac{R}{S}\cos kl+iZ\sin kl}\cos kx-i\sin kx\right] \tag{23}$$

This latter equation † indicates numerous methods of determining Z (and g) experimentally. For instance, it may be measured from the values of $\dot{\xi}_0$ and p at any point in the tube, including the driving point as a particular case. It may also be deduced from the pressures at two known positions in the tube, say at positions of maximum and minimum pressure (p. 211), or at the reflecting surface $x=l$ and half a wave-length in front of it $(x=l-\lambda/2)$; the latter arrangement leads to the simple relationship $p_l/p_{l-\lambda/2}=ZS/R$. Other methods include measurements of the pressure at a point—say the driving point—for constant piston velocity when two different lengths of tube are employed, or when a known impedance is substituted for the unknown one.

Special uses to which a closed tube have been put include measurements of the impedance of human ears (p. 248) and of the absorbing power of materials (p. 210).

Measurement of Acoustical Impedance. The earliest measurements of acoustical impedance as such were made by Kennelly and Kurokawa, who measured the motional impedance of a telephone receiver with and without an attached acoustic impedance. The method was inaccurate, except near resonance, because the acoustic impedance was associated with a relatively

* E. C. Wente and E. H. Bedell, *Bell Sys. Tech. J.* 7, 1, 1928.

† If there is dissipation in the air of the tube, so that waves are propagated with an attenuation constant $P=\alpha+i\beta$, the medium having a specific acoustic impedance Z_0, the pressure p is given by

$$p=\frac{Z_0}{S}\dot{X}_0e^{i\omega t}\left[\frac{Z_0\cosh Pl+Z\sinh Pl}{Z\cosh Pl+Z_0\sinh Pl}\cosh Px-\sinh Px\right]$$

large mechanical impedance. Stewart * described a direct method in which the change in acoustic transmission through a long uniform tube was measured both as regards amplitude and phase, when the unknown impedance was inserted as a branch. His method was specially suited to middle frequencies. Mason † measured the propagation constants of acoustic filters and sound tubes by inserting them in an electro-acoustic transmission circuit, and measuring the resultant changes in the magnitude and phase of the transmitted disturbance.

P. B. Flanders ‡ has described a method in which the unknown impedance is attached to the end of a tube having an electrically excited source of sound at the other end. Flanders makes measurements, by means of an exploring canal communicating with a microphone, of the magnitude and relative phase of the pressure e_1 set up at the point where the unknown impedance is attached to the tube. He then substitutes a perfectly reflecting plug § for the known impedance (fig. 77), and obtains a new

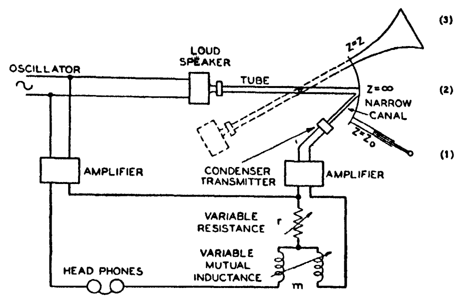


FIG. 77.—Apparatus for measurement of acoustical impedance (Flanders)

* G. W. Stewart, *Phys. Rev.*, 28, 1038, 1926.

† W. P. Mason, *Phys. Rev.*, 31, 283, 1928.

‡ P. B. Flanders, *Acous. Soc. Am. J.*, 4, 402, 1932.

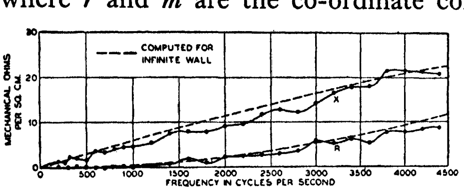
§ Flanders makes use of the acoustic analogue of Thevenin's theorem, which states that in any invariable electrical network the current in any branch is equal to the current that would flow in a simple series circuit composed of an electromotive force and two impedances. The electromotive force is the voltage that would obtain at the branch terminals on open circuit. The impedances are the impedance at the terminals looking back into the source of power, and the impedance of the branch. By the acoustical analogue (see also Mason, *Bell Sys. Tech. J.*, 6, 29, 291, 1927) Flanders's tube, loud-speaker, oscillator, etc., may be replaced by one pressure and one impedance. The pressure is the 'open-circuit' pressure at the end of the tube, or, in other words, the pressure that would be exerted on a rigid wall placed there. The impedance is the complex ratio of pressure to velocity at the end of the tube which would exist if acoustic energy were sent into it toward the loud-speaker, the oscillator being shut off. In electrical terms this would be called the impedance looking into the source. The velocity or acoustic current that flows into an impedance attached to the end is then the current that would flow in an analogous circuit composed of this vibromotive force or pressure and the two impedances in series.

reading e_2 of the pressure. He further substitutes a pure acoustical reactance for the reflector and obtains a value e_3. All these pressures are complex, and are represented by the corresponding e.m.f.'s given by the microphone. Their phases as well as amplitudes are measured by a potentiometer of the Larsen type, fed by a.c. derived from the oscillator which drives the source.

It may be shown that if Z is the unknown impedance and Z_0 is the known impedance of the pure resistance (ρc per unit area)

$$\frac{Z}{Z_0}=\frac{e_1-e_2}{e_3-e_2}=\frac{(r_1-r_2)+i\omega(m_1-m_2)}{(r_3-r_2)+i\omega(m_3-m_2)} \tag{24}$$

where r and m are the co-ordinate components of the complex e.m.f. e. The pure reactance is obtained from a closed tube of uniform bore, whose length is one-eighth of the wave-length of the sound concerned (see equation, p. 207).



Acoustic radiation impedance of hole, 0.7 inch in diameter, in flange having diameter of 6 inches.

FIG. 78.—Acoustical impedance of piston source

As an example of impedance measurement fig. 78 shows the radiation impedance of a hole 0·7 in. diameter and surrounded by a flange which approximates to an infinite wall for frequencies of interest. The dotted lines are the resistance (R) and reactance (X) calculated by Rayleigh.* The figure is due to Flanders.

'Stationary-wave' methods of measuring reflecting power have been applied for some time to the study of the absorption of sound by materials. In recent work phase changes on reflection as well as amplitude changes have been measured, and such stationary-wave experiments constitute methods of measuring acoustical impedance. They are referred to below with special reference to the measurement of the sound-absorbing powers of materials.

Stationary-wave Measurement of Acoustical Impedance, and of the Absorption of Sound by Materials (Normal Incidence). In the stationary-wave method of determining acoustical impedance, a long pipe is provided with a source of sound at one end and is closed at the other by the test specimen. Sound waves from the source travel down the pipe and, incident normally upon the specimen, are reflected to an extent depending on its absorbing

* Rayleigh, *Theory of Sound*, 2, 164, 1896.

power. The superposition of the incident and reflected waves gives rise to a stationary-wave system, and the pressure amplitude varies continuously along the pipe, going through a series of maximum and minimum values. The same description applies to the velocity amplitude, with the difference that the pressure maxima coincide in position with the velocity minima and *vice versa.*

To determine the absorption coefficient 'a' of a material it is sufficient to determine the ratio N/M which the minimum amplitude—either of pressure or of velocity—bears to the maximum. The absorption coefficient, defined as the ratio of absorbed to incident energy, is then given by

$$a = 4\Big/\left(2 + \frac{\mathrm{M}}{\mathrm{N}} + \frac{\mathrm{N}}{\mathrm{M}}\right) \tag{25}$$

Various methods of measuring this ratio have been adopted. Tuma,* who first suggested the stationary-wave method of measuring absorbing power, used the ear as detector; Weisbach † used a telephone ear-piece; Hawley Taylor ‡ employed a Rayleigh disc placed outside the pipe and communicating with the interior by means of a narrow tube; Paris § used a hot-wire microphone; Eckhardt and Chrisler || used a telephone ear-piece communicating with the interior of the pipe by means of a long narrow tube; Heimburger ¶ employed a Rayleigh disc placed in the pipe, the pipe being tuned. Wente and Bedell,** using as source an electrically driven diaphragm occupying the whole section of the pipe, measured the pressure amplitude immediately in front of the diaphragm by a short narrow tube leading to a microphone. The specimen was moved along the pipe and the maximum and minimum pressure amplitude determined. They assumed that, in consequence of the specially massive diaphragm used, the velocity of the diaphragm was unchanged when the specimen was moved along the tube. They did not find it necessary to correct for absorption at the walls of the pipe.

* Tuma, *Wien. Ber.*, 111, 402, 1902.

† Weisbach, *Ann. d. Phys.*, 33, 763, 1910.

‡ Taylor, *Phys. Rev.*, 2, 270, 1913.

§ Paris, *Phys. Soc., Proc.*, 39, 269, 1927.

|| E. A. Eckhardt and V. L. Chrisler, *Bur. Stds. Sci. Papers*, No. 526, 1926; see also *N.P.L. Annual Report*, p. 67, 1924.

¶ G. Heimburger, *Phys. Rev.*, 31, 275, 1926.

** E. C. Wente and Bedell, *Bell Sys. Tech. J.*, 7, 1, 1928.

Theory of the Stationary-wave Method. It will be convenient to have the following brief statement of theory. Neglecting, in the first instance, dissipative effects at the walls of the pipe, and taking the face of the test impedance as the origin $x=0$, let the wave incident upon the specimen be the real part of

$$\phi_1 = Ae^{i\beta x}e^{i\omega t} \tag{26}$$

Then confining our attention throughout to real parts the reflected wave may be written

$$\phi_2 = Be^{-i\beta x}e^{i\omega t} \tag{27}$$

where ϕ is the velocity potential, $\omega = 2\pi \times$ frequency, $\beta = \omega/c = 2\pi/\lambda$, $c =$ velocity of sound, and $\lambda =$ wave-length. The resultant potential in the pipe is

$$\phi = \phi_1 + \phi_2 = Ae^{i(\omega t + \beta x)} + Be^{i(\omega t - \beta x)} \tag{28}$$

If A and B are wholly real, so that there is no phase change on reflection, the pressure amplitude P at any point is

$$P = \left| \rho \frac{\partial \phi}{\partial t} \right| = \rho\omega[A^2 + B^2 + 2AB \cos 2\beta x]^{\frac{1}{2}} \tag{29}$$

Thus P^2 varies harmonically with x, maximum values $\rho^2\omega^2(A+B)^2$ occurring at $x=0$, $\frac{1}{2}\lambda$, λ, etc., and minimum values $\rho^2\omega^2(A-B)^2$ at $x=\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, etc.

The absorption coefficient

$$a = 1 - \frac{B^2}{A^2} = 4\Big/\left(2 + \frac{P_{max}}{P_{min}} + \frac{P_{min}}{P_{max}}\right) \tag{30}$$

If a change of phase occurs on reflection, B is complex ($B = B_0e^{-i\beta\delta}$), and the velocity potential is given by the real part of

$$\phi = Ae^{i(\omega t + \beta x)} + B_0e^{i(\omega t - \overline{\beta x + \delta})} \tag{31}$$

If we substitute $x_1 = x + \delta/2$, $t_1 = t - \beta\delta/2\omega$, we find, as pointed out by Paris,

$$\phi = Ae^{i(\omega t_1 + \beta x_1)} + B_0e^{i(\omega t_1 - \beta x_1)}$$

which is identical in form with (14), and A and B_0 are both real. Thus the effect of the phase change δ is to shift the whole of the stationary-wave system a distance of $\frac{1}{2}\delta$ towards the origin. Accordingly, the phase change can be determined from a knowledge of the position of the stationary system relative to the

specimen. Since the reflection coefficient is thus known in amplitude and phase, the acoustical impedance is calculable (p. 208).

One arrangement employed * in measurements of the absorbing power of materials is that shown in fig. 79. The iron test pipe, 250 cm. long, had a flange at one end to receive the test specimen and a steel plate which backed it. The pipe diameter,

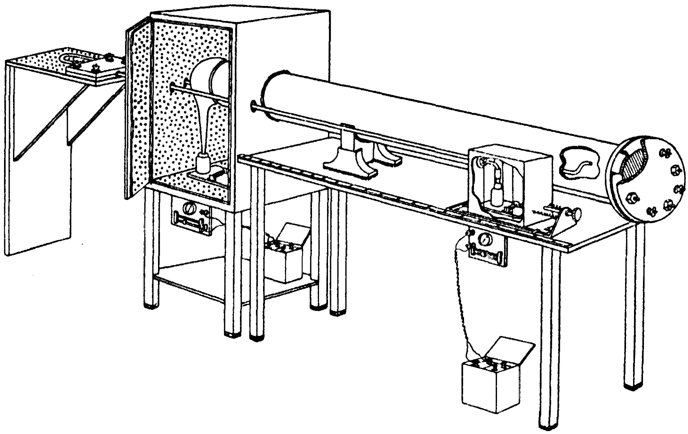


FIG. 79.—Stationary-wave apparatus for measuring amplitude and phase changes which accompany reflection of sound

15 cm., was such that radial vibrations of the contained air might be set up for a number of frequencies above 1290 cycles per second, and might thus impose an upper limit on the working of the apparatus. A moving-coil horn-type loud-speaker, excited by a valve oscillator, was used as source. The mouth of the horn, which was of the same diameter as the unflanged end of the pipe, was placed close to that end of the pipe, but not in actual contact. The loud-speaker, together with the adjacent end of the pipe, was enclosed in a felt-lined box. For the measurements of sound amplitude at points within the pipe an exploring tube of 1·2 cm. internal diameter and 2 mm. wall was employed, communicating with a microphone in a travelling carriage with micrometer adjustment, situated on the table outside the pipe. By bending the exploring tube into U-form (like a trombone slide) space was saved. To suppress certain lateral vibrations of the exploring tube which were found to give rise to sound

* A. H. Davis and E. J. Evans, *Roy. Soc., Proc.*, 127, 89, 1930.

leakage into the conduit, the tube was passed through grooves tightly packed with cotton-wool, and separately supported just behind the loud-speaker cabinet. Special arrangements prevented leakage of sound and vibration to the microphone. In taking readings at various points the whole microphone system was moved along. A second microphone and tube (not shown) was used as a check in order to detect any change of the loud-speaker output when the exploring microphone system was moved. For each frequency this second tube was in a fixed position near a maximum at the loud-speaker end.

In practice some correction is often necessary for the absorption of sound at the walls of the pipe, an effect which causes both maxima and minima to increase with their distance from the closed end. To obtain an estimate of the effect of dissipation at the walls it is assumed * that the amplitude of a progressive wave in the pipe decays exponentially, and the velocity potentials in the incident and reflected wave are written respectively

$$\phi_1 = Ae^{(a+i\beta)x}e^{i\omega t} \quad \text{and} \quad \phi_2 = Be^{-(a+i\beta)x}e^{i\omega t}$$

where a is the attenuation coefficient, assumed to be small. The total potential is

$$\phi = \phi_1 + \phi_2 = Ae^{(a+i\beta)x}e^{i\omega t} + Be^{-(a+i\beta)x}e^{i\omega t}$$

and the pressure amplitude

$$P = \rho\omega[A^2e^{2ax} + B^2e^{-2ax} + 2AB\cos 2\beta x]^{\frac{1}{2}}$$

The positions of the maximum and minimum values of P, given by $\partial(P^2)/\partial x = 0$, are found to be very nearly the same as for no dissipation.

Let $\rho\omega A = P_i$ and $\rho\omega B_0 = P_r$; let M_1, M_2, M_3, etc., denote the 1st, 2nd, and 3rd . . . maxima, and N_1, N_2, N_3 the 1st, 2nd, and 3rd . . . minima. Then

$$M_1 = P_i + P_r, \qquad M_2 = P_i + P_r + \tfrac{1}{2}a\lambda(P_i - P_r),$$
$$M_n = P_i + P_r + \tfrac{1}{2}(n-1)a\lambda(P_i - P_r)$$

and

$$N_1 = (P_i - P_r) + \tfrac{1}{4}a\lambda(P_i + P_r), \qquad N_2 = (P_i - P_r) + \tfrac{3}{4}a\lambda(P_i + P_r),$$
$$N_n = (P_i - P_r) + (2n-1)\tfrac{1}{4}a\lambda(P_1 + P_r)$$

* I. B. Crandall, *Theory of Vibrating Systems and Sound*, p. 95; Eckhardt and Chrisler, *Bur. Stds. Sci. Papers*, No. 526, 1926.

We see, then, that the effect of absorption is to cause both the maxima and minima to increase with their distance from the closed end, this increase being negligible in the case of the maxima, but appreciable for the minima, especially with highly reflecting specimens. The correction $\frac{1}{4}\alpha\lambda$ is therefore conveniently determined by preliminary measurements with the highly reflecting steel plate at the closed end of the tube. The correction may then be applied to the minima measured under other conditions.

Davis and Evans * found α (*i.e.* m' of p. 226) to be of the order of magnitude expected on theoretical grounds.

It is interesting to note that the absorption of sound in ventilating ducts could be measured by exploring the distribution of sound along their length, just as the distribution of sound along the stationary-wave pipe is explored.

Comparison between Stationary-wave and Reverberation Measurements of Absorbing Power of Materials. Measurements of the absorbing power of a material by the reverberation method yield results which relate to sound incident upon the specimen in a random manner from all directions. They are directly applicable to the calculation of the degree of reverberation of sound in an auditorium. They should be distinguished from absorbing powers which are determined by the stationary-wave method, in which the sound is incident perpendicularly upon the absorbent. Paris (p. 228) has calculated the relation to be expected between the two. The equation must be regarded as tentative, as no experimental confirmation of the formula has been given.

Apart altogether from the question of the variation of sound absorption with the angle of incidence of the sound, it is to be noted that in reverberation tests large areas of sample can be employed, and mounted upon the wall of a room on studding or in panels or in any other manner which would be employed in actual building practice. Thus any absorption due to the vibration of the panels is allowed for. In the stationary-wave measurements, however, the material cannot be used in sufficient size for this, so the material is usually attached rigidly to the backing-plate and the effect of panel vibration is eliminated. Thus no universal relations are to be expected between the two types of measurement. However, since surface absorption often plays a predominating part, and in view of the importance of reverberation figures, fig. 80 * has been compiled from such data

* A. H. Davis and E. J. Evans, *loc. cit.*

as are available, showing the actual experimental ratios between the two. The results have been compiled from various sources, and whilst the materials are proprietary articles which have the same name, the samples were tested at widely different times in different countries and may well differ appreciably. An inference is that the reverberation coefficient in the ordinary practical range is generally greater than the stationary-wave coefficient, the

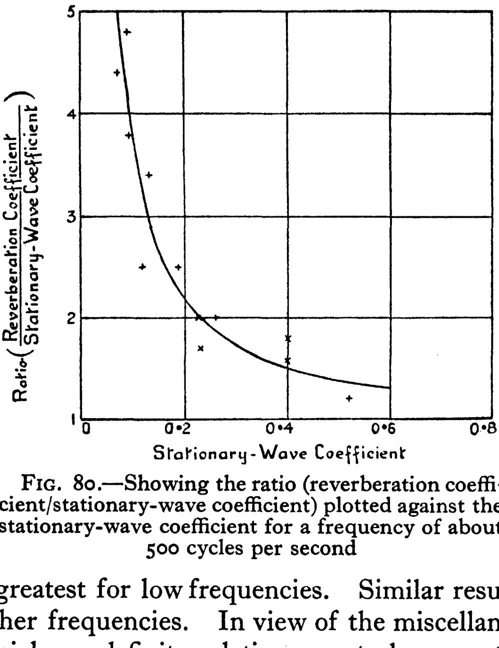


FIG. 80.—Showing the ratio (reverberation coefficient/stationary-wave coefficient) plotted against the stationary-wave coefficient for a frequency of about 500 cycles per second

ratio being greatest for low frequencies. Similar results were obtained at other frequencies. In view of the miscellaneous nature of the materials no definite relations are to be expected.

Transmission of Sound at a Plane Junction of Two Media. If a sound wave in a medium of specific acoustic resistance $R_1(=\rho_1 c_1)$ is incidental normally upon a plane surface bounding a second medium $R_2(=\rho_2 c_2)$, some of the sound will be transmitted and some reflected. Let the wave displacements at the interface in the incident, reflected, and transmitted waves be respectively $A_i e^{i\omega t}$, $A_r e^{i\omega t}$, and $A_t e^{i\omega t}$.

Pressures in the waves are calculable from the specific acoustic impedance relations $p_i = R_i \xi_i$, etc. The boundary conditions at the interface are:

(1) Continuity of pressure, *i.e.*

$$p_i + p_r = p_t \quad \text{whence} \quad R_1 A_i + R_1 A_r = R_2 A_t \tag{32}$$

(2) Continuity of volume displacement, *i.e.*

$$\xi_i = \xi_r + \xi_t \quad \text{whence} \quad A_i = A_r + A_t \tag{33}$$

These relations yield immediately for the amplitude coefficients of reflection and transmission:

$$\frac{A_r}{A_i} = \frac{R_2 - R_1}{R_2 + R_1}; \qquad \frac{A_t}{A_i} = \frac{2R_1}{R_2 + R_1} \tag{34}$$

Energy ratios are proportional to the squares of these quantities. It may be noted that for air $R = 41$; for ordinary metals R varies from about $1{\cdot}37 \times 10^6$ (lead) to $4{\cdot}23 \times 10^6$ (nickel); for water R is about $1{\cdot}43 \times 10^5$. It will be clear from these results, on substituting in the equations above, that the fraction of sound transmitted across an interface from air to metals or to water, or from water or metals to air, is extremely small.*

Transmission of Sound through a Slab of Finite Thickness. In the transmission of sound through a slab of material, the sound falls upon the first face of the slab and is there partly reflected and partly transmitted. The transmitted sound proceeds to the second face, and is itself divided. The part that is reflected will, in due course, be again partially reflected from the front surface, and so on. The total acoustical disturbance due to multiple reflections in the plate can be expressed as the sum of two waves, one travelling forwards and one backwards in the slab. The total emission from the back of the slab—*i.e.* the total transmission—can be expressed as one emergent wave. From these considerations, and from continuity of pressure and velocity at each interface, the sound reflected from the slab and that transmitted through it can be calculated. Rayleigh † shows that the fraction of incident sound energy reflected in air from a rigid plate of non-absorbent material is given by

$$\left(\frac{R_1}{R_2} - \frac{R_2}{R_1}\right)^2 \Big/ \left\{4\cot^2\frac{2\pi l}{\lambda} + \left(\frac{R_1}{R_2} + \frac{R_2}{R_1}\right)^2\right\} \tag{35}$$

* A fuller proof would take the incident, reflected, and transmitted waves to be represented by the velocity potentials

$$\varphi_i e^{i\left(\omega t - \frac{x}{c_1}\right)}, \qquad \varphi_r e^{i\left(\omega t + \frac{x}{c_1}\right)}, \qquad \varphi_t e^{i\left(\omega t - \frac{x}{c_2}\right)}$$

and would introduce boundary conditions (continuity of pressure and velocity) at the interface, assumed to be, say, at $x = 0$.

† Rayleigh, *Theory of Sound*, **2**, 88, 1896.

where l is the thickness of the plate, λ the wave-length of sound in the plate, and R_1 and R_2 are respectively the radiation resistances ($\rho_1 c_1$ and $\rho_2 c_2$) of the air and of the material of the plate. Assuming that there is no absorption of sound in the plate the remainder is transmitted and may be shown to be

$$\frac{4+4\cot^2\left(\frac{2\pi l}{\lambda}\right)}{\left(\frac{R_1}{R_2}+\frac{R_2}{R_1}\right)^2+4\cot^2\left(\frac{2\pi l}{\lambda}\right)} \tag{36}$$

The reciprocal of this is the sound-reduction factor for the plate, and may be written

$$1+\tfrac{1}{4}\left(\frac{R_1}{R_2}-\frac{R_2}{R_1}\right)^2\sin^2\frac{2\pi l}{\lambda} \tag{37}$$

It will be noted that a plate half a wave-length thick (or any integral number of half wave-lengths) transmits all and reflects none of the incident energy. When the thickness is an odd multiple of $\lambda/4$ the reflected sound is a maximum, and the transmitted a minimum. Boyle and Lehman * have verified the relations experimentally for a flat plate immersed in water. In their experiments the incident waves were supersonic, being of frequencies of the order of 135,000 cycles per second.†

When the slab is not a thick one compared with the wave-length of sound in the material, *i.e.* when $2\pi l/\lambda$ is small, we may write $4\pi^2 l^2/\lambda^2$ for $\sin^2(2\pi l/\lambda)$. If, further, R_2 is large compared with R_1, as it is when R_2 relates to a solid and R_1 to air, we have ‡

$$\text{Reduction factor } 1+\frac{\pi^2 l^2 R_2{}^2}{\lambda^2 R_1{}^2} \text{ approx.}$$

which yields when rewritten

$$1+\frac{\omega^2 m^2}{4R_1{}^2} \tag{38}$$

where $m(=\rho_2 l)$ is the mass per unit area of the plate. It is seen that the reduction factor tends to vary as the square of the

* R. W. Boyle and J. F. Lehman, *Roy. Soc. Canada, Trans.*, 21, 115, 1927.

† R. W. Boyle and W. F. Rawlinson (*Roy. Soc. Canada, Trans.*, 3, 233, 1928) have extended the analysis to angles of incidence other than normal.

‡ A. H. Davis, *Phil. Mag.*, 15, 309, 1933.

frequency of the incident sound except for materials so light as paper, for which the additive term in unity is important: less variation then occurs. The only property of the partition which

TABLE VIII

Transmission of Sound through Homogeneous Panels. (Comparison of Experimental Results with Calculated Values)

N.B.—R_1 (air) = 41. $\therefore 4R_1^2 = 6700$

Panel		Pulsatance of Sound, ω	Calculated Reduction Factor, $1 + (\omega^2 m^2/6700)$	Reduction Factor in Decibels	
Type	Weight, m			Calculated	Observed
	grm./cm.^{2}				
Paper . .	0·015	1880	1·1	0·8	0·5
		6280	2·3	4	2
		10000	4·4	6	4
Sailcloth .	0·044	1880	2·0	3	3
		6280	12	11	10
		10000	30	15	9
Fibre board .	0·32	1880	54	17	14
		6280	600	28	27
		10000	1500	32	32
Mahogany 2 in.	2·4	1880	3000	35	26
		6280	34000	45	36
		10000	86000	49	39
Brick wall .	20	1880	210000	53	38
		6280	2400000	64	59
		10000	6000000	68	55

appears in this formula is its mass per unit area. The preponderating effect of mass in determining transmission through building partitions is commented upon elsewhere, from an experimental basis. The equation agrees in general magnitude with experimental results for transmission of sound through paper, sailcloth, light boards, etc., at a series of frequencies, but there are evidences of variations at particular frequencies, possibly due to resonances (Table VIII). Transmission through thicker partitions, such as brick walls, appears to exceed calculated values. This is possibly

due to flexural resonances, the wall vibrating as an elastic plate clamped at the edges, a factor which has not been considered in developing the formula.

Transmission of Sound by a Thin Slab, under Elastic Restraint. The theory of sound transmission through a thin partition, capable of flexural vibration and clamped at the edges, presents many complexities, owing to the many modes of vibration possible. A simple case which will be considered * as an example is that of an infinite, thin, but rigid piston that can vibrate as a whole under the action of elastic restraints.

Let an infinite rigid slab of mass m per unit area be located in the plane $x=0$. Let a sound wave $\xi_1=\xi_{01}e^{i\omega\left(t+\frac{x}{c_1}\right)}$ be incident normally upon the diaphragm from the medium to the left of the origin, where ξ_1 represents particle displacement at a time t in the plane $x=x$, c_1 being the velocity of sound in the medium. Under the action of this sound the plate will be displaced and will move in a manner represented by $\xi_2=\xi_{02}e^{i\omega t}$. There will also be a transmitted wave sent out into the air from the other side of the diaphragm, and a reflected wave † sent back towards the sending region.

The following equations therefore represent the conditions:—

In the first medium

$$\xi_1=\xi_{01}e^{i\omega\left(t-\frac{x}{c_1}\right)}+\xi_{01}{}'e^{i\omega\left(t+\frac{x}{c_1}\right)}$$

the second term representing the reflected wave.

For the partition

$$\xi_2=\xi_{02}e^{i\omega t}$$

For the wave transmitted on the other side of the partition

$$\xi_3=\xi_{03}e^{i\omega\left(t-\frac{x}{c}\right)}$$

At the boundary between the first medium and the panel velocity must be continuous, and we have

$$\xi_{01}+\xi_{01}{}'=\xi_{02}$$

similarly, considering the second medium,

$$\xi_{03}=\xi_{02}$$

---

* A. H. Davis, *loc. cit.*

† This arises from the fact that the solutions of the equation $\frac{\delta^2\theta}{\delta t^2}=C\frac{\delta^2\theta}{\delta x^2}$ must be of the type $\theta=A.f(ct-x)+B.F(ct+x)$.

The pressure variations causing the motion of the panel arise from the sum of the effects of the pressure variations associated with the incident, reflected, and transmitted waves. The total force on unit area of the plate may be written

$$\delta p_{01} + \delta p_{01}' - \delta p_{03}, \ i.e.\ \rho c(\dot{\xi}_{01} - \dot{\xi}_{01}' - \dot{\xi}_{03})e^{i\omega t}$$

since $\delta p = \pm c\rho\dot{\xi}$. Particular attention has been paid to the sign of c, which depends upon the direction of propagation. It is convenient to write R for $c\rho$, and we thus arrive at the following equation to represent the motion of the panel:—

$$m\ddot{\xi}_2 + r\dot{\xi}_2 + s\xi_2 = \mathrm{R}(\dot{\xi}_{01} - \dot{\xi}_{01}' - \dot{\xi}_{03})e^{i\omega t} \tag{39}$$

where s defines the elastic restraint of the panel and r the dissipative resistance.

This may be rewritten when the relations between $\dot{\xi}_{01}$, $\dot{\xi}_{01}'$, and $\dot{\xi}_{03}$ are considered,

$$m\ddot{\xi}_2 + (r + 2\mathrm{R})\dot{\xi}_2 + s\xi_2 = 2\mathrm{R}\dot{\xi}_{01}e^{i\omega t} \tag{40}$$

Solving this gives

$$\dot{\xi}_2 = \frac{2\mathrm{R}\dot{\xi}_{01} \,.\, e^{i\omega t}}{\mathrm{Z}} = \dot{\xi}_{02}e^{i\omega t} \tag{41}$$

where

$$\mathrm{Z} = (r + 2\mathrm{R}) + i\left(m\omega - \frac{s}{\omega}\right) \quad i.e. \quad \mathrm{Z}^2 = (r + 2\mathrm{R})^2 + \left(m\omega - \frac{s}{\omega}\right)^2$$

The energy transmission coefficient for the panel is given by $\left(\dfrac{\dot{\xi}_{03}}{\dot{\xi}_{01}}\right)^2$, and its reciprocal, the reduction factor as ordinarily defined, becomes

$$\left(\frac{\dot{\xi}_{01}}{\dot{\xi}_{03}}\right)^2 = \left(\frac{\dot{\xi}_{01}}{\dot{\xi}_{02}}\right)^2 = \frac{\mathrm{Z}^2}{4\mathrm{R}^2} = \frac{(r + 2\mathrm{R})^2 + \left(m\omega - \dfrac{s}{\omega}\right)^2}{4\mathrm{R}^2} \tag{42}$$

The expression for the reduction factor may be put in another form by writing ω_0 for the natural pulsatance $\sqrt{\dfrac{s}{m}}$ of the panel, and becomes

$$\frac{(r + 2\mathrm{R})^2 + m^2\omega^2(1 - \omega^2{}_0/\omega^2)^2}{4\mathrm{R}^2} \tag{43}$$

When there is no appreciable dissipation in the panel itself may be neglected and the reduction factor becomes

$$1+\frac{(m\omega - s/\omega)^2}{4R^2} = 1+\frac{m^2\omega^2(1-\omega_0{}^2/\omega^2)^2}{4R^2} \qquad (44)$$

This reduces to the form given earlier for a thin unconstrained panel when s is put equal to 0, or when ω_0 is small compared with ω. Also it should be noted that the effect of resonance at the frequency determined by ω_0 is to reduce the reduction factor below what it would be if resonance did not occur. The new formulæ are thus consistent with the observed fact that the sound-reduction factor of a brick wall or of a mahogany board falls below those calculable by Rayleigh's formula. When, however, calculation is made of the values to be attributed to ω_0 to account for the observed reduction factors these partitions at various frequencies, it is necessary to assign some value to the damping r. Assuming it to be negligible in comparison with $m\omega$—a reasonable assumption—it appears that a single resonance will not suffice to reconcile theory with experiment at all test frequencies. The inference is that each partition as tested has several resonant frequencies, a conclusion which is consistent with the well-known fact that a clamped elastic plate has several modes of vibration, and with observations of the resonance of actual panels.

Transmission of Sound through Apertures, and Diffraction by Screens. For recent work on transmission of sound through slits and apertures reference may be made to papers by Strutt on transmission through slits of finite width,* by Ritchie,† and by Wintergerst and Knecht ‡ on transmission through apertures. Sivian and O'Neil § have dealt with the diffraction of sound by variously shaped screens.

* M. J. O. Strutt, *Zeits. f. tech. Phys.*, 69, 597, 1931.
† E. Ritchie, *Acous. Soc. Am. J.*, 3, 402, 1932.
‡ E. Wintergerst and W. Knecht, *V.D.I.*, 76, 777, 1932.
§ L. J. Sivian and H. T. O'Neil, *Acous. Soc. Am. J.*, 3, 483, 1932.

CHAPTER XIII

DISSIPATION AND ABSORPTION OF SOUND

Viscous Dissipation in Free Air. In previous chapters the effects of the viscosity of the medium in which sound is propagated have been largely ignored, and they are usually so small as to be negligible. The effects of viscosity upon the propagation of sound in a medium have, however, been investigated by Stokes * (1845).

The differential equation for plane waves corresponding to equation (6) on p. 53 for inviscid media becomes

$$\frac{\partial^2 \xi}{dt^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} + \tfrac{4}{3}\nu \frac{\partial^3 \xi}{\partial x^2 \partial t} \qquad (1)$$

Here ξ is the particle displacement at a time t at a point distant x from the origin ; ν is the kinematic viscosity of the medium ; and c the velocity of sound in an inviscid medium of the same density and elasticity.

In the case of free waves the amplitude dies away exponentially with time t, the displacement being proportional at any point to $e^{-t/\tau} \cos(\omega t + \epsilon)$ where $\tau = 3/2\nu k^2$, k having the usual value of $2\pi/\lambda$. Actually $\omega^2 = k^2 c^2 - 1/\tau^2$. In all cases of aerial sounds $1/kc\tau$ is small, so that $k = \omega/c$ practically, the viscosity having no appreciable effect on the relation between frequency and wave-length.

For the case of forced harmonic waves arising from a constant disturbance $\xi = a \cos \omega t$ † maintained in the plane $x = 0$, we find $\xi = ae^{-x/l} \cos \omega(t - x/c)$ where $l = 3c^3/2\nu\omega^2 = c\tau$, on the assumption that $\nu\omega/c^2$ (*i.e.* $3/2k\tau$) is small. The wave has (sensibly) the usual velocity c, but diminishes exponentially in amplitude as

* See Rayleigh, *Theory of Sound*, **2**, 316; Lamb, *Dynamical Theory of Sound*, p. 188.

† The solution is readily effected by means of complex quantities by assuming as a solution $\xi = ae^{i\omega t + mx}$, and evaluating m.

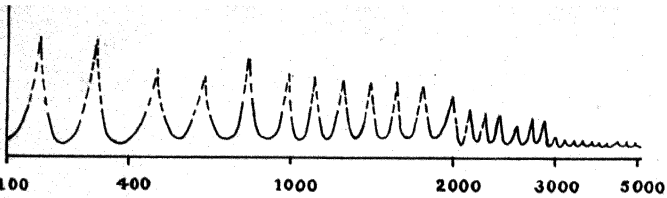
it proceeds. It will be noted that since $l = c\tau$ the decay in amplitude suffered in reaching a distance x from the source is that calculable from the time modulus τ and the time interval t involved. If the wave-length of the sound is expressed in centimetres, we find $l = 9{\cdot}56 \times \lambda^2 \times 10^3$ cm. It follows that the effect of viscosity on amplitude is very slight except for sounds of very short wave-length.

Kirchhoff showed that a further cause of dissipation of sound energy is to be found in the thermal processes consequent upon alternate rarefactions and condensations of air. A complete investigation taking into account this thermal effect as well as viscosity shows that the effect is equivalent to an increase in the kinematic viscosity ν, but the order of the effect is unchanged.

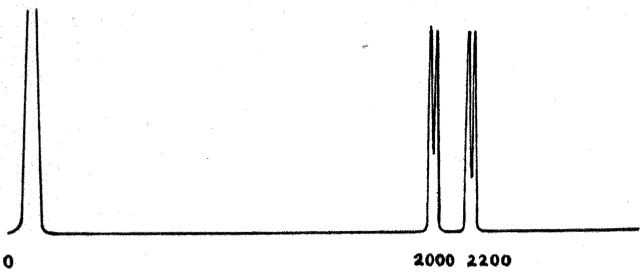
The calculated sound absorption in air due to viscosity and heat conduction fails, however, to account for the anomalous absorption noted in moist air by Knudsen * and others in reverberation measurements. The absorption calculated on the above basis increases proportionally with the square of the frequency of the sound, and depends very little on small amounts of admixed foreign gases. The observed absorption in moist air, however, is about 10–100 times as large as the value predicted by classical theory, increases only linearly with frequency, and depends in a very characteristic way on the amount of water vapour present. The absorption is lowest for dry air, reaches a maximum at a fairly low moisture content which depends upon the frequency, and falls again as the humidity is further increased. The extent of the absorption depends upon the temperature (p. 155).

It appears that at least part of the special absorption actually observed in moist air may arise from intermolecular exchange of energy, in which a finite period is involved in the adjustment of equilibrium. Maximum absorption is to be anticipated for a sound wave for which the frequency equals approximately the rate of adjustment of thermal equilibrium, but no absorption at very high or very low frequencies. Theories by Einstein, Herzfeld and Rice, Bourgin, Kneser, Henry, and Rutgers have dealt with absorption arising from the transfer of energy between translational and either rotational or vibrational forms of energy

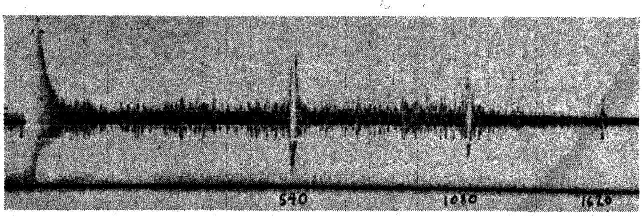
* V. O. Knudsen, *Acous. Soc. Am. J.*, 3, 126, 1931; 5, 112, 1933.



Buzzer note, 160 cycles per sec. (Wegel and Moore). See p. 174



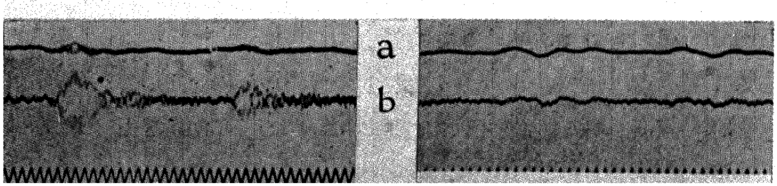
Two pure tones (Grutzmacher). See p. 178



Airscrew, six-bladed (Theodorsen). See p. 181

Analyser Records of Various Noises

(Frequency distribution and intensity of component sounds)



Without silencer With silencer

(*a*, Amplifier uniform ; *b*, Amplifier " weighted ")

Oscillograms of noise of motor-car engine (Trendelenburg). See p. 263

[*p.* 225

during molecular collisions.* Kneser † suggested as an explanation of Knudsen's results that a large part of the absorption was attributable to collisions between water and oxygen molecules. The later experiments by Knudsen showed that this explanation is probably correct, for, as regards absorption of sound, water vapour has about five times the effect on oxygen that it has on air (a mixture of which about $\frac{1}{5}$ is oxygen), and is without appreciable effect on nitrogen. Indeed the absorption in nitrogen, whether dry or moist, does not greatly exceed the classical value, and is about the same as that for dry air.

Rocard ‡ has discussed damping, in air, due to the reciprocal diffusion of the constituent elements, and later that which would arise from the evaporation and condensation of water in the regions where a sound wave heats and cools the air. In the latter work he finds that the consequent absorption of sound would be independent of the frequency within the audible range, and would have a definite relation to the optical transparency of the atmosphere in good weather.

Sound in Tubes: Effect of Viscosity. The differential equation for the propagation of sound waves in a viscous gas in a tube of uniform cross-section is §

$$\left(1+\frac{0}{\mathrm{S}}\sqrt{\frac{\nu}{2\omega}}\right)\frac{\partial^2\xi}{\partial t^2}+\frac{0}{\mathrm{S}}\sqrt{\frac{\nu\omega}{2}}\frac{\partial\xi}{\partial t}=c^2\frac{\partial^2\xi}{\partial x^2} \qquad (2)$$

where ξ denotes the displacement of the fluid at a distance x from one end of the tube,

ν = the coefficient of kinematic viscosity of the medium,
0 = perimeter and S = cross-sectional area of pipe,
ω = pulsatance of the sound,
c = velocity of sound in the unrestricted medium.

This equation is valid for tube diameters and frequencies such that

$$\sqrt{\frac{\omega}{2\nu}}\frac{\mathrm{S}}{0}>1$$

* A. Einstein, *Ber. d. Berl. Akad.*, 19, 380, 1920; K. F. Herzfeld and F. O. Rice, *Phys. Rev.*, 31, 691, 1928; D. G. Bourgin, *Nature*, 122, 133, 1928; *Phil. Mag.*, 7, 821, 1929; *Phys. Rev.*, 34, 521, 1929; 42, 721, 1932; *Acous. Soc. Am. J.*, 4, 108, 1932; H. O. Kneser, *Ann. d. Phys.*, 11, 761, 777, 1931; 16, 337, 350, 1933; P. S. H. Henry, *Camb. Phil. Soc., Proc.*, 28, 249, 1932; A. J. Rutgers, *Ann. d. Phys.*, 16, 350, 1933.

† H. O. Kneser, *Acous. Soc. Am. J.*, 5, 122, 1933.

‡ Y. Rocard, *J. de Phys. et le Radium*, 1, 426, 1930; 4, 118, 1933.

§ Rayleigh, *Theory of Sound*, 2, 319, 1896.

and hence can be used for all frequencies of interest in connection with acoustic filters.

The velocity of sound in the tube is

$$c\left\{1-\frac{1}{2}\frac{0}{S}\sqrt{\left(\frac{\nu}{2\omega}\right)}\right\}$$

which in the case of a circular tube of radius r is

$$c\left\{1-\frac{1}{r}\sqrt{\left(\frac{\nu}{2\omega}\right)}\right\}$$

This result was first obtained by Helmholtz.*

Kirchhoff † (Rayleigh, p. 325) extended the theory to take account of the losses due to heat conduction in the medium, and showed that in the above equation $\sqrt{\nu}$ should be replaced by a quantity γ' where

$$\gamma'=\sqrt{\nu}+\left(\sqrt{\gamma}-\frac{1}{\sqrt{\gamma}}\right)\sqrt{\kappa}$$

where κ is the coefficient of thermal diffusivity of the medium and γ is the ratio of specific heats. By the kinetic theory of gases κ has the value $5\nu/2$ for monatomic gases, and $1{\cdot}90\nu$ for diatomic gases.

Rayleigh gives as the solution for the velocity $u(=\partial\xi/\partial t)$ at points at a finite distance from the walls

$$u=C_1e^{m'x}\sin(\omega t+m''x+\delta_1)+C_2e^{-m'x}\sin(\omega t-m''x+\delta_2) \quad (3)$$

where C_1, C_2, δ_1, and δ_2 are four arbitrary constants.

$$m'=\frac{\gamma'}{cr}\frac{\omega}{\sqrt{2}}, \qquad m''=\frac{\omega}{c}+\frac{\gamma'}{cr}\frac{\sqrt{\omega}}{\sqrt{2}}$$

Clearly m' determines the attenuation the waves suffer and m'' the velocity of propagation. This velocity is

$$c_1=\omega/m''=c\left\{1-\frac{\gamma'}{r\sqrt{2\omega}}\right\}$$

Experiments on the velocity of sound in pipes confirm this equation to the extent that the correction to the velocity does vary inversely as the radius of the pipe and inversely as the square root of the frequency, although the actual constant may

* H. Helmholtz, *Crelles J.*, 57, 1, 1859.
† G. Kirchhoff, *Pogg. Ann.*, 134, 177, 1868.

not be identical with the calculated value. Consequently the velocity of sound in free air may be deduced, from measurements in two tubes of different radii r_1 and r_2, from the relations $c_1 = c(1 - k/r_1)$ and $c_2 = c(1 - k/r_2)$, which together yield

$$c = (r_2 c_2 - r_1 c_1)/(r_2 - r_1)$$

Mason * gives the following solution of the differential equation as one which is of value in the theory of combinations of acoustic conduits :—

$$\xi = e^{i\omega t}(\mathrm{A} \cosh \alpha x + \mathrm{B} \sinh \alpha x) \tag{4}$$

where A and B are constants, and α by analogy with an electric line is the propagation constant of the tube. It is found that the latter equation is a solution of (2), when

$$\alpha = a + ib = \frac{1}{2}\frac{0}{c\mathrm{S}}\sqrt{\frac{\gamma'^2\omega}{2}} + i\frac{\omega}{c}\left[1 + \frac{1}{2}\frac{0}{\mathrm{S}}\sqrt{\frac{\gamma'^2}{2\omega}}\right] \quad \text{approximately}$$

provided $0\gamma'/\mathrm{S}\sqrt{2\omega}$ is a small quantity. He finds the acoustical impedance of the tube to be

$$\mathrm{Z} = \frac{\rho c}{\mathrm{S}}\left[\left(1 + \frac{1}{2}\frac{0}{\mathrm{S}}\sqrt{\frac{\gamma'^2}{2\omega}}\right) - i\frac{1}{2}\frac{0}{\mathrm{S}}\sqrt{\frac{\gamma'^2}{2\omega}}\right]$$

This reduces to the acoustical impedance $\rho c/\mathrm{S}$ when the effects of viscosity and of heat conductivity are small, *i.e.* when $\gamma' = 0$.

In a later paper Mason † discusses a method of measuring the attenuation characteristics of tubes and filters.

Among recent papers dealing with the absorption of sound in transmission through tubes, reference may be made to measurements by Carnac,‡ who studied tubes of rubber and flexible metal, by Davis and Evans (p. 215), and by Kaye and Sherratt,§ who were concerned ultimately with tubes of graphite, etc., which could be employed in furnace construction.

Absorption of Sound by Porous Surfaces. Rayleigh || first calculated the absorption of sound at the face of a rigid porous surface, from considerations of the dissipation of sound energy

* W. P. Mason, *Bell Sys. Tech. J.*, 6, 258, 1927.
† W. P. Mason, *Phys. Rev.*, 31, 283, 1928.
‡ F. Carnac, *Rev. d'Acoustique*, 1, 52, 1932.
§ G. W. C. Kaye and G. G. Sherratt, *Roy. Soc., Proc.*, 141, 123, 1933.
|| Rayleigh, *Theory of Sound*, 2, 328, 1896.

into heat through viscous and thermal actions in capillary channels into which the sound waves penetrate. When the channels are cylindrical, small (less than 0·01 cm. dia.), perpendicular to the surface, and of a depth small compared with the wave-length but large relative to the diameter, and sufficient to ensure that the sound that enters does not return, then the absorbing power per unit area of a rigid porous surface normal to the incident waves was calculated to be given by

$$a = \frac{4M}{2M^2 + 2M + 1} \tag{5}$$

where

$$M = \frac{2(1+g)\sqrt{\nu\gamma}}{r\sqrt{\omega}},$$

g = ratio of unperforated to perforated area of porous surface,
ν = kinematic viscosity of the gas,
γ = ratio of the specific heats of the gas,
r = radius of the pores,
$\omega = 2\pi \times$ frequency.

When the holes are very small (say $r = 0{\cdot}001$ cm.) reasonable values of M are fairly large, and the absorption coefficient tends to be ultimately proportional to the square root of the frequency of the sound. If the holes are only moderately small ($r = 0{\cdot}01$ cm.) M may be even less than unity. Now it may be shown that the absorption coefficient 'a' above is a maximum when $M = 1/\sqrt{2}$. Consistent with this result, some porous materials often have a region of maximum absorption in the audible range.* A fairly appreciable thickness would seem to be necessary (say 10 cm.) in this case to prevent return of sound waves from the channels.

In a later paper Rayleigh † dealt with sound waves incident at any angle upon a porous wall. The amplitude of the reflected wave depends upon the angle of incidence, and in certain circumstances at certain angles $\left(\cos\theta = \frac{1}{1+g}\right)$ the reflected wave vanishes and the absorption is complete.

Paris derived formulæ by which the absorption coefficient of a material at any angle of incidence can be inferred, if a special

* See I. B. Crandall, *Theory of Vibrating Systems and Sound*, p. 188.
† Rayleigh, *Phil. Mag.*, 39, 225, 1910.

property which he defined * as its 'acoustical admittance' (represented by Ω below) is known. Paris made no special assumption as to the nature of the absorbent, except one which implies that disturbances are not propagated laterally, from one part of the porous medium to the other, to any appreciable extent.

If the velocity potential of the wave incident at an angle θ upon a reflecting surface in the plane $x=0$, and moving towards $-x$, is represented by

$$\phi = \mathrm{A}e^{ik(vt+x\cos\theta+y\sin\theta)}$$

and that of the reflected wave by

$$\phi = \mathrm{B}e^{ik(vt-x\cos\theta+y\sin\theta)}$$

the ratio

$$\frac{\mathrm{B}}{\mathrm{A}} = \frac{\cos\theta - v\Omega}{\cos\theta + v\Omega} \quad \text{whence} \quad a_\theta = 1 - \left|\frac{\cos\theta - v\Omega}{\cos\theta + v\Omega}\right|^2 \tag{6}$$

a_θ being the absorption coefficient of the material appropriate to the angle of incidence. In general Ω is complex; in other words, there is on reflection a change of phase as well as a reduction in amplitude.

On the basis of the above formula Paris deduced from stationary-wave measurements at normal incidence that a certain plaster would exhibit maximum absorption (approx. 0·8) at an angle of incidence about 8° removed from grazing, the absorption falling to zero at grazing incidence, and to about 0·3 at normal incidence.

Incidentally Rayleigh's results for oblique reflection from a perforated porous wall are identical with the above if

$$\Omega = \frac{1}{v} \cdot \frac{1}{1+g} \cdot \frac{k' \tan k'l}{ik}$$

where $k'^2 = k^2 - i\omega h/v^2$, $k = 2\pi/\lambda$, h is Rayleigh's dissipation factor, and l is the depth of the perforations. This therefore is the theoretical value of Ω for Rayleigh's problem. When the channels are so long that no sound returns from them, a very simple equation results, viz. $\Omega = 1/v(1+g)$. Absorption is unity (complete) when $\cos\theta = v\Omega$, *i.e.* when $\cos\theta = 1/(1+g)$.

V. Kühl and E. Meyer † have indicated qualitative agreement between experiment and the above theory for variation of absorbing power with angle of incidence.

* E. T. Paris, *Roy. Soc., Proc.*, 115, 407, 1927; "acoustical admittance" is equal to ρ times the reciprocal of the acoustical impedance.

† V. Kühl and E. Meyer, *Nature*, 130, 580, 1932.

The admittance Ω can be calculated if the ratio of the amplitudes of the incident and reflected waves are measured at some given angle of incidence, together with the change of phase on reflection. Paris proposed making the measurements at normal incidence by the stationary-wave method. He has since proceeded further,* and shown how the reverberation absorption coefficient (p. 156)—*i.e.* the coefficient for miscellaneous incidence—can be inferred. His equation is

$$a_r = 8v\Omega\left[\frac{1+2v\Omega}{1+v\Omega} - 2v\Omega\log_e\left(\frac{1+v\Omega}{v\Omega}\right)\right] \tag{7}$$

Experimental confirmation of this formula is not yet forthcoming.

Variation of Absorption Coefficient with Thickness (Normal Incidence). Crandall,† from certain assumptions, has worked out a formula for the variation of the absorbing power of a porous material with thickness. The formula relates to normal incidence, and the material is supposed to be backed by a rigid, perfectly reflecting plate. He regards the transmission through the material as being represented by the formula $\xi = \xi_0 e^{-(\alpha+i\beta)x}e^{i\omega t}$, where α and β are attenuation and phase factors which have to be determined experimentally. He considers the case of a unit incident wave. Allowing for reflection to and fro within the material, Crandall arrives finally at the result for a thickness l:

$$\xi = \frac{r - e^{-2(\alpha+i\beta)l}}{1 - re^{-2(\alpha+i\beta)l}} \tag{8}$$

where ξ is the (complex) displacement amplitude of the reflected wave, and r the reflected amplitude for infinite thickness. If pressure amplitudes are considered instead of displacements the formula becomes

$$p = \frac{r + e^{-2(\alpha+i\beta)l}}{1 + re^{-2(\alpha+i\beta)l}}$$

Experimental data corresponding to these formulæ have been obtained by stationary-wave method.‡ Measurements were made both of the amplitude and of the phase displacement of the wave reflected from various thicknesses of cotton-waste. From these data the constants r, α, and β were deduced, and reflection coefficients for various thicknesses thereby calculated. The

* E. T. Paris, *Phil. Mag.*, 5, 489, 1928.
† I. B. Crandall, *Theory of Vibrating Systems and Sound*, p. 195.
‡ A. H. Davis and E. J. Evans, *Roy. Soc., Proc.*, 127, 89, 1930.

theoretical curves thus obtained for a series of frequencies are exhibited in fig. 81, and show good agreement with the experimental results plotted upon them as points. It is of interest to note that the velocity of sound in cotton-waste at 1200 cycles per second, calculated from β, is approximately 21,000 cm. per second. For certain regions of thickness, an increase of thickness produced a decrease of absorption coefficient. This effect,

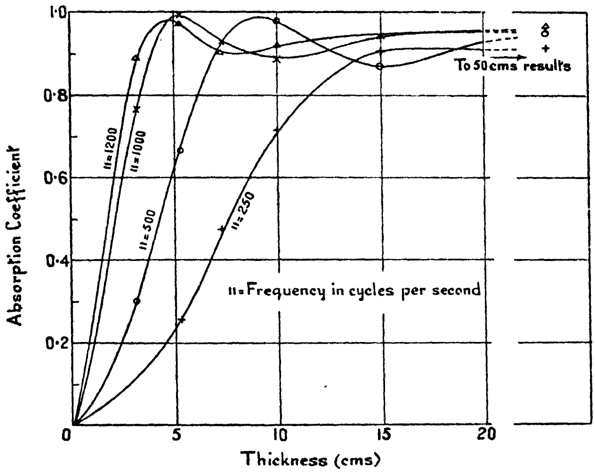


FIG. 81.—Absorption coefficient of cotton-waste variation with thickness at different frequencies

which had not been noted before, is due to the interference between sound reflected directly from the front surface and that emerging from the material after reflection at the backing surface.

Results were also obtained for felt and for cotton-wool in layers. The behaviour is generally the same in all cases, the absorption rising to a maximum at roughly $\frac{1}{4}\lambda$ for each frequency, then falling away and finally becoming constant.

Variation of Absorption with Distance of a Material Layer from a Backing-plate (Normal Incidence). Davis and Evans * derived a formula for the absorbing power of a layer of material set with its back surface at various distances l from a rigid reflecting backing-plate, the sound being incident perpendicularly. On incidence upon the absorbent, part of the sound is reflected and part transmitted. The fraction transmitted is reflected to and fro between the backing-plate and the material, and at each arrival at the material a fraction emerges again on the side of the

* A. H. Davis and E. J. Evans, *loc. cit.*

incident wave. At a point x cm. in front of the specimen the pressure in the composite reflected wave is found to be given by

$$p = e^{-i\beta x} e^{i\omega t} \left[\mathrm{R} + \frac{\mathrm{T}^2 e^{-2i\beta l}}{1 - \mathrm{R}e^{-2i\beta l}} \right]$$

where R and T are the coefficients of reflection and transmission for the layer of absorbent itself, and are in general complex.

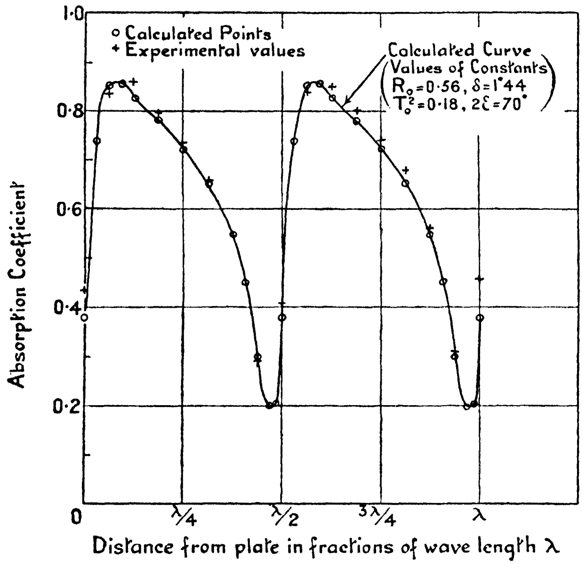


FIG. 82.—Absorption coefficient of ½-inch felt at different distances from backing-plate (frequency of 1200 cycles per second)

Writing $\mathrm{R} = \mathrm{R}_0 e^{-i\delta}$ and $\mathrm{T} = \mathrm{T}_0 e^{-i\epsilon}$, it is found that the constants of the equation can be determined from the values $\mathrm{P}_{2\pi}$ and P_π of the amplitude of the pressure of the reflected wave for distances $l = \lambda/2$ and $l = \lambda/4$ between the material and the back plate. Thus

$$\mathrm{R}_0 e^{-i\delta} = \frac{\mathrm{P}_{2\pi} + \mathrm{P}_\pi}{2 + \mathrm{P}_{2\pi} - \mathrm{P}_\pi}$$

$$\mathrm{T}_0{}^2 e^{-2i\epsilon} = \tfrac{1}{2}(\mathrm{P}_{2\pi} - \mathrm{P}_\pi)(1 - \mathrm{R}_0{}^2 e^{-2i\delta})$$

Thus from a knowledge of $\mathrm{P}_{2\pi}$ and P_π one can calculate R_0, δ, T_0, ϵ, and hence find the reflected wave for any position of the absorbing material.

As a test of the formulæ, measurements were made of the

amplitude and phase of the sound reflected from $\frac{1}{2}$-in. felt placed at different distances from the backing-plate of a stationary-wave apparatus. In each case the theoretical curves for variation were calculated from measurements at two distances by formulæ as indicated above. Fig. 82 shows the comparison between the theoretical and the experimental results, at a frequency of 1200 cycles per second. It will be noted that both theory and experiment agree that the absorbing power increases very rapidly within a short distance of the plate, and that the results are repeated for positions $\frac{1}{2}\lambda$ apart.

Absorption of Sound by Non-rigid Materials. Sound can be absorbed by materials which are not porous, if the incident sound sets the material in vibration. The extent of the absorption depends upon the damping to which the vibration is subjected by the viscous and similar processes which occur in the material itself. The equations for this case are of the type of those given earlier in connection with the transmission of sound through a thin slab of material under elastic restraint. For a given incident sound the response $\dot{\xi}_2$ of the panel is given by equation (41) (p. 221), so far as this equation is applicable. The rate of dissipation of energy in the panel is $r\dot{\xi}_2{}^2$, a quantity which is absorbed from the incident sound, and neither reflected nor transmitted.

CHAPTER XIV

THE EAR AND HEARING

The Human Ear. The human ear may be regarded as divided into three parts : the outer, the middle, and the inner ears. The outer ear consists of the external parts (pinna) and the ear canal (external auditory meatus), and it terminates at the ear-drum (membrana tympani). The middle ear contains three small bones (ossicles), the hammer, anvil, and stirrup, which connect the drum with the small oval diaphragm or window (fenestra ovalis) of the inner ear. The inner ear is a complex structure, and reference should be made to appropriate textbooks for details.* It contains the semicircular canals which are concerned with the maintenance of equilibrium, as well as the cochlea which is really the end organ of hearing. The cochlea is a spiral cavity in the bone, and it is filled with liquid ; into it a spiral ledge projects, and the liquid above the ledge is separated from that below by a flexible membrane along which the nerves of hearing are distributed. Two windows, the oval window and the round window, retain the liquid at the base of the spiral, but at the apex there is a small hole (helicotrema) through the membrane which allows liquid to flow from the upper side to the lower.

Sound entering the ear sets up minute changes in the air pressure in the ear canal, and these cause the ear-drum to vibrate. This vibration is communicated to the oval window, and thus to the liquid in the inner ear by means of the chain of ossicles. By their lever action the ossicles act as a transformer for facilitating communication of vibrational energy from the light medium (air) in the outer ear to the denser (liquid) in the inner ear. The processes within the inner ear are not fully understood, and there are various forms of the theory of hearing. For details reference

* See also *Phys. Soc., Discussion on Audition,* 1931.

must be made to other works. We may perhaps suppose that the vibrations of the liquid in the inner ear affect the central membrane in different positions depending upon the frequency of the sound, the high tones disturbing the thick end of the membrane—*i.e.* the end near the oval window—and the low tones affecting the other. In some way the pattern of the disturbance of the membrane is conveyed to the brain and there interpreted.*

When the aerial waves entering the ear consist of a regular succession of vibrations the sensation is that of a tone, but an irregular disturbance results in an unpitched sensation. The aerial waves giving rise to a tone may differ in frequency, in amplitude, and in wave-form. Amplitude determines the loudness of the sound sensation, frequency determines pitch, and wave-form determines the quality of the tone.

Pitch. The relation of the frequency of the vibration to the sensation of pitch is well known. Doubling the frequency results in raising the pitch by an octave, as is seen from Table IX.

TABLE IX

Frequency (vibrations per second)	Pitch
16	C_{II}
32	C_{I}
64	C
128	*c*
256	c' †
512	c''
1024	c'''
2048	c^{iv}
4096	c^{v}
8192	c^{vi}
16384	c^{vii}
32768	c^{viii}

More generally equal ratios of frequency give rise to equal intervals of pitch. Clearly if frequency is plotted on a logarithmic

* See H. Fletcher, *Speech and Hearing*, 1929; H. Fletcher, *Frank. Inst. J.*, 196, 289, 1923; V. O. Knudsen and I. H. Jones, *Ann. of Otol. Rhin. and Laryngology*, December 1925 and March 1926; H. E. Roaf, *Phil. Mag.*, 43, 349, 1922.

† Middle *c* on piano.

scale,* equal increments of the scale correspond to equal frequency ratios, *i.e.* to equal intervals of pitch. Such a method of plotting is almost universally adopted, and different notes are spaced along it in the manner in which they are spaced along a pianoforte keyboard. On the 'tempered' scale a semitone—of which there are 12 to the octave—is associated with a frequency ratio of $2^{1/12} = 1{\cdot}05946$-fold. A unit which is often convenient is the centi-octave, defined by the ratio $2^{1/100}$ or 1·0069 approximately. There are, of course, $100/12 = 8\frac{1}{3}$ intervals of a centi-octave in a semitone.

The range of pitch to which the human ear is sensitive depends somewhat upon the individual. The lowest frequency sensed as a note is given variously, but may be taken perhaps as 16 vibrations per second. The upper limit depends upon age, and is about 20,000 vibrations per second. This is a total range of some ten octaves. The range of the piano is seven and a half octaves.

In the ten octaves to which the ear is sensitive, about 1500 gradations of pitch can be distinguished when notes are sounded successively. For sounds of moderate loudness and of frequencies above 400 per second the minimum perceptible frequency change is about 0·3 per cent. (*i.e.* about 1/20 of a semitone or 0·4 centi-octaves). For lower notes a rather greater ratio is necessary. Thus the ear can just barely distinguish between tones of 128 and 129 d.v. (*i.e.* about 1 centi-octave) and between tones of 50 and $50\frac{1}{2}$ d.v. (about $1\frac{1}{2}$ centi-octaves). Probably a greater ratio than 0·3 per cent. is also necessary at higher audible frequencies above, say, 10,000.†

Apparently training and musical talent do not enable smaller differences of pitch to be detected, but increase the nicety of judgment as to whether a tone is sharpened or flattened. Disease of the ear does not impair discrimination of pitch.

Quality. The quality of a tone of given pitch is determined by the wave-form—that is, by the manner in which the aerial pressures at the ear rise and fall during each vibration. The

* It has been proposed that the logarithm, to the base 2, of the frequency shall be given the name of frequency level, and Fletcher (*Speech and Hearing*) has employed a scale in which pitch is expressed in terms of the number of octaves by which the note lies above a zero pitch taken at a frequency of 1000 cycles per second.

† V. O. Knudsen, *Phys. Rev.*, **21**, 84, 1923; see also E. G. Shower and R. Biddulph, *Acous. Soc. Am. J.*, **3**, 275, 1931.

simple pendular motion of the prong of a tuning-fork in free vibration gives rise to a wave-form represented by fig. 9 (*a*) (p. 30), which is the characteristic 'sine wave' associated with pure tones. In the violin, however, where the string vibrations are constrained by the bow, or in, say, a reed-organ pipe, a more complicated wave-form is obtained (fig. 9 (*c*)). In this figure it is seen that the more leisurely fundamental vibration is accompanied by subsidiary fluctuations of higher frequency evidenced by the wavelets present. Expressed in another manner, the note contains a number of overtones in addition to the fundamental note. In general, overtones consist of tones having frequencies exactly 2, 3, 4, 5, etc., times the frequency of the fundamental tone, and in general any note of accurately repeating wave-form can be shown mathematically to be composed of a fundamental tone in combination with selected overtones of suitable strength. By sounding suitable harmonic combinations of tuning-forks the total qualities of the violin or of the reed-organ pipe or of other instruments may be built up.

Limits of Audibility. Before a tone can give rise to the sensation of hearing, the vibrations must attain a certain minimum amplitude. This minimum amplitude varies greatly with the pitch of the tone. Low tones require considerable amplitude as also do tones of highest pitch, and the ear is most sensitive for notes in the region of c''' to c^{v} on the musical scale, corresponding to a frequency range of 1000–5000 vibrations per second.

As the amplitude of the sound is increased, the sensation becomes progressively louder until the sensation of feeling or tickling supervenes in the ear and, shortly afterwards, pain is experienced. There is no doubt that the character of the sensation alters at this point. For it has been found that even deaf-mutes, who have no sensation of tone, may, with such intense sounds, experience the sensation of feeling in the ear. Also, if the same intensity of sound is impressed against the finger, it excites the tactile nerves. The intensity at which the sensation changes from hearing to feeling is regarded as the maximum utilisable for hearing.

The amplitude of a sound wave is frequently expressed in terms of the variations of aerial pressure. The upper and lower curves of fig. 83, based upon curves by Fletcher and Wegel,*

* H. Fletcher and R. L. Wegel, *Phys. Rev.*, 19, 553, 1922.

represent the upper and lower limits of audible sound as a function of pitch. The unit of pressure—1 dyne per sq. cm.—corresponds to about one millionth of an atmosphere, and is of the order of the pressure variations in conversation as ordinarily heard. It is to be noted that enormously greater amplitude is required with low notes than with notes of middle pitch before the ear perceives the note at all, but also that low notes become painful

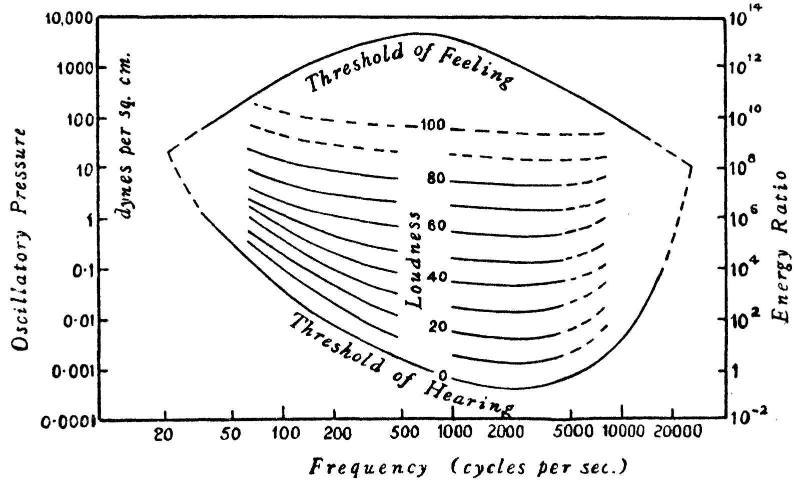


FIG. 83.—Auditory sensation curves and equal loudness contours

sooner than middle ones. Acoustical power and energy ratios are proportional to the square of amplitude ratios, and the scale to the right of fig. 83 shows the enormous range of power—a million million-fold—to which the ear is adaptable. In the region of painful sounds it may be calculated that an instrument giving a painfully loud note of middle C in pitch, at a distance of 10 ft., would be sending some 5 h.p. of sound energy across a spherical boundary at this distance. For moderate everyday sounds it has been calculated that if a million people were to talk steadily, and the energy of their voices were converted into heat, they would need to talk for an hour and a half to produce enough heat to make a cup of tea. At the threshold of audibility the energy output of a source of medium pitch is almost too small for description: it would take a million such sources about two centuries to produce enough heat to make a cup of tea.

Various methods have been employed in determining the minimal audible intensity of sound, and bibliographies of early

work were given by J. P. Minton * and C. M. Swan.† In 1877 Rayleigh made estimates from the energy consumed in a whistle of high pitch and the distance at which the whistle became inaudible. Rayleigh ‡ also studied the question by measuring the currents in a telephone receiver which gave sounds on the borderline of audibility. Wien § also published results obtained with telephone receivers. This early work suffers from the fact that the ear's characteristics were marked by those of the receiver. Advances in connection with the theory of receivers has, however, made it possible to obtain reliable results with these instruments. Fletcher and Wegel || carried out an extensive research over the range 60–4000 cycles per second, using a special air-damped telephone receiver pressed to the ear, and excited by alternating current supplied from a valve oscillator and controlled in magnitude by a calibrated attenuator. J. P. Minton * employed a telephone receiver tuned to a natural frequency of 5215 cycles per second. He controlled the current supplied to the receiver by means of a form of Wheatstone's bridge, and calculated the acoustical energy at various frequencies from a knowledge of this current, and of the electrical and vibrational characteristics of the receiver as determined by impedance analysis. In 1921 Kranz ¶ employed as a source of sound a thermophone pressed against the ear, and Lane,** using a Hewlett generator as a source of calculable intensity, conducted measurements out of doors over the range 2000–14,000 cycles per second.††

For relative measurements audiometers are available; in these an oscillator is arranged to actuate a telephone receiver with notes of frequency 64, 128, 256, 512, 1024, 2048, 4096, or 8192 cycles per second. The intensity of the note is varied by means of a calibrated attenuator.

Intensity Discrimination (pure tones). Between the upper and lower limits of audibility the human ear can distinguish some 270 gradations of loudness at a frequency of 512 vibrations per

* J. P. Minton, *Phys. Rev.*, 19, 80, 1922.
† C. M. Swan, *Proc. Am. Acad.*, 58, 425, 1923.
‡ Rayleigh, *Phil. Mag.*, 38, 285, 1894.
§ M. Wien, *Phys. Zeits.*, 4, 69, 1902.
|| H. Fletcher and R. L. Wegel, *Phys. Rev.*, 19, 553, 1922.
¶ F. W. Kranz, *Phys. Rev.*, 17, 384, 1921; also 21, 573, 1923.
** C. E. Lane, *Phys. Rev.*, 19, 492, 1922.
†† L. J. Sivian and S. D. White (*Acous. Soc. Am. J.*, 4, 288, 1933) give free air thresholds some 10 decibels lower than those in fig. 83. They give a useful bibliography.

second, but rather fewer at low frequencies and high frequencies. It has been found that at ordinary loudness levels, *i.e.* for intensities (energy units) greater than 10^4 times the minimum audible, the minimum change of sound intensity just perceptible is approximately the same for all frequencies, and is about 10 per cent.* For fainter sounds rather larger changes are needed, and a 20 per cent. change is necessary in the case of a sound having only 10 times the energy of minimum audibility.

It may be mentioned that these figures apply to two notes sounded successively. If a silent interval of only one half second elapses between the two notes, double the above loudness changes are required, and the number of just perceptible steps of loudness falls to, say, 120 at 512 cycles per second.

The total number of *pure* tones which the human ear can distinguish from each other in pitch or in loudness has been estimated from available data on pitch and intensity discrimination, and appears to be somewhat over 300,000.

Relation between Loudness and Stimulus. It will be noted that, except at the various limits of audibility, a percentage change in the energy of the stimulus is the minimum perceptible to the ear. It is not the actual increase in sound energy that matters, but the fractional increase. It follows that the loudness level in just perceptible steps above threshold is closely related, within certain limits, to the logarithm of the intensity of the exciting sound,† and a logarithmic unit called a 'bel' is adopted. If I and I_0 are two different values of the sound energy, the difference in energy level is given by

$$L = \log_{10} (I/I_0) \text{ bels}$$

A unit (the 'decibel'), which is 1/10 of the 'bel,' is more convenient in acoustical work, and the difference in energy L may be expressed in decibels thus:

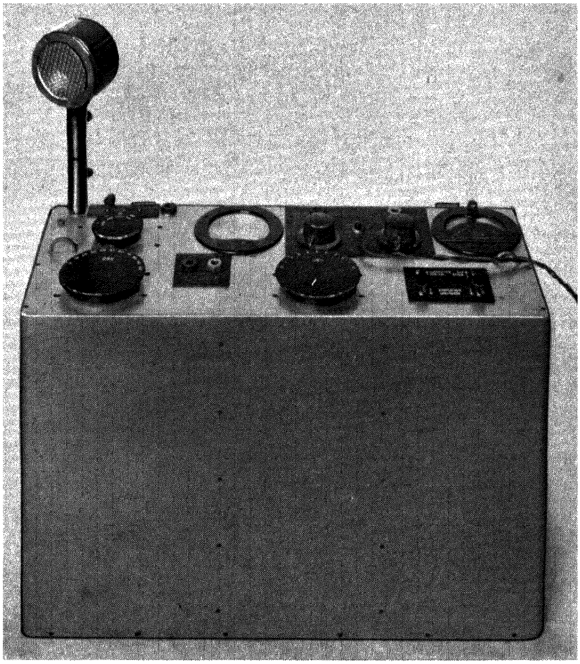
$$L = 10 \log_{10} (I/I_0) \text{ decibels}$$

From this it is found that a 26 per cent. change in intensity alters the energy level by 1 decibel. The 'decibel,' equal to the well-known 'transmission unit' of telephony, is purely a unit

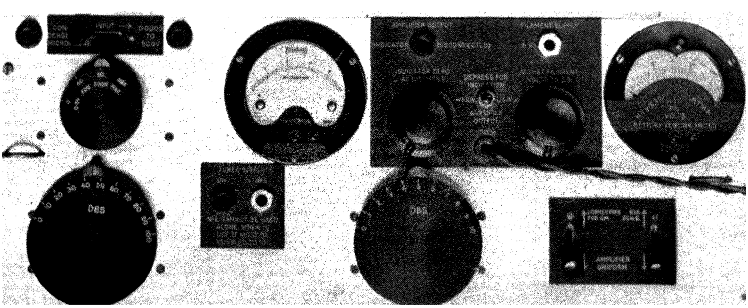
* V. O. Knudsen, *Phys. Rev.*, **21**, 84, 1923; see also R. R. Riesz, *Phys. Rev.*, **31**, 867, 1928.

† The Weber-Fechner law of psycho-physics, which applies within limits to all the sense organs, states that equal increments of sensation are associated with equal increments in the logarithm of the stimulus.

PLATE VI



General view



Top : microphone removed

Portable Acoustimeter (Davis). See p. 264

of power ratio.* It is important because it is practically the smallest change in energy level that the ear can ordinarily detect. Actually the 10 per cent. energy change detectable under best laboratory conditions is about 0·4 decibel.

Observations have been made of the intensity differences which exist when observers judge one sound to be 'half as loud' as another of the same pitch. Results are interesting, but not very concordant at the present time.†

Comparison of the Loudness of Pure Tones of Different Frequencies. Fig. 83 shows, in addition to the maximal and minimal audible intensities of pure tones of various frequencies, some curves based upon data by B. A. Kingsbury ‡ connecting the intensities of sounds of different frequencies which are judged to be equally loud by the ear. There is no significant sex difference. It is seen that for frequencies above about 700 cycles per second, if two pure tones of different pitch are to be equally loud, they remain equally loud when the amplitude of both are increased in the same ratio. It appears from the closer spacing of the curves at low frequencies that low notes require less increase of amplitude than high notes in order to raise their level of loudness. These facts are expressed in another manner in fig. 84, where curves for various notes show the necessary intensity, in decibels above threshold, to yield a loudness equal to a given intensity of note of 800 cycles per second. The curves are barely distinguishable for frequencies above 700 cycles per second. As a consequence of the rapid increase in the loudness of low notes when they are amplified, amplification of complex notes is liable to give greater prominence to low-pitched constituents.

* There is a need for a mathematical terminology for expressing any quantities (not merely power) in common logarithmic form. The writer finds of value, in acoustics and in other sciences as well, the following terminology based on the name of the inventor of common logarithms:—

Brig.—Two numbers N_1 and N_2 are said to differ by n brigs (or $10n$ decibrigs) when $n = \log_{10}(N_1/N_2)$.

As the term bel is used in this book it is simply a change of 1 brig in acoustical energy; moreover, since intensity or energy is usually mentioned for completeness wherever the bel is employed, the term bel could nearly always be replaced by the term brig, and decibel by decibrig, within the pages of the present work.

† L. F. Richardson and Ross, *J. Gen. Psycho.*, April 1930; Laird, Taylor, and Wille, *Acous. Soc. Am. J.*, 3, 393, 1932; L. B. Ham and J. S. Parkinson, *ibid.*, 3, 511, 1932; P. H. Geiger and F. A. Firestone, *ibid.*, 5, 25, 1933; B. G. Churcher and A. J. King, *Nature*, 131, 760, 1933.

‡ B. A. Kingsbury, *Phys. Rev.*, 29, 588, 1927.

From the curves of fig. 83 it is clear that the loudness of any pure note may be expressed as a number by stating the intensity, in decibels above threshold, of an equally loud note of chosen standard frequency—say 800 ~. Moreover, it would probably be a convenience to choose the pitch of the standard note in the region above about 500 ~, where a decibel has approximately a constant loudness effect. The figures on the contours in fig. 83

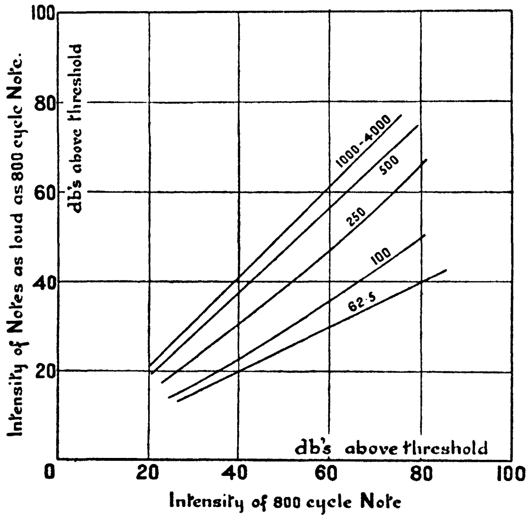


FIG. 84.—Comparison of loudness value of decibel for notes of different pitches

(Curves showing, for various notes, the necessary intensity (in decibels above threshold) to yield a loudness equal to that of an 800-cycle note of given intensity)

express the loudness of the various notes by stating the loudness in decibels above threshold of an equally loud 800-cycle note.

It is to be noted that in expressing the loudness of a sound no mention is made of its intensity, but the intensity of a comparison note of equal loudness is given in decibels above threshold. Now in the absence of a separate term for loudness many writers treat the term 'decibel' as applicable equally to 'intensity' and 'loudness.' When the form of expression of these different ideas is exact it is cumbrous. When the form of expression is loose, persons but slightly acquainted with the subject are apt to think that a noise having a loudness of *n* decibels, has also an intensity of *n* decibels above its own threshold. This is not true and leads to confusion. In what follows the word 'decibels' will be reserved for expressing intensity ratios. When a sound has a loudness

equal to that of a standard audiometer note at a level of n decibels above threshold, it will be said to have a loudness of n 'phon.' *

The Loudness of Complex Tones. The loudness of a pure note on such a scale can clearly be inferred from the curves when its pitch and physical intensity are known. There appears, however, to be no accepted method of deducing the loudness of a complex sound from a knowledge of its constituents, although it is true that complicated relations have been proposed. Consequently, at present the only method of expressing the loudness of a complex note or of a noise would appear to be the direct experimental comparison with a note of standard pitch and controllable intensity. Within limits such comparison appears to be possible.

As regards formulæ for the loudness of complex notes Barkhausen † and his collaborators suggested as a simple rule that the loudness was equal to that of the physiologically loudest overtone, the remaining partials contributing nothing to the loudness if they differed by more than 20 per cent. in frequency. This rule, however, was found later to be inadequate.‡

Steinberg § found that the subsidiary partials make a contribution, and that the loudness of certain complex notes could be assessed from a weighted summation of the effects of the various constituents. His formula, however, proved to be inadequate to cover a wide range of conditions.

Steudel,|| following Barkhausen, has proposed for sounds of loudness exceeding 50 phon a different type of formula. It is based upon experiments with notes, sound pulses, and noises. The loudness L in phon (referred to a 1000-cycle note as standard) is said to be given by

$$L = 20 \log \left[\frac{1}{P_0} \frac{1}{\tau} \left\{ \int_{t_0}^{t_0+\tau} (p - p_0) dt \right\}_{\max} \right]$$

where p is the instantaneous sound-pressure in dynes per sq. cm.

* In Germany a 'phon' used to be an intensity ratio defined by a twofold change of amplitude, *i.e.* a fourfold change of intensity or 6 db. Now the 'phon' is used as the equivalent of the decibel. In this book the word decibel will be used for intensity relations, phon for expressing loudness.

† H. Barkhausen and G. Lewicki, *Phys. Zeits.*, 25, 537, 1924; H. Barkhausen and H. Tischner, *Zeits. f. tech. Phys.*, 8, 215, 1927.

‡ H. Barkhausen and U. Steudel, *Hochfrequenztech. u. Elektroakustik*, 41, 115, 1933.

§ J. C. Steinberg, *Phys. Rev.*, 26, 507, 1925.

|| U. Steudel, *Hochfrequenztech. u. Elektroakustik*, 41, 116, 1933.

at the entrance to the ear, p_0 the pressure at time t_0, and τ a short interval of time ($\tau = 3 \times 10^{-4}$ seconds). It is also necessary to know, or to determine, the threshold pressure P_0 for the sound concerned. According to the formula the loudness is determined by the integrated oscillatory pressure over an interval of 3×10^{-4} seconds in the region of the peak pressure; in other words, by the maximum impulse delivered to the ear in the interval mentioned. If the pressure impulse is repeated more than once per second an addition must be made to L; the additional loudness is about 5 phon for 5 repetitions per second, and rises to 10 phon for 50 or more repetitions per second.

Fletcher and Munson* have proposed the following formula for the loudness (L) of steady sounds having n components:—

$$G(L) = \sum_{k=1}^{k=n} b_k G(L_k).$$

The loudness L_k of the k^{th} component is calculated in phon from its frequency and intensity. Its contribution to the total loudness (L) is determined partly by a function $G(L_k)$ which is tabulated in the paper for various values of L_k, and partly by the factor b_k. This latter quantity depends in a specified manner upon the frequency, intensity, and loudness of the k^{th} component, and upon the frequency and loudness of its 'masking' component (usually the neighbouring component of lower pitch). All constituents in certain specified frequency bands have to be treated as one. The formula has been tested for steady complex sounds in which the partials were harmonically related. Work is said to be in progress on the loudness of unsteady sounds.

Annoyance. Many persons have suspected that the annoyance experienced as a result of noise is a function of pitch as well as of intensity. Many people unhesitatingly state that high notes are to them more annoying than low notes. On the other hand, some observers claim that prolonged exposure to low notes is especially troublesome.

Laird and Coye† conducted some experiments (not fully described) on the loudness of notes of different pitches which were judged to be equally 'annoying' to a number of observers trained in acoustical work. Preliminary work showed that there was not a single pitch which was not 'annoying' to some observer,

* H. Fletcher and W. A. Munson, *Acous. Soc. Am. J.*, 5, 82, 1933.
† D. A. Laird and K. Coye, *Acous. Soc. Am. J.*, 1, 158, 1929.

but the least annoying were notes of 256, 512, and 1024 cycles per second. Laird and Coye plotted lines of equal annoyance on a graph upon which contours of equal loudness were also shown. For notes below, say, 500–700 cycles per second they found the 'annoyance' and the loudness curves were parallel, and inferred that for low notes annoyance is determined by loudness alone. But for notes above 500 cycles per second they found a tendency for the high notes to be specially annoying, particularly when they were very loud. It was not, however, possible to find a method of measuring annoyance. The offensiveness of some motor-horns with loud high-pitched constituents—to which reference will be made later—together with other observations of everyday life, tend to confirm Laird and Coye's conclusions. They themselves claimed that the irritation produced by certain tenor and soprano voices is in harmony with their findings.

Beats, and Combination Tones. It is well known that two tones differing slightly in pitch produce 'beats' when sounded together, *i.e.* they unite so that the resultant tone periodically augments and diminishes in loudness. The frequency of the fluctuation of the loudness is equal to the difference of the frequencies of vibration of the two tones. If the frequency of the fluctuations (beats) is not greater than about 20 per second, the fluctuations are sufficiently slow to be followed by the ear.

Usually as the frequency of the beats is increased beyond this limit the ear senses a musical tone having the frequency of the beats. This tone is not simply a beat phenomenon, but is a combination tone owing its existence to the fact that the response of the ear is not strictly proportional to the amplitude of the exciting sound. The ear-drum is not symmetrically loaded, and the response depends to some extent upon the square of the amplitude of the sound as well as upon the first power. We have seen, when considering forced vibrations (p. 23) and in the electrical case (p. 179), that this leads to the production of summation and difference tones in the transmitted disturbance, and the apparent 'beat tone' is in reality the difference tone. Consistent with this explanation, it is to be noted that the ear also senses the summation tone having a frequency equal to the sum of the frequencies of the constituent tones. For the satisfactory hearing of combination tones the primary tones should be loud and sustained, and it is an advantage if the pitch of the combination is in the middle of the audible range. Combination

tones are readily observed when two loud high-pitched Edelmann whistles are sounded simultaneously and the pitch of one is altered slowly.

The combination tones mentioned are subjective, for they are produced by the behaviour of the ear and have no existence in the incident sound. Combination tones may also be produced

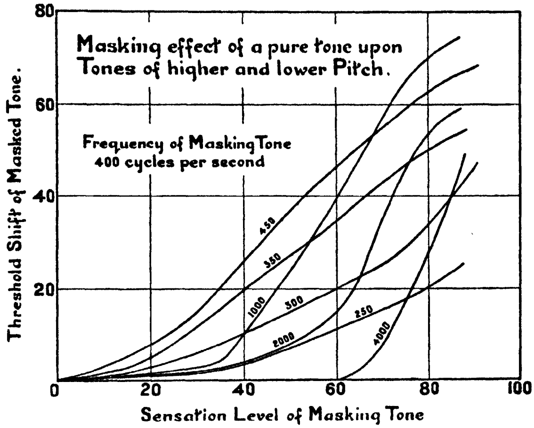


FIG. 85.—Masking effect of a pure tone upon tones of higher and lower pitch

in the air itself in the case of loud sounds, for there is not strict proportionality between pressure and condensation unless the amplitude of the sound is very small. It is to be noted also that detecting instruments other than the ear—microphones, for instance—may fail in proportionality for loud sounds, with a consequent production of combination tones.

Masking of one Sound by Another. Experiments have been carried out upon the extent to which one sound interferes with the hearing of another.* The effect, known as masking, may be measured by noting the amount by which the threshold of audibility for the masked note is raised by the interference. Experiments on the masking effect of one pure sound upon the audibility of another have revealed that, in general, a sound is most easily masked by a note of approximately the same pitch. Where the masking tone differs in pitch from the masked it is found that a pure tone more easily masks a tone of higher pitch than one of lower, particularly when the masking tone is comparatively loud, say as loud as, or louder, than conversational speech. This is illustrated in fig. 85, where the masking effect of a tone of

* R. L. Wegel and C. E. Lane, *Phys. Rev.*, 23, 266, 1924.

400 cycles per second is shown for masked tones of higher and lower pitch. Very similar results are obtained for other frequencies of the masking tone.

Very loud tones give rise in the ear not only to the fundamental tone, but also, owing to the non-linear behaviour of the ear, to combination tones (harmonics) as well. In consequence, the masking effect of a very loud note shows signs of masking by the subjective octave ($2n$) and by the twelfth ($3n$) superposed upon the masking effect of the fundamental (n). The masking effect of complex tones shows signs of the masking by the components, and also by the subjective harmonics and combination tones.

Perception of Direction. Whilst a single ear can give some information concerning the direction of a source of sound, the use of two ears is necessary if any great accuracy is to be obtained. It is not known exactly how the judgment of direction is made, but it must be due to some difference between the impressions received by the two ears. These impressions differ in loudness and in phase relation. On the whole, it appears that the phase relation is the predominating feature for sounds below about 1000 cycles per second. Above that frequency the distance between the ears soon becomes greater than the wave-length of the sound, so several directions may be found in which the path differences differ only in an integral number of wave-lengths. Since the phase relations associated with these various directions are the same, the phase effect becomes ambiguous as a criterion in direction finding. For these frequencies, however, the head has a definite shielding value, and the intensities received by the two ears will differ by an appreciable amount which might assist the location of direction. For the region 100–1000 cycles per second for which the phase effect would appear adequate, Stewart * found results which could be represented by the formula $\phi/\theta = 0{\cdot}0034\text{N} + 0{\cdot}8$ approx., where ϕ is the phase difference in degrees at the two ears, θ is the number of degrees to the right or left of his front that the observer locates the source of sound, and N is the frequency of the sound in cycles per second.

In sound locators † which are used for anti-aircraft defence, listening trumpets are employed some half-metre apart, one communicating by means of a stethoscope tube with the right

* G. W. Stewart, *Phys. Rev.*, 25, 425, 1920.
† E. T. Paris, *Science Progress*, p. 457, 1933.

ear of the observer, one with the left. In this way, since the observing base is much greater than the distance between ears, time differences at the ears are enhanced and accuracy of location increased.

Mechanical Data Concerning the Ear. The ear-canal, which has a length of some 2–2·6 cm. and an opening of area $\frac{1}{3}$–$\frac{1}{2}$ sq. cm., has a volume of about 1 c.c. The drum has an area of about $\frac{2}{3}$ sq. cm., and is slightly elliptical, the vertical and horizontal diameters being about 0·85 and 1·0 cm. respectively. The mechanical impedance of the ear-drum, according to Wegel and Lane, is of the order of 20–30 c.g.s. units over the frequency range 200–4000 cycles per second. Bekesy estimates the logarithmic decrement of the ear as a whole to be 0·1—representing a considerable degree of damping. Barkhausen * found $\Delta = 0{\cdot}72$ over the range 500–1500 cycles per second. W. West † and J. Tröger ‡ have estimated the impedance of the ear at various frequencies. The determination involves the development of standing waves in a tube, the end of which is closed by the ear or ear-drum. The resistance component is deduced from the damping of the tube and the reactive component from the phase change.

West, who was interested in telephonic aspects of the question, studied the impedance of the ear as seen through the orifice of the cap of a telephone ear-piece. He placed the ear-cap and ear at the end of the pipe and noted the resonance of the pipe. He expressed the reactance component in terms of the equivalent volume of an adjustable brass vessel which on substitution for the ear gave rise to the same resonance frequency of the pipe, *i.e.* to the same change of phase on reflection. He compared the absorption of sound on reflection by the ear with the absorption by a long tube closed at the far end by a totally absorbing plug. The data obtained express the impedance Z of the ear in terms of a resistance and a reactance in parallel thus: $1/Z = 1/R + 1/iX$.

West observed that a semi-infinite rigid-walled tube of $\frac{1}{4}$-inch internal diameter provides a good match to a real ear, having an acoustical impedance (alternating pressure/rate of volume displacement, *i.e.* $p/S\dot{\xi}$) of about 130, a rough average value for the acoustical resistance of the ear. West constructed an artificial ear

* H. Barkhausen, *Phys. Zeits.*, 25, 537, 1924.
† W. West, *P.O.E.E., J.*, 21, 293, 1929.
‡ J. Tröger, *Phys. Zeits.*, 31, 26, 1930.

based upon this fact.* It was for use in assessing the pressures set up in human ears by telephone receivers under test. His construction made use of 9 ft. of a $\frac{1}{4}$-inch metal tube (having strands of wool in the last foot or two to act as absorbent) for providing the necessary dissipation of sound energy, some lack of rigidity in the wall being permitted locally for operating the condenser microphone with which the pressures set up were measured. The essential details are shown in fig. 86. The

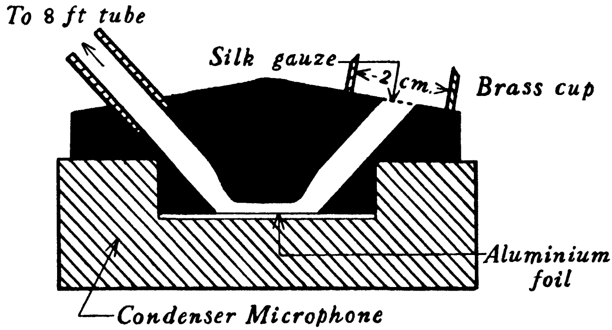


FIG. 86.—Artificial ear (West)

telephone receiver under test is simply placed over the rim of the brass cup. On test the resistance of the artificial ear was found to be about 100, and the equivalent volume (to give the reactance measured) was about 2·8 c.c. as against some 1–5 c.c. for real ears.

Tröger, interested in the mechanics of the ear, studied its impedance when employed as the termination of a tube having a diameter approximately equal to that of the ear-canal. No telephone ear-cap was involved, and his experimental conditions were therefore different from West's. The impedance is expressed in his case in the more usual form of additive components thus: $Z = R_1 + iX_1$. Trögers results, when expressed in the same form as West's, give 1·7–2·7 c.c. for the equivalent volume and 1100–1500 for the equivalent resistance.

Inglis, Gray, and Jenkins † have recently described a new form of artificial ear, and have given a number of further measurements of the acoustical resistance and reactance of the human ear. When the ears were studied as seen through the aperture of a cap of a telephone receiver, the acoustical resistance varied

* W. West, *P.O.E.E., J.*, *22*, 260, 1930.

† A. H. Inglis, C. H. G. Gray, and R. T. Jenkins, *Bell Sys. Tech. J.*, *11*, 293, 1932.

from about 1 to 200 acoustic ohms and the reactance from about -300 to $+100$ acoustic ohms. For an average ear the reactance was about -100 at a frequency of 500 cycles per second, a figure which corresponds to a volume of some 6 c.c. Individual ears vary considerably from the average. The average impedance of the auditory canal was found to have a resistance component varying over the range 10–100 as the test frequency was altered, the reactance varying from -500 to $+200$, and being about -200 at 1000 cycles per second. This latter figure corresponds to the reactance of a volume of about 1·5 c.c.

Speech. In the production of speech air is forced through the vocal passages by the lungs. In the larynx, situated at the upper end of the windpipe, the air-stream passes through a straight slit between two muscular ledges, known as the vocal cords. Through vibration of the cords the slit is alternately opened and closed, so that a train of sound waves is set up in the throat. The frequency of the waves varies from about 90 cycles per second for a deep-voiced man to about 300 for a high-voiced woman. As the waves pass out of the mouth certain resonant characteristics are imposed upon them by the vocal cavities of the nose, throat, and mouth, and certain impulsive characteristics by the movements of the tongue and lips. These variations constitute the voiced sounds of speech. There are also, however, a few speech sounds in which the vocal cords play no part, and these (p, k, t, f, s, ch, sh, th) are known as unvoiced sounds. Both unvoiced and voiced speech sounds may be divided into two classes. In one of these, the continuants, the sounds (such as o, z, and s) are produced by a continuous flow of air, whilst in the other, the stops (such as p, b, and t) are due to sudden stoppage of the air.

Vowel sounds consist essentially of vibrations which build up to a certain loudness, are sustained for a short interval, and then decay to inaudibility. Each vowel has two characteristic frequency regions (fig. 87), and the nasal continuants n, ng, m have three. The frequencies of vowel sounds embrace a range from, say, 300–3000 cycles per second.* J. Obata and C. Satta † find evidence that the falsetto voice is produced by reed-like

* R. A. S. Paget, *Roy. Soc., Proc.*, 102, 752, 1903; H. Fletcher, *Speech and Hearing*, 1929.

† J. Obata and C. Satta, *Phys. Math. Soc. Japan, Proc.*, 14, 341, 1932.

vibrations of the edges of the vocal chords, whereas the chest voice is produced by the opening and closing of the air passage. Consonants, which are not sustained to any great extent, consist mainly of high-frequency sounds above 2000 cycles per second, or in some cases—the 'hiss sounds'—even above 5000 cycles per second. Thus, to transmit speech naturally and distinctly

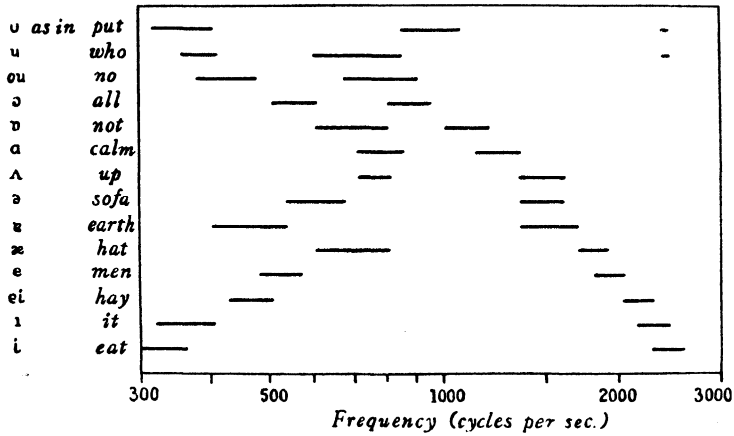


FIG. 87.—Characteristic frequency regions of vowel sounds (Paget)

a telephone system must deal faithfully with a range from 60–5000 cycles per second. However, although the naturalness may suffer, there is but little loss of intelligibility if a narrower range of 200–4000 cycles per second is adopted.

It is important to notice that although the vocal chords give quality and energy to the voice, they are not responsible to any great degree for the distinguishing characteristics of speech. The fact that whispered speech is readily intelligible is evidence of this. Experiments with filtered telephone circuits have shown that the main energy of the voice is of comparatively low pitch, the normal male voice having a pronounced constituent of frequency about 120 cycles per second, whilst the female voice has one about an octave higher. The characteristics essential to intelligibility are, however, of high pitch. For instance, if speech currents in a high-quality telephone line are passed through a filter which removes all sounds higher in pitch than 1500 cycles per second, although only 7 per cent. of the sound energy is suppressed, the percentage of unconnected syllables received correctly by a hearer drops to 62 per cent.—a result well below the limit for satisfactory conversations. On the other hand, a

filter removing all sounds below 500 cycles per second removes 62 per cent. of the energy and results in a somewhat unnatural quality of voice, but leaves the distinctness practically unaffected. Table X is a summary of the effect of filtering high- and low-frequency ranges from speech; the intelligibility results were obtained directly by applying filters to a high-quality telephone circuit,* but the energy data were deduced from an analysis of the distribution of energy in speech carried out by Crandall and MacKenzie.†

TABLE X

Showing the Proportions in which the Energy Content and the Intelligibility of Speech are affected by removing from the Speech all Frequencies above or below certain Limits of Pitch

Limiting Frequency, n	Frequencies above n removed		Frequencies below n removed	
	Per Cent. of Energy remaining	Per Cent. of Unconnected Syllables understood by Hearer	Per Cent. of Energy remaining	Per Cent. of Unconnected Syllables understood by Hearer
250	40	2	60	98
500	62	7	38	96
1000	86	40	14	85
1500	93	62	7	68
2000	96	75	4	40
3000	98	82	2	10
4000	100	87	0	

The Intelligibility of Speech. The method used in obtaining the figures for the distinctness of speech is known as an 'articulation test,' and is that adopted by telephone engineers for testing speech transmission over telephone lines. Several lists of meaningless syllables are drawn up, each list containing the sounds of the language in approximately the proportions which occur in conversation. A reader calls each syllable separately, and a hearer records what he has heard. The percentage of sounds correctly recorded is known as the 'percentage articulation'

* H. Fletcher, *Frank. Inst. J.*, 193, 6, 1922.
† I. B. Crandall and D. MacKenzie, *Phys. Rev.*, 19, 221, 1922.

obtained under the conditions. An articulation of 96 per cent. is the greatest perfection usually obtainable in ordinary conversation. Figures in excess of 85 per cent. represent quite good conditions, which do not necessitate specially attentive hearing; an articulation of 70 per cent. represents the limit taken by telephone engineers as practicable for speech. The subject is a specialised one, and lists of suitable syllables are in regular use. A simplified test, in which lists of simple monosyllabic words are employed, is preferable with untrained observers (p. 307). The relation of articulation to the intelligibility of sentences has been investigated.*

The Effects of Loudness and Distortion upon Speech. The loudness of speech affects distinctness, which is best for the ordinary loudness of conversation. Under these circumstances the oscillatory pressure in the sound wave at one foot from the mouth is about one dyne per sq. cm. Increasing or decreasing the amplitude to a tenfold extent has but little effect upon distinctness, but a decrease in amplitude to one-thousandth reduces the intelligibility of isolated syllables to about 60 per cent. Fig. 88 (*a*) (due to H. Fletcher) expresses the articulation obtained at various intensity levels, in terms of a factor k_i referred to later† (p. 256). It is found that consonants are more seriously affected than vowels by reduction of speech intensity, and in ordinary conversation 50 per cent. of mistakes can be traced to confusion over three consonants alone.

A special type of distortion occurs when, in gramophone reproduction, the speed differs from that employed in recording. This raises the pitch of the speech, and also increases the rapidity of enunciation. It is found that such changes of speed have but little effect upon intelligibility unless the change exceeds 10 per cent. Above this limit the articulation falls off rapidly with further deviation from the normal speed, especially if the speed is being decreased below normal.

Resonances in a telephone or other system of reproduction

* H. Fletcher, *Frank. Inst. J.*, 193, 729, 1922; *Speech and Hearing*, 1929; B. S. Cohen, *I.E.E., J.*, 66, 165, 1927; J. Collard, *Electrical Communication*, 1928; H. Fletcher and J. C. Steinberg, *Acous. Soc. Am. J.*, 1, Supplement 1930.

† The percentage articulation is equal to $100k_i$. The zero of the intensity scale is at a level corresponding to an energy flux in a plane wave of 10^{-9} ergs per sq. cm. per sec. (*i.e.* $2{\cdot}9 \times 10^{-14}$ ergs or $2{\cdot}9 \times 10^{-15}$ microwatts per c.c.). From this it follows (p. 109) that the 70-db. level corresponds to an acoustical pressure of 0·9 dyne per sq. cm.

affect the transmission of speech, but not seriously if the resonance is in the region 900–2000 cycles per second. The impairment is greatest at high intensities, presumably because the frequencies near the resonance become overpoweringly loud. This is said to be particularly noticeable in the case of deaf aids, when the user complains of having his ear 'banged' by certain vowels before the loudness is sufficient for him to hear the consonants.

The effect of removing certain frequency ranges from telephonic speech has been referred to. It is interesting that Collard * has shown that the articulation of a telephone circuit can be calculated from a knowledge of air-to-air attenuation of the circuit for a series of bands, each 200 cycles wide, over the range 0–3000 cycles per second. He can allow for frequency distortion, loudness, and interference by noise within the bands. The full calculation is of course complicated, but it is of value in telephone work, and is understood to be promising. The possibility of the calculation is due to the fact that each speech sound is carried in general by two or three frequency bands, so that if the transmission of the bands is measured it is reasonable to anticipate that calculation can be made of the articulation of sounds in which these bands are involved. The matter is of some importance as providing a link between physical measurements of the response curves of telephones, loud-speakers, etc., and an interpretation in terms of the intelligibility of reproduced speech by the instruments concerned.

Speech and Interfering Effect of Noise and of Reverberation. The effects of extraneous noise on the hearing of speech have received some attention from experimenters, and, for small or moderate noises, the effect is as though the ear were deafened by an amount equivalent to something of the order of the average deafening effect the noise has on the ear for a series of pure notes (500, 1000, and 2000 cycles per second respectively) in the speech range.† In other words, it is as though the loudness of speech were reduced by this amount. Loud noises affect speech to a greater extent than this. No experiments appear to have been carried out to elucidate the type of indiscriminate noise—high pitch or low pitch—which is most disastrous to the hearing of speech, but observations in noisy aeroplanes ‡ suggest that

* J. Collard, *Electrical Communication*, 1929.
† H. Fletcher, *Speech and Hearing*, pp. 185 and 297, 1929.
‡ A. H. Davis, *Roy. Aero. Soc. J.*, 35, 675, 1931.

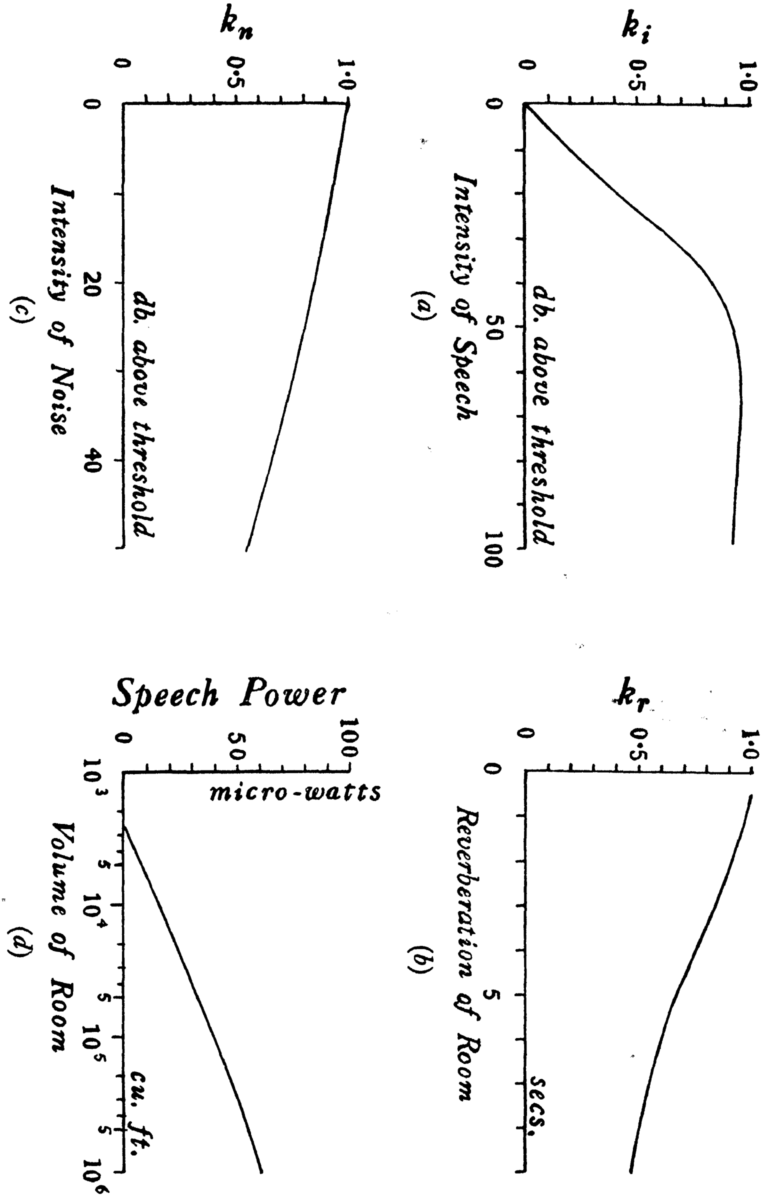


FIG. 88.—Knudsen's data for calculating the probable percentage articulation of speech in a room

high-pitched noises interfere most. With pure tones as the interfering sound, Knudsen * found that the interference was almost independent of pitch when the loudness did not exceed the loudness of speech, but that for louder tones greater interference was caused by low-pitched tones. He also found that the interfering effect of noise was greater than that of tones of any pitch, but no quantitative experiments appear to have been carried out to find the effects of different types of noise.

Knudsen † has evolved a formula, which must be regarded as somewhat tentative, for expressing the effects of intensity, reverberation, and noise upon the hearing of speech in auditoriums. He calculates the percentage articulation to be expected in an auditorium by means of the equation

$$\text{Percentage articulation} = k_i k_r k_n k_s \times 100$$

where k_i, k_r, k_n, and k_s are reduction factors expressing respectively the effects of average intensity, of interference by reverberation, and by noise, and of effects due to unusual shape of the auditorium.

The intensity factor ‡ k_i is given by the curves of fig. 88 (*a*). The average intensity of public speech as heard in auditoriums was found by Knudsen to be about 51 decibels above threshold in small auditoriums (27,000 cu. ft.), and about 46 decibels in large ones (240,000 cu. ft.). This intensity is at a critically low level, so that slight disturbances from reverberation or noise are deleterious.

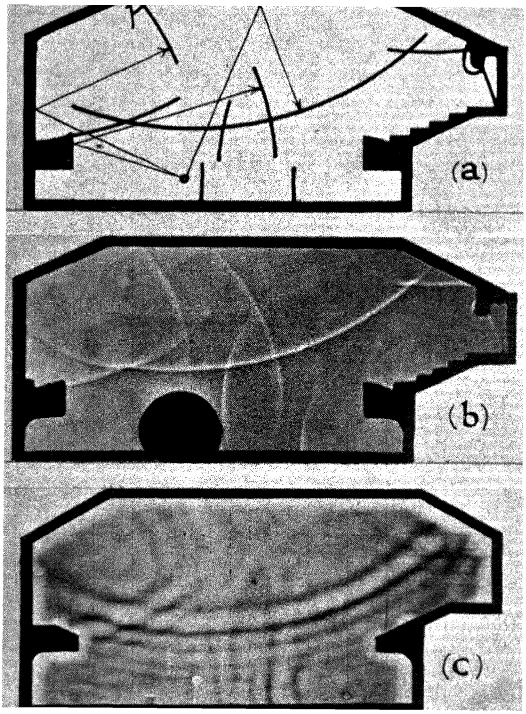
Actually the average intensity built up in a reverberant room depends upon the absorbing power and size of the room. The above are average values, but for any particular case the probable intensity may be calculated from the following formula based upon reverberation theory :—

$$\left.\begin{array}{l}\text{Speech intensity in}\\ \text{decibels above}\\ \text{threshold.}\end{array}\right\} = 10\log_{10}\left(\frac{\mathrm{ET}}{13{\cdot}8\mathrm{VI}_0}\right) = 10\log_{10}\left(\frac{10^8\times\mathrm{ET}}{\mathrm{V}}\right) \text{ approx.}$$

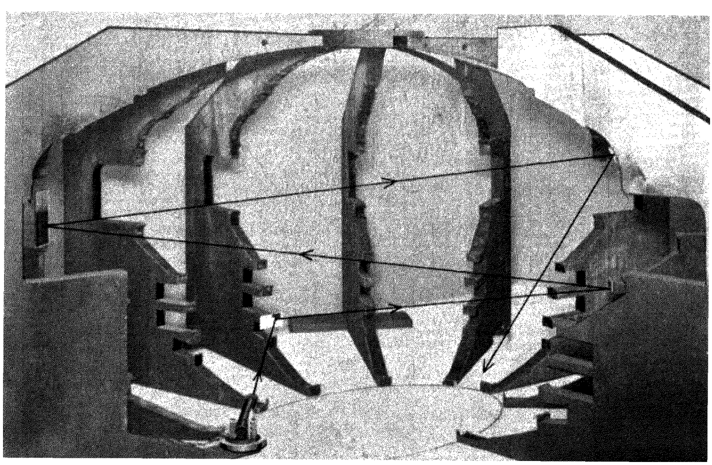
where T = period of reverberation of room, of volume V (cu. ft.),
E = average speech power (microwatts) in a room of volume V (values given in fig. 88 (*d*) due to Knudsen),
I_0 = minimum audible intensity of speech in microjoules per cu. ft. (taken to be $8{\cdot}2\times10^{-10}$).

* V. O. Knudsen, *Phys. Rev.*, 26, 133, 1925.
† V. O. Knudsen, *Acous. Soc. Am. J.*, 1, 56, 1929.
‡ Knudsen speaks of the loudness of the speech, but strictly it is the intensity to which Fletcher's curve, fig. 88 (*a*), refers.

PLATE VII



Geometrical, sound-pulse, and ripple-tank studies of the reflection of sound in a section of an auditorium
See pp. 281, 282



Skeleton model of an auditorium. See p. 282

Knudsen's results for the extent to which reverberation reduces intelligibility are given in fig. 88 (*b*). They were obtained in a series of auditoriums of the general shape and size (200,000–300,000 cu. ft.) but different periods of reverberation. The reverberation was measured in each case for a note of pitch 512 cycles per second. The articulation decreases by about 6 per cent. for each increase of 1 second in the reverberation period. Knudsen's curves for the interfering effect of noise in auditoriums were obtained in a room of 15,000 cu. ft. capacity having a reverberation period of 1·3 seconds, the speech being maintained at a level of 47 db. above threshold. The results are shown in fig. 88 (*c*). Knudsen's formula makes no allowance for the character of the interfering noises, nor for unusual characteristics of the room. Indeed, for the shape factor k_s Knudsen, in the absence of information, takes a factor of unity. The formula is, however, an attempt to apply existing information in a broad manner to a calculation of the probable articulation of speech in auditoriums under various conditions.

Speech Power. The average speech power delivered by an average speaker in ordinary conversation is 10 microwatts,* or 100 ergs per second. Some 15 per cent. of speakers use powers less than 1/10 of the average, and another 15 per cent. use more than twice the average. In loud shouting the average emission of energy is raised a hundredfold, and in soft (but intelligible) whispering it falls to about one ten-thousandth. Data for the average emission by speakers in public halls are given on p. 255. The greatest power is associated with some of the vowels such as 'aw'; the least with certain consonants such as 'th' in thin.

Music. The analysis of musical sounds has received attention at various times since the days of Helmholtz, who carried out measurements with resonators. Early results were largely concerned with the determination of the frequencies of the essential components responsible for the characteristic quality of the instrument. In later work with electrical analysers and filters it has been possible to measure the magnitude of the components, and to assess the total power of the instruments.†

* L. J. Sivian, *Bell Sys. Tech. J.*, 8, 646, 1929.

† E. Meyer and P. Just, *Zeits. f. tech. Phys.*, 10, 8, 1929; L. J. Sivian, *Bell Sys. Tech. J.*, October 1929; L. J. Sivian, H. K. Dunn, S. D. White, *Acous. Soc. Am. J.*, 2, 330, 1931; E. Meyer, *Zeits. f. tech. Phys.*, 12, 606, 1931; H. Lueder, *Wiss. Veröff. a. d. Siemens-Konzern*, May 1930.

Fig. 89, due to W. B. Snow,* shows the actual tone range of various instruments, the minimum range which could be employed in hearing without 80 per cent. of the hearers detecting the introduction of the limiting filter, and the range of noises which accompany certain of the instruments. Ranges for a few other noises and for speech are also given. The lower limit for music

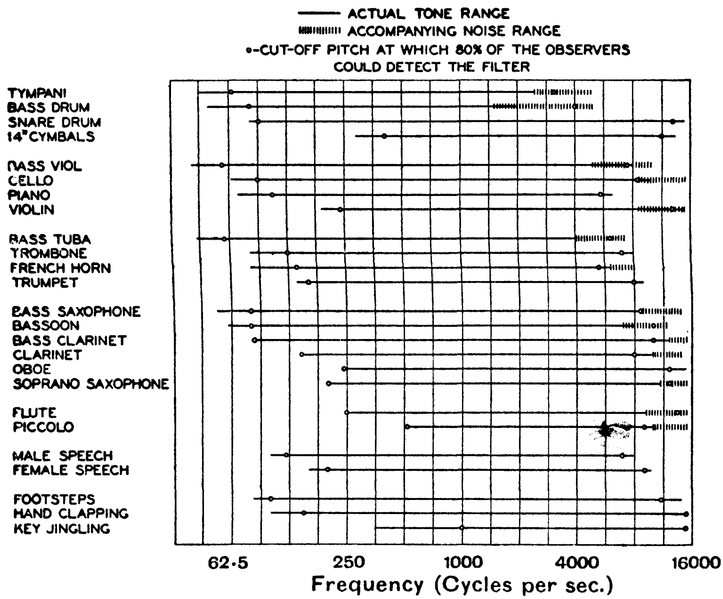


FIG. 89.—Audible frequency range for speech, music, and noise (Snow)

is determined by the bass tuba, the bass viol, and the kettle drum, and is in the region of 40 cycles per second. The upper limit is seen to be about 15,000 cycles per second, as required by the violin, the cymbals, and the snare drum. The range of hearing necessary for the full appreciation of music is thus much wider than that required for the hearing of speech. The important region is said to be from 50–5000 cycles per second.

Measurement of the maximum instantaneous emission of acoustical power from typical musical instruments has yielded, for fortissimo playing, the results given in Table XI.† Generally speaking, the emission from a given instrument is reduced by

* W. B. Snow, *Acous. Soc. Am. J.*, 3, 155, 1931; see also H. Fletcher, *Acous. Soc. Am. J.*, 3, No. 2, part 2, 1931.

† L. J. Sivian, H. K. Dunn, and S. D. White, *Acous. Soc. Am. J.*, 2, 330, 1931.

about fourfold to tenfold from these levels by pianissimo playing.* To find the lowest level used in orchestral music a violin was played as softly as is ever customary when playing in public, and

TABLE XI

Peak Power of Musical Instruments (Fortissimo Playing)

Instrument	Peak Power (Watts)
Heavy Orchestra	70
Large Bass Drum	25
Pipe Organ	13
Snare Drum	12
Cymbals	10
Trombone	6
Piano	0·4
Trumpet	0·3
Bass Saxophone	0·3
Bass Tuba	0·2
Bass Viol	0·16
Piccolo	0·08
Flute	0·06
Clarinet	0·05
French Horn	0·05
Triangle	0·05

the power emission was found to be about 4 microwatts. This is about one twenty-millionth of the peak power of the sound from an orchestra of 75 players.

Figures given by Fletcher for the power radiated by a singer having a voice of ordinary choir strength show that the power varies from about 1000 microwatts for alto pianissimo singing, to 30,000 microwatts for fortissimo bass. These emissions are considerably higher than those used in conversational speech.

The mean value of the sound during the rendition of an orchestral selection sometimes varies by as much as 100,000-fold—a fact which makes it difficult to deal with the telephonic transmission of such music.

Acoustic spectra of various musical instruments have been given by Sivian, Dunn, and White,† and by Meyer.‡

* H. Fletcher, *Speech and Hearing*, p. 97, 1929.

† L. J. Sivian, H. K. Dunn, and S. D. White, *Acous. Soc. Am. J.*, 2, 330, 1931.

‡ E. Meyer, *Zeits. f. tech. Phys.*, 12, 606, 1931; *Phys. Soc., Discussion on Audition*, p. 53, 1931; E. Meyer and G. Buchmann, *Preuss. Akad. Wiss. Berlin, Ber.*, 52, 735, 1931.

CHAPTER XV

NOISE: ITS MEASUREMENT AND SUPPRESSION

Noise and its Effects. Noise is becoming of increasing importance and interest in modern life, largely through the increasing use of mechanical devices for transport, and for operations of all kinds in industrial, business, and even domestic occupations. When complaints are analysed it is found that the offending noises consist largely of loud traffic sounds, loud radio at home or in streets, sleep-disturbing noises, and certain specially loud sources—such as pneumatic drills—for which the number of complaints is limited because the noises are not frequent. It is found, however, that some persons complain of noises which others find innocuous, and this, together with the fact that one person's radio music can be a neighbour's noise, suggest that for ordinary purposes noise should be defined simply as any sound undesired by the hearer. Many of the noises enumerated above, however, are of the irregular non-musical type of sound which may perhaps be called unpitched sound, and which was formerly the only type which was technically regarded as noise by physicists. A wider use of the term noise is now general.

The question of the effect of noise upon human beings is a psychological and medical one, and lies outside the scope of the pure physicist. Some reference may be made, however, to a paper by Laird,* who has summarised experimental data on the effect of sound on living beings. According to his review, there is evidence that sound affects motor functions such as strength of grip and reaction times, and interferes with cerebral functions such as ability to perform psychological intelligence tests and to carry out multiplication sums at speed. Breathing rate and pulse are altered and blood-pressure increased. Stomach contractions are affected.† There is interesting evidence that the effects are

* D. A. Laird, *Acous. Soc. Am. J.*, 1, 256, 1930.

† E. L. Smith and D. A. Laird, *Acous. Soc. Am. J.*, 2, 94, 1930.

essentially biological, not only in that the functions affected are sometimes beyond voluntary control, but also in that they are exhibited alike by animals which have had their cerebral hemispheres removed as well as by those with intact nervous systems. Tests in daily work are said to have revealed improved output by certain workers due to reduction of noise.

The effect of noise upon working efficiency is under investigation by the Industrial Health Research Board. It is complicated by the possible adaptability of the worker. A series of tests carried out by K. G. Pollock and F. C. Bartlett,* in which 80 undergraduates took part, indicated that noise tends to produce slight but recoverable diminution of motor and mental efficiency. Any appreciably greater effect upon mental as opposed to motor performance is probably due to the relative complexity of the former. The investigators comment that noise tends to be disliked particularly by people unaccustomed to it, and that mental work of any difficulty is best done in silence. In another test made under industrial conditions, H. C. Weston and S. Adams * studied the output of weavers, a class of operators who work in the extremely noisy environment associated with looms. The results suggest that the output increases slightly when the weavers are protected from the noise by ear defenders. Even after years of work in the noisy conditions, the worker does not become completely adapted to the noise, but goes through the process of adaptation daily, the adaptation appearing to wear off when fatigue sets in.

Pollock and Bartlett found that loud mechanical noise was more disturbing when it was discontinuous than when it was continuous, but soft gramophone noises were almost as distracting, especially when the subject-matter was interesting. Eminent members of the medical profession have referred to the serious cumulative effects of disturbed sleep which make the question of disturbing noises one of importance. The noises most disturbing to sleep are said to be all kinds of sudden unexpected shocks, such as uncontrolled exhausts, drills, vibrations, whistles, barking dogs, milk cans, and all kinds of strident horns.

Physical Measurements of Noise. In measuring sound with physical apparatus such as a microphone-amplifier system it is possible—

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* *Industrial Health Research Board, Report No.* 65, 1932.

(1) To make an estimate of the average sound pressures measured by a microphone.

(2) To ascertain the wave-form of the noise by taking an oscillograph record of the variations of acoustical pressure.

(3) To analyse the noise and thus determine either (*a*) the intensity and frequency of its components—a procedure of special value in identifying the sources of machine noises by correlating constituents with, say, the frequency of meshing of teeth on gear-wheels—or (*b*) the distribution of energy in various frequency bands.

The above methods of measurement have been dealt with elsewhere, and are generally applicable to noise.

Simulating the Response of the Ear. In view, however, of the different sensitivities of the ear at various frequencies the results of physical measurements cannot be interpreted as loudness to the ear, except perhaps in the case of the analyses. For closer estimates of the aural importance of a noise the amplifier of any of the above types of apparatus may be adjusted so that it has an ear-like frequency-response curve. It is impracticable to arrange this for all loudness levels, but it is comparatively simple by suitable choice of the intervalve coupling units when the approximate loudness level is known. For instance, a small coupling condenser reduces low-frequency response, and a condenser in parallel with the grid leak weakens high-frequency response. The graphs of fig. 83 (p. 238) show equal loudness curves, and it is possible to arrange an amplifier and valve voltmeter to give equal deflections for sounds of different pitch as defined by one of these curves. A convenient curve for moderate sounds is that corresponding to a loudness level of, say, 40 phon above threshold.* For fairly loud sounds the 80-phon curve is useful. It should be noted, however, that the 80-phon curve is practically flat, so that for fairly loud sounds it is fairly satisfactory to employ an equipment with a uniform response curve. It is also satisfactory as a rule if very high or very low tones are not involved. F. Trendelenburg † has arranged an amplifier so that only one valve is used for loud sounds, but additional valves are introduced to deal with quieter sounds. By suitable choice of intervalve coupling units it is possible for the amplifier as used for loud sounds to have a frequency characteristic corresponding with that

* See E. E. Free, *Acous. Soc. Am. J.*, *2*, 18, 1930.

† F. Trendelenburg, *Phys. Soc.*, *Discussion on Audition*, p. 92, 1931.

of the ear at, say, 60 phon above threshold; as used for moderate sounds to correspond with the 40-phon curve; and as used for threshold sounds to correspond with the threshold curve.

Trendelenburg obtained oscillograph records using amplifiers adjusted to have an ear-like frequency response. An interesting example is given in Pl. V, p. 225, relating to the noises of a motor-car engine exhaust when used without and with a silencer. The upper curves (*a*) were recorded by means of a uniformly sensitive amplifier, the lower (*b*) with the aid of an amplifier of ear-like frequency response. In the upper curves low frequencies having the period of the ignition cycle are distinctly evident in the sound of the unsilenced exhaust, and the superposed higher components are relatively weak. As judged by the upper curves fitting the silencer causes but little improvement. Lower components have been practically unaffected. To the ear, however, which is not very sensitive to low notes, the silencer does reduce the exhaust noise, by reducing the higher components, to which the ear is relatively sensitive. This improvement in high frequencies is brought out (lower curves) when the amplifier is weighted to have ear-like characteristics. The unsilenced exhaust is then seen to be rich in high frequencies, and their amplitudes are considerably reduced when the silencer is fitted.

One of the most useful applications of the ear-like response was revealed when oscillograms of 'heart murmurs' were made. There was little correlation between the aural results of ausculation and the oscillograph records until the oscillograph amplifier was adjusted to have appropriately reduced sensitivity to low-pitched sounds.

It should be realised that an instrument adjusted to have an ear-like frequency characteristic at one or two levels has definite limitations, and its readings must be interpreted with discrimination. Particularly it should be noted that a small change in the intensity of a note of very low frequency is equivalent in loudness to a considerably greater change in the intensity of notes of medium pitch. An instrument adjusted for only one or two loudness levels takes but little cognisance of this fact, and is therefore unlikely to give very reliable estimates of overall loudness where low-pitched notes and medium-pitched notes occur together.

Mention should also be made of the fact that the ear differs

from high-grade physical apparatus in certain other respects. The ear has a non-linear characteristic to changes of intensity, and as a consequence subjective harmonic and combination tones are formed and sensed as new sounds. Again when a complex sound strikes the ear certain loud components may mask others, so that components which exist objectively do not exist subjectively. These facts need to be borne in mind in interpreting physical analyses and measurements.

Noise Meters. Various meters for the measurement and analysis of noise have been described. In a convenient portable microphone and amplifier unit in use at the National Physical Laboratory * (Pl. VI, p. 240) for general acoustical work, which will measure the overall acoustical pressures, dials are provided

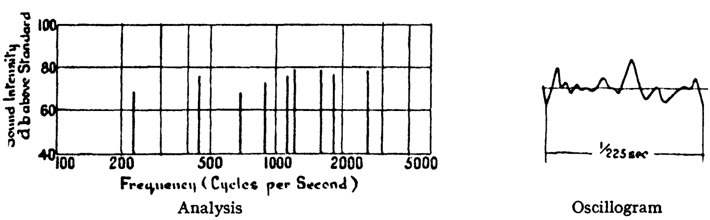


FIG. 90.—Analysis and oscillogram of motor-horn note

for altering the sensitivity of the instrument in decibel steps over the range of 110 decibels. By plugging into the amplifier circuit one or two tunable electrical circuits or filter circuits the instrument will measure the intensity of components, or the energy level in selected frequency bands. Where greater selectivity is desired the apparatus may be used in connection with an auxiliary input stage, for analysis by Grutzmacher's method. An output plug allows a cathode-ray oscillograph to be connected to the output of the amplifier when required.

Fig. 90 is an analysis of the motor-horn note as obtained with the instrument illustrated, in conjunction with a single tuning unit. It is shown in conjunction with an oscillogram obtained by connecting the output of the amplifier to a cathode ray oscillograph. The oscillogram was drawn from an average of several superimposed curves. This horn had components which did not fit in a harmonic series and was rich in powerful high-frequency notes. Apparently these factors are both conducive to stridency.

Simple switches throw into circuit (*a*) a network giving to

* A. H. Davis, *Roy. Aero. Soc.*, *J.*, 35, 675, 1931.

the apparatus a response curve corresponding to that of the human ear at a level of moderate noise (say 40 phon above threshold), and (*b*) a network correcting a resonance due to the microphone used with the equipment. It is also possible to substitute for the microphone an instrument for detecting vibration, constructed on the lines of a gramophone pick-up. When this is held in contact with the parts of a machine or the walls of a building, vibrations may be detected and relative measurements and analyses made. Such procedure assists in elucidating the actual areas responsible for emission of sound.

Aural Measurement of Noise. Since it is not yet possible to deduce accurately the loudness of a complex noise even when its constituents are known, it is often almost compulsory to conduct some part of any study of noise by an aural method. Moreover, very simple apparatus may be used for these measurements, which are themselves often adequate for the purpose in view.

From the curves of fig. 83 it is clear that the loudness of any *pure* note can be expressed as a number (in phon) by stating the intensity in decibels above threshold of an equally loud note of chosen standard frequency. In expressing the loudness of a *complex* sound or noise it would appear desirable to conduct a direct experimental comparison with a note of standard pitch. When the intensity of the equally loud standard note is n decibels above threshold, the noise itself, as stated earlier, will be said to have a loudness of n phon.

Experimental data giving the masking effect of a sound or noise upon a note of standard pitch—that is, the extent to which one sound interferes with the audibility of another—may also be used as a measure of a noise, in that they show the reduction of the capacity of the ear in the presence of the noise, and thus express the interference the noise causes. It is often illuminating to observe the deafening due to the noise for each of a series of notes in the audible range. A spectrum of the deafening effect is thus obtained.

Aural measurements are somewhat speculative when the noise fluctuates rapidly, because of the short time available for obtaining the match between the noise and the audiometer tone.

Electrical Audiometers. Various forms of apparatus are in use. In all forms a standard note is varied in strength until it is either just drowned by the noise, or is judged to be equally

loud. In the Siemens Barkhausen * audiometer a comparison note is produced in a telephone ear-piece by means of suitable buzzer and electrical circuits. The magnitude of the current through the telephone, and thus the intensity of sound produced, may be varied by means of a calibrated potentiometer. With the telephone ear-piece held to one ear the current is adjusted until the sound emitted is judged to be equal in loudness to the sound heard in the other ear. Alternatively, or better still, in addition, the current is adjusted until the telephone note is just inaudible in the presence of the noise. The instrument is supplied so adjusted that zero reading corresponds to the average threshold of audibility for the note concerned.

A useful procedure in adjusting for equality of loudness is to adjust the telephone note first to be clearly too loud, then to be clearly not loud enough, and thus to approach the point at which the observer is equally 'aware' of the noise and of the telephone note as they are sounding together.

The sound output from the telephone ear-piece may be calibrated in absolute units † to avoid the indefiniteness of the human threshold of hearing. For purposes of calibration the telephone receiver is placed tightly over an 'artificial ear-canal' (1 cm. diam. and 3 cm. length) communicating with the diaphragm of a calibrated microphone. Measurements are made in dynes per sq. cm. of the oscillatory acoustical pressure set up in the canal. The instrument may then be calibrated to read in decibels above 1 millidyne per sq. cm., a convenient round figure representing approximately the minimum audible note for a frequency between, say, 600 and 900 cycles per second. With some instruments the calibration remains constant to 1 decibel or so for several years. Tests in quietest conditions may reveal that rather a lower intensity than the zero on the scale can be detected by observers accustomed to such work (some decibels lower), but the round figure stated is a convenient practical zero.‡

In an audiometer due to the Metropolitan-Vickers Electrical Co., a valve oscillator is used instead of an electrical buzzer;

* H. Barkhausen, *Zeits. f. tech. Phys.*, 7, 599, 1926.

† A. H. Davis, *Roy. Aero. Soc. J.*, 35, 675, 1931.

‡ There is at present no agreed pitch or zero for audiometers used in aural measurement of noise; see E. E. Free, *Rev. Sci. Insts.*, 4, 368, 1933. A note of frequency 1000 cycles per second, and of an intensity corresponding to a pressure of about 0·2 or 0·3 millidynes per sq. cm. is, however, becoming usual as a reference standard.

the instrument has the advantage that it is quite silent except for the note produced in the telephone receiver.

A more elaborate instrument, due to the Western Electric Co. of America, employs several valve oscillators instead of a buzzer, and is adjusted to give a choice of eight different notes ranging in pitch from 64 to 8192 cycles per second. In this case the telephone ear-piece is so constructed that the noise under investigation enters the same ear as the audiometer note ; the cap of the ear-piece has the usual central hole to permit sound to pass from

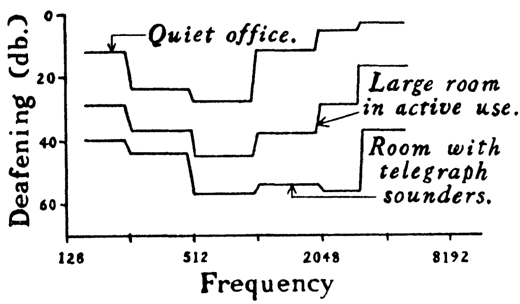


FIG. 91.—Room-noise audiograms (Galt)

the diaphragm to the ear, but in addition circumferential slots are provided to admit external noise with a minimum escape of diaphragm sounds.

Another form of audiometer uses a gramophone record and electrical pick-up as the source of the note in the telephone ear-piece. In this case, instead of employing single-frequency notes, 'warbling' notes are employed in which the frequency varies, say, about an octave some six times a second. A series of six notes on the record will thus cover the greater part of the audio-frequency range. The object of the 'warbling' note is to avoid the beats which may occur if the noise under investigation contains a component nearly equal in pitch to a single-frequency testing-tone. Such occasions are infrequent, but we have seen that unusually great masking occurs when two notes are of approximately the same pitch (p. 246).

Fig. 91 represents some audiograms of room noise obtained by Galt * by means of a phonograph audiometer having a series of notes of the warble type. Galt mentions that variations of from 5–10 db. in the threshold setting in the presence of the noise may occur with different observers or with the same

* R. H. Galt, *Acous. Soc. Am. J.*, I, 147, 1929.

observer at different times. These differences are subjective, and the variations due to the apparatus are smaller than this. The effect of needle variations is ordinarily less than 1 db., but the reproducer may vary by 3 db.

Tuning-fork Audiometer. As a very convenient portable audiometer it is possible to use a tuning-fork.* The fork is struck in a standard manner, and note is made of the time which elapses before the loudness of the fork, when placed as close to the ear as possible with the flat of the spring facing the auditory meatus, falls to the loudness of the observed noise. The total interval which elapses before the fork is masked by the noise is also taken for check purposes. Readings are facilitated in practice if, as the fork approaches the matching value, it is moved to and from the ear so that it is alternately louder and softer than the noise. A particular fork used had a frequency of 640 vibrations per second, the intensity of the note when first struck was about 90 db. above the threshold, and the measured rate of decay was on an average about $1\frac{1}{2}$ db. per second, a simple relation which arose from the fact that the sensation-stimulus law of the ear and the law of decay of the fork are both practically logarithmic. It is important to realise that no very special precautions are necessary, in striking the fork, in order to attain a standard initial intensity correct to a decibel or two. In calibrating the fork the loudness may be compared, over a very wide range, with that of a note of variable intensity in the telephone receiver of a calibrated audiometer. Usually the rate of decay of the fork is greater for large amplitudes than small. Calibrations of the rate of decay may also be effected by holding the fork near a recording microphone-amplifier equipment, for experiment shows that the two methods give similar results. Over a restricted range optical methods of calibration may be adopted.

In measurements, by means of the tuning-fork, of everyday noises of various kinds, an approximately linear average relation was found between 'equality' values and 'masking' results. By extrapolating this relation it was possible to use masking values alone in assessing noises which were louder than the fork, and the scale was thus extended to 110 phon above the threshold. The fork yielded results similar to those obtained by an audiometer of the same pitch.

* A. H. Davis, *Nature*, 125, 48, 1930; *Phys. Soc., Discussion on Audition*, p. 82, 1931. The latter paper gives details of technique.

E. E. Free* and R. H. Galt† have verified that tuning-forks yield results similar to those obtained with other types of audiometer. In America sets of forks of frequency 128, 256, 512, and 1024 cycles per second are now made at the Riverbank Laboratories for noise measurements. They are designed to have a duration of audibility of 60–70 seconds, and are supplied already calibrated, with the rate of decay—as well as the frequency—stamped upon each fork. E. Z. Stowell ‡ has described some work carried out with an elaborated tuning-fork audiometer; simplicity and easy portability are of course lost by elaborations.

Fig. 92 presents a series of noise measurements made in England by the writer, either by means of tuning-fork of frequency 640 cycles per second, or by means of a Barkhausen audiometer of the same pitch. The threshold of audibility was taken to be at a pressure of 1 millidyne per sq. cm. as set up in an artificial ear-canal; actually it was a few decibels lower. Table XII, compiled from various sources, presents results for noise in buildings.

TABLE XII

Some Approximate Noise Levels in Buildings

100		45	Noisy residence
	Boiler factory		
95		40	
			Quiet office
90		35	
			Average residence
85	Various factories	30	
80		25	
75		20	
			Quiet residence
70	Typing room	15	
65	Average factory	10	
60		5	
	Noisy office		
55		0	
50			

* E. E. Free, *Acous. Soc. Am. J.*, **2**, 18, 1930.
† R. H. Galt, *Acous. Soc. Am. J.*, **2**, 30, 1930.
‡ E. Z. Stowell, *Acous. Soc. Am. J.*, **4**, 344, 1933.

Comparison of Methods of Noise Measurement. Some comments upon the differences between the above methods of aural measurement would seem to be desirable. On the whole, 'masking' values require a more definite judgment than 'equality' values; they are not necessarily more reliable. Moreover, they would appear to have the disadvantage that an appreciable level of sound is sometimes necessary before masking occurs. This fact is illustrated in fig. 85, where an appreciable level of sound at frequency 400 cycles per second is seen to be necessary to mask in the same ear a note of frequency, say, a thousand cycles per second. In consequence the masking method is inapplicable to measuring the loudness of comparatively quiet sounds, and the 'equality' values give the best proportional indication of the energy content of the noise concerned.*

There does not seem to be much difference in practice whether the noise and the audiometer note are introduced into the same ear or into different ears. The use of different ears requires either that the two ears of the observer shall be substantially alike in sensitivity, or that the observation shall be repeated after changing the ear to which the audiometer is applied. For normal observers the interchange appears to be usually an unnecessary refinement. On the other hand, the use of different ears for observing equality between the noise and the audiometer note has the advantage that it protects the ear which is listening to the noise from interfering stimulation due to the presence of the audiometer note. A balance between two separately heard sounds can thus be obtained. As regards masking values, however, if it is specially desired to ascertain the extent to which the presence of the noise interferes with the hearing of the audiometer, clearly the sounds should be applied to the same ear.

J. Obata and S. Morita † have studied the accuracy of aural methods from various points of view. They found a rather unexpected high accuracy, and conclude that aural measurements of noise, if conducted with the necessary care, have a sufficient and important value. Accuracy was markedly increased by the observer's skill or familiarity with the characteristics of the sounds, and steady noise levels were more readily determined than those which were changing rapidly through a wide range.

* This is also evident from the fact that the masking curves of fig. 93 are more complicated than the equality curves of fig. 92.

† J. Obata and S. Morita, *Acous. Soc. Am. J.*, 4, 129, 1932.

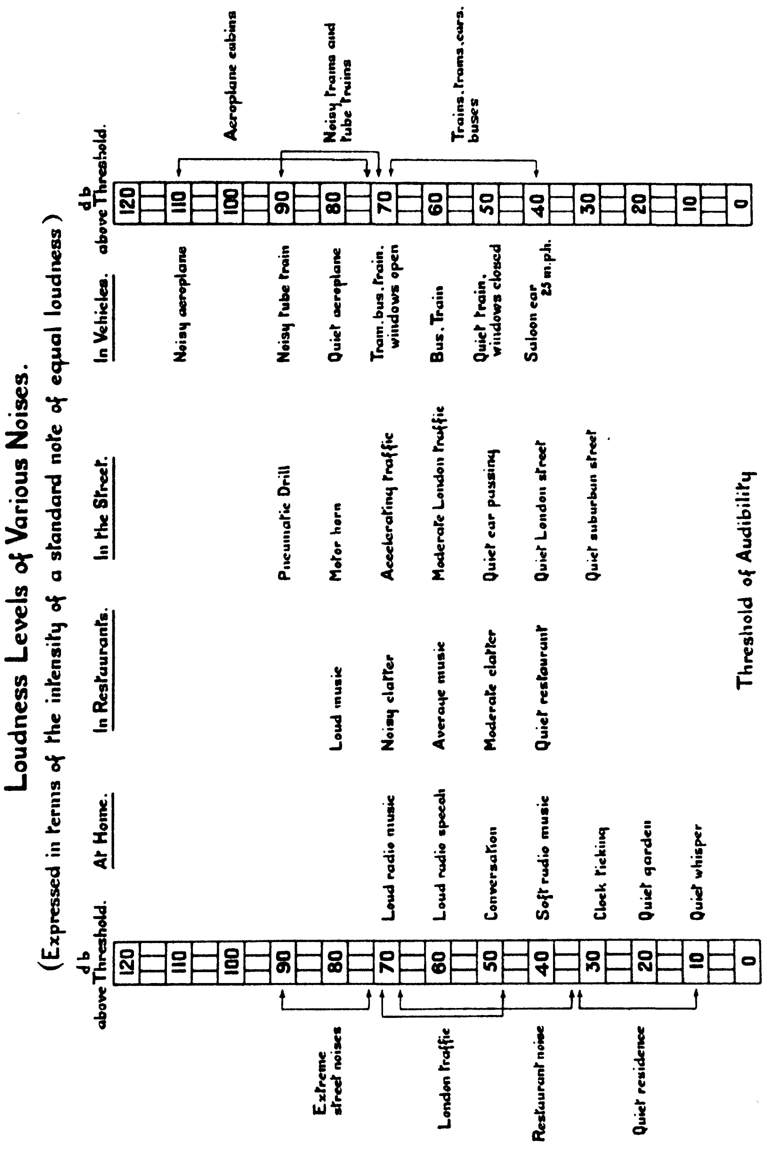


FIG. 92.—London noise (Davis)

Fatigue had no very important effect. Obata and Morita comment also that a physical noise meter in which the frequency characteristic corresponded to the hearing curve, was not—contrary to its apparent trustworthiness—free from error in measuring a composite sound in which the masking effects of various sound components played an important rôle.

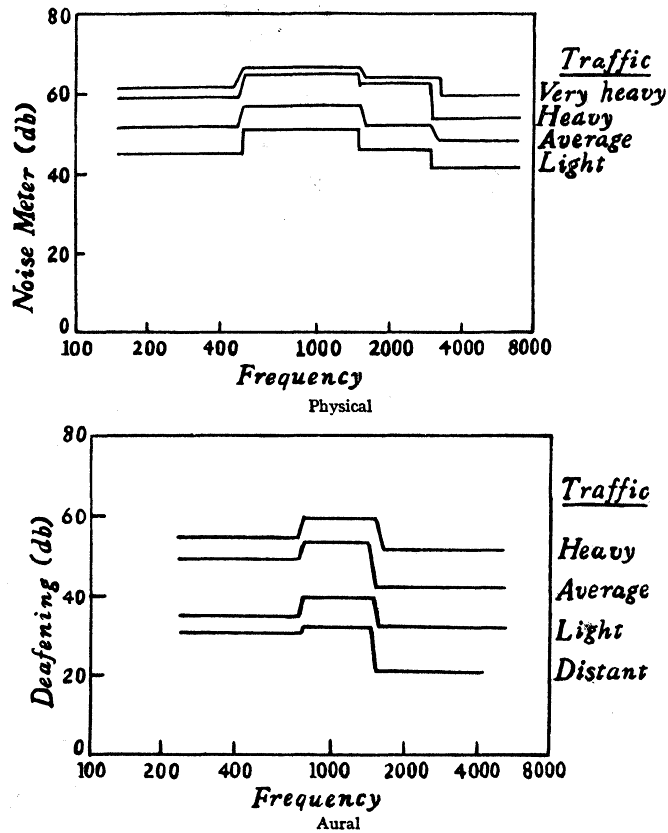
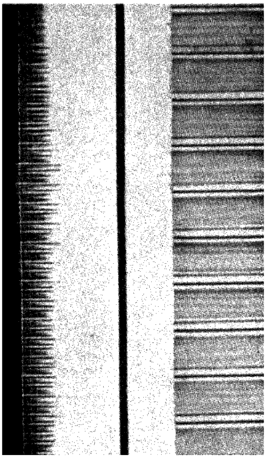


FIG. 93.—Physical and aural measurements of street noise (Galt)

Whilst in general any relationship that may exist between the masking and equality values depends upon the pitch and constitutions of the sounds concerned (*cf.* figs. 84 and 85) in practice, for audiometers of medium pitch and for everyday noises, the equality value is related to the masking value in an approximately linear manner, and, indeed, by a simple difference which is nearly constant. With the notes and noises introduced into the same



Types of photographic sound records
See p. 298
(*a*) Variable area ; (*b*) Variable density

Open-air site for acoustical tests (N.P.L.). See p. 310

ear, a tuning-fork of medium pitch is masked when its intensity is some 16–30 db. below the noise equality level, according to the loudness of the noise observed. In measurements of intense noises with the audiometer note and the noise applied to different ears, the difference is about 20–30 db.*

R. H. Galt † indicated that a microphone amplifier apparatus, fitted with a series of four band-pass filters, and in addition weighted so that in each band the instrument had a sensitivity corresponding to that of the ear at 30 phon above threshold, gave noise measurements comparable with aural values obtained by means of an audiometer (fig. 93). Actually he found that the level of noise as measured by the meter was some 15 db. above the masking value obtained with an audiometer.

E. Meyer ‡ has compared audiograms of various types of noise with physical analyses obtained by Grutzmacher's method of analysis. Fig. 94 (*a*) shows the analysis and audiogram of a warbling tone of 800–1000 cycles per second. The objective spectrum (upper curve) contains only this one frequency range, whilst the subjective audiogram includes neighbouring frequencies because they also are influenced by the masking sound. Fig. 94 (*b*) relates to the noise produced by a wooden hammer striking a wooden board. The results of the two methods are seen to have a general resemblance, the deviations observed being natural.

The Principles of Noise Investigation. In connection with the suppression of noise at its source, it must be realised that if several sources of different loudness exist together, no appreciable improvement can be attained by suppressing any but the loudest.§ When that is reduced the next loudest dominates and must be suppressed in turn. A useful procedure in noise investigation is therefore to ascertain which causes are responsible for the loudest components of the noise under study.

In some cases, particularly where sources of noise can be separated and tested alone, simple audiometer measurements of total loudness can be of value. Such a case is illustrated by the Aeronautical Research Committee's work on noise in passenger cabins of aircraft.* Aural measurements of noise were made near

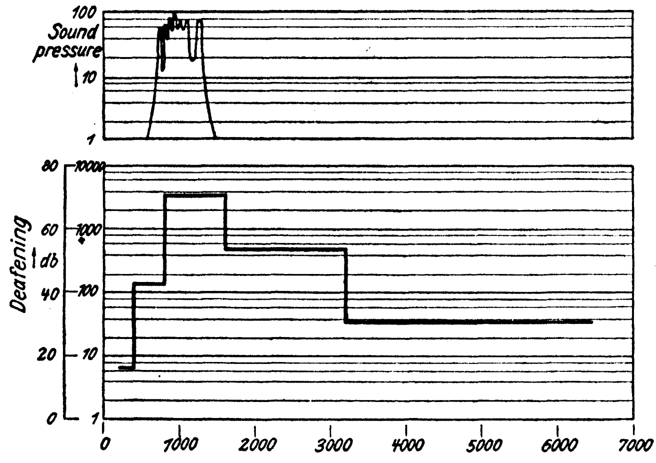
* A. H. Davis, *Roy. Aero. Soc. J.*, 35, 675, 1931; *Aero. Rch. Cttee. Rpts. and Memo.*, No. 1540, 1933.

† R. H. Galt, *Acous. Soc. Am. J.*, 2, 30, 1930.

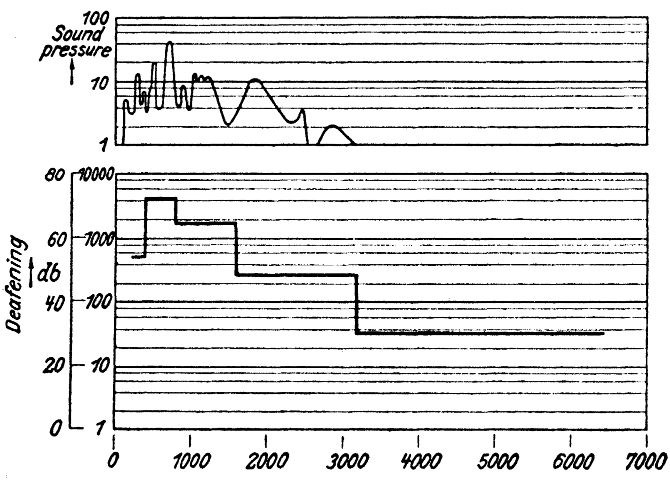
‡ E. Meyer, *Phys. Soc.*, *Discussion on Audition*, 1931.

§ Even removing one of two equal sounds would only lower the energy by twofold, a step of 3 db.

air-screws, exhausts, and engines when they were running in static experiments upon the test benches. It was found that high-speed



(*a*) Warble tone



(*b*) Sound of a wooden hammer striking a wooden board

FIG. 94.—Objective and subjective analyses (Meyer)

air-screws were the most important single source of noise ; that the noise of a driven air-screw of medium tip speed was approximately equal to that of an aircraft engine, say, 100 phon ; that to minimise air-screw noise large air-screws of steep pitch and

slow speed should be employed; and that the effect of a silencer was to reduce exhaust noise by some 10 phon. Experiments on light structures suitable for the walls of aircraft cabins indicated that the transmitted sound could be reduced in loudness by some 20–30 phon, provided filled double walls of some $\frac{3}{4}$ lb. to 1 lb. weight per square foot were employed. Cabin walls of this type would reduce the noise level (90–100 phon) of the power plant outside the cabin to 70–80 phon inside. A further reduction could be attained by locating passenger accommodation away from close proximity to the power plant and screening it from the engines and exhausts, say, by interposing wings. Simple measurements in the cabins of passenger aircraft were generally consistent with the separate experimental findings. Consequently the reduction of cabin noise to practically that (60 phon) in a railway train was regarded as within the bounds of early developments, and considerable strides have already been made.

In other cases, where the possible causes cannot be investigated separately, the elucidation of causes is achieved by analysis. For instance, when machinery noises are studied by this means, components are often found which have frequencies which are recognisable, such as frequencies corresponding to the fundamental frequency of rotation of the motor, to the frequencies of the passage of fan-blades, of the meshing of gear-wheel teeth, of the contact of armature sections with the commutator, or of the opposition of stator and rotor teeth in synchronous motors. Components may also be found which have the same pitch as the note which is set up when parts of the machine are struck with a hammer. Such results clearly indicate the causes of the noise, and show where improvements are desirable. If the analyser is adjusted to weight components in an ear-like manner, the relative importance of the sources as regards loudness are immediately evident.

Since emission of sound is likely to be enhanced by resonances of parts of the machine, such parts should be discovered. A convenient method of search is to explore the surfaces with a vibration-detecting element (p. 265). Resonant parts should be thickened with a view to suppressing the resonance.

Applications of Analysis to the Study of Machine Noise. The following brief survey of sources of noise in machines may assist investigation in particular cases :—

1. *Gears.* Noises may be due to (*a*) the regular meshing of

teeth, (*b*) the passage of occasional defective of imperfectly cut teeth, (*c*) the ringing of gear-wheels when there is no load on the machine.*

Good gearing, correctly mounted and in perfect order, produces a smooth, continuous sound. High-pitched screeching may arise from rough tooth faces. Intermittent or fluctuating noise may be due to irregular tooth spacing, misshapen teeth, (hobbing noise), or eccentricity of gears on spindles. Inadequate strength of the shaft or of its supports is a common defect, and may cause pulsating sound of varying pitch. Accuracy in the machinery and finish of the tooth faces tend to reduce noise, and in general fine-pitched teeth and wide faces are advantageous. Accurate mounting is essential. Noise is often greatly reduced when one gear-wheel of a pair is made of raw hide or other substitute for metal. Noises and vibration due to the regular meshing of metal teeth are largely eliminated when double helical gearing is employed. The teeth are then in continuous engagement, and considerable power can be transferred smoothly from tooth to tooth. Spiral gears, well designed and constructed, are relatively smooth and noiseless in operation.

2. *Electric Motors and Generators*. There are several causes of noise from rotating electrical machinery, viz. (*a*) unbalance, (*b*) the passage of armature sections past the commutator, (*c*) the passage of rotor teeth past stator teeth, (*d*) windage and siren frequencies from fans or even from the rotor, (*e*) ringing of frames and parts, which usually arises from magnetic forces or from resonances at certain speeds.†

Direct current motors may usually be made comparatively quiet. Induction motors are frequently a cause of annoyance, as a consequence of the pulsations of magnetic force. Such pulsations occur as rotor teeth pass the teeth of the stator, and magnetic noise may also arise at twice the frequency of the supply mains. The part of the structure set in vibration by the forces is usually the stator, but may be the rotor, and is accentuated by mechanical resonance.‡ Such resonance should be avoided in design.

3. *Transformers and Induction Regulators*. The noise emitted by transformers and induction regulators is usually due to

* T. Spooner and J. P. Foltz, *A.I.E.E.*, *J.*, 48, 199, 1929.

† B. A. G. Churcher and A. J. King, *I.E.E.*, *J.*, 68, 97, 1930.

‡ Q. Graham, S. Beckwith, and F. H. Milliken, *A.I.E.E.*, *Trans.*, 50, 1056, 1931.

magnetically excited vibration of the core, coil, or stator, transmitted by the cooling oil to the tank wall. It is mainly of twice the frequency of the electric supply.*

4. *Vacuum Cleaners.* Noise is observed due to (*a*) unbalance, (*b*) motor noises, (*c*) fan siren noise, and (*d*) ringing noises from fan blades or other parts.

5. *Machinery Generally.* Noises may arise from (*a*) unbalance, (*b*) reciprocating parts, (*c*) gears, (*d*) defective parts, bearings, supports, or alignment, (*e*) exhausts (p. 193), (*f*) ringing noises.

6. *Vehicles.*—Noise observed in vehicles may be due to (*a*) attached engines, exhausts or machinery, (*b*) wheel and track impacts, friction and vibration, (*c*) turbulent air flow, (*d*) worn, rattling or loose parts, (*e*) resonant wheels, rails, panels or other parts.†

The Mitigation of Noise. The methods of noise suppression are primarily threefold:

(*a*) To suppress at the source the causes of noise.
(*b*) To confine the noise by enclosing the source.
(*c*) To exclude the noise from regions where quiet is desired.

The suppression at the source involves the elucidation of the causes, either by the methods referred to above or, as is often possible, by simple inspection. It may then prove to be possible to eliminate the causes of the noise, or to stiffen or damp any resonant parts which amplify it.

If the cause cannot be suppressed, confinement of noise at the source is clearly preferable to attempting to exclude it separately from the many and varied situations to which it may gain access. The mitigation of noise from electrical convertors and transformers in electrical sub-stations may, for instance, be tackled from the standpoint of confinement.‡ It may be mentioned, however, that the introduction of the mercury-vapour rectifier has provided a comparatively noiseless substitute for convertors.

The question of the confinement of noise is but a different aspect of the problem of its exclusion, and the principles are the same as those of insulation in buildings (p. 285). To know the

* T. Spooner and J. P. Foltz, *A.I.E.E., J.*, 48, 199, 1929; J. P. Foltz and W. Shirk, *A.I.E.E., Trans.*, 50, 1052, 1931; R. B. George, *A.I.E.E., Trans.*, 50, 347, 1931; M. Krondl, *Rev. d'Acous.*, 2, 358, 1933.

† *Engineer*, 146, 414, 1928; *V.D.I.*, 77, 955, 1933; *Engineering*, Mar. 2, 1934.

‡ E. A. Bishop, *A.I.E.E., Trans.*, 50, 1069, 1931.

degree of insulation required it is necessary to have some knowledge of the loudness of the noise to be excluded. Since high-pitched sounds are easier to exclude than low, information as to the pitch of the noise is also desirable. Many relevant data for everyday sounds have been indicated above.* It is desirable to know in addition the general loudness levels which are usually accepted in buildings without comment, and Table XIII, based on data due to Knudsen, gives an estimate of such levels.

TABLE XIII

Tolerable Noise Levels in Buildings

(Based on Data given by Knudsen)

	Phon
Studios for recording sound or broadcasting	6–10
Hospitals	8–12
Music studios	10–15
Apartments, hotels, and homes	10–20
Auditoriums (including theatres, cinemas, churches, classrooms) and libraries	12–25
Private offices	20–30
Public offices, banking rooms, etc.	25–40

Some mitigation of noise may often be achieved, either near the source or in the room where quiet is desired, by rendering the internal ceiling, walls, and other surfaces of the room as absorbent as possible. Experiments which have been made upon the value of absorbent linings for the walls of London's tubes are an example of this principle. Certain London banks and houses have adopted specially absorbent ceilings to reduce the noise in offices—with success. The provision of soft carpets, draperies, and upholstered furniture all tend to improvement. It must be understood, however, that, while the improvement obtained by such means is perceptible, it is limited to a few phon if the enclosure concerned is already moderately absorbent.†

* See also *City Noise*, Noise Abatement Commission, Dept. of Health, New York City, 1930.

† Bibliographies relating to various particular cases of the suppression of noise are given by A. B. Eason, *The Prevention of Vibration and Noise*, 1923; S. E. Slocum, *Noise and Vibration Engineering*, 1931; F. R. Watson, *Acous. Soc. Am. J.*, 3, 14, 1931; Larmabroehr, *V.D.I.*, 1933; M. Krondl, *Rev. d'Acous.*, 2, 358, 1933.

CHAPTER XVI

ACOUSTICS OF BUILDINGS

The Acoustics of Auditoriums.* The chief conditions for good hearing in an auditorium are (*a*) that loudness should be adequate; (*b*) that there should be no perceptible echoes or focusing; (*c*) that there should be no undue reverberation, *i.e.* each speech sound should die away quickly enough to be inappreciable by the time the next is uttered; (*d*) that where best music is concerned the hall should be non-resonant and suitably reverberant for sounds of all musical pitches, in order to preserve the proper relative proportions of the components of a complex sound; and (*e*) that the boundaries be sufficiently sound-proof to exclude extraneous noise.

Loudness. The question of loudness does not often arise in connection with orchestral or choral performances, and auditoriums for such purposes may be larger than those satisfactory for ordinary speech. Sir Christopher Wren, in designing churches, was guided by the principle that voices of moderate strength and distinct pronunciation are usually loud enough to reach hearers at distances not exceeding about 50 ft. in front of a speaker or 30 ft. to the side, provided the hearers have an uninterrupted view of the speaker. This limitation leads, in auditoriums where the hearing of speech is the primary consideration, to a size that will not usually accommodate more than 1000 persons, to the adoption of a raised platform or of raked seating so that a direct view of the speaker may be ensured, and to galleries for accommodating as many persons as possible within range of the voice. Other authorities have given ranges for the voice about twice as great as those adopted by Wren, and Fowke, the designer of the Royal Albert Hall, took 204 ft. and 82 ft. as the corresponding figures. Wren's figures are safe, but the others make more demands upon the speaker, and, particularly where the hall is not carefully designed, are often excessive.

* See also A. H. Davis and G. W. C. Kaye, *Acoustics of Buildings*.

A calculation of the articulation of speech at various distances is of interest. Assuming the sound to spread out equally in all directions above the floor in front of a speaker and not to be propagated appreciably to the rear, and accepting the acoustical output of the speaker to be 24 microwatts, it is found that the intensity falls at 70 ft. to a level such that the percentage articulation in the absence of noise is only 75 per cent. This articulation is reached at 110 ft. if the speaker has a stronger voice and emits 60 microwatts. Incidental noise may easily reduce the articulation to 70 per cent.—a low value (see p. 254).

Reflecting boards near the speaker are sometimes employed to direct sound to regions where it is most required, but since audible sounds may have a wave-length of several feet, small reflectors have only slight value. The wall behind the speaker, splayed walls at the side, and a splayed ceiling above may, however, be usefully employed for the purpose. Loudness is also enhanced by keeping the ceiling of a hall low so that it may act as a reflector to strengthen sounds reaching remote seats. In order that an audience in galleries may benefit from the reflection, the galleries should be under the main ceiling of the hall rather than shut off in a separate alcove. In regions where intelligibility is falling through lack of loudness, a twofold (3 db.) increase in intensity raises the percentage articulation by about 10 per cent. (p. 255), a clear indication of the value of a few reflecting surfaces suitably placed.

Electrical amplifying equipment, in conjunction with large loud-speakers, is now used successfully to increase the loudness of speech. Amplification, however, must not be excessive, for excessive loudness gives undue prominence to low-pitched sounds. It is necessary, therefore, to amplify only to an extent such that remote listeners can hear with comfort, to place the projectors well above the speaker's head, and so to direct them that the sound is not excessive for hearers near the platform. Loudness is then fairly uniform over the floor space, and the majority of the audience has the impression of listening to only one source of sound—the speaker himself.

Echoes. As a matter of general experience, it appears that echoes are not noticeable when the time-interval between the arrival of the direct and reflected sounds is less than about one-fifteenth of a second, or, in other words, when the path difference is less than 75 ft. In fact, it may be considered that reflected

sounds arriving within this interval contribute usefully in raising the level of loudness, whereas those arriving later are undesirable and should be enfeebled as far as possible by absorption or scattering. Echo effects should be avoided in design, or minimised by applying absorbents to and breaking up the continuity of the surfaces which give rise to them.

In general, where ceilings are flat, it is desirable to avoid a greater height than about 40 ft. Ceilings 60 ft. or so in height often lead to echo effects which are perceptible only in the middle of the hall. Actually in such a case the sound reflected from the ceiling lags by more than 1/15 of a second behind the direct sound throughout the whole of the first 40 or 50 ft. of the hall, but it is not troublesome within, say, 20 ft. of the speaker, because near the speaker the direct sound is relatively loud. Greater ceiling height than 40 ft. may be employed in the main body of the hall without introducing echo effects, provided that the ceiling is lower over the speaker's platform and its vicinity and is splayed to direct the reflected sound away from the front of the hall. Frequently it is useful to apply absorbents to back walls, and to the upper parts of walls, and so to suppress sound which otherwise would be reflected upwards and returned to the floor of the auditorium later as an indirect echo. Echoes from high ceilings are largely eliminated when electrical amplifying equipment is used for increasing loudness, since the speech projectors are usually placed at least 25 ft. above floor level.

The general direction of likely echoes and the desirable situations for absorbents may usually be inferred from an inspection of sections of the chamber. For instance, on the assumption that sound obeys the simple laws of reflection of light, Pl. VII (*a*), p. 257, is an analysis of sound reflection within the vertical longitudinal section of a council chamber. Floor reflections are omitted. On the scale adopted, it shows portions of a sound wave after travelling a total distance of 60 ft. from the source ●, which represents the speaker on the floor of the chamber. For convenience, arrows have been added to indicate the tracks of a number of the waves. In interpreting such drawings, however, it must be borne in mind that, while for large surfaces sound tends to obey the ordinary laws of optical reflection, it usually spreads considerably beyond the limits applicable to light owing to its relatively greater wave-length.

Experimental methods of analysis have been developed in which observation is made of the progress of a sound pulse within a model or within model outlines of suitable sections of the building. This was first carried out by W. C. Sabine * in America, using the technique of sound-pulse photography which had been developed previously.

Pl. VII (*b*), p. 257, shows a sound-pulse photograph taken with the apparatus described on p. 4, and it relates to a section of a council chamber closely similar to that analysed geometrically in Pl. VII (*a*), the floor being absent. General correspondence with Pl. VII (*a*) is marked, but the spreading of sound beyond optical limits is clearly brought out. In another method of study, which is simpler in technique, use is made of the approximate similarity between sound waves and water waves.† A model section of a building is laid flat in a tank of shallow water (Pl. I, p. 16). Ripples are produced at a point corresponding to a speaker's position by withdrawing a small plunger from the water; on reflection at the boundary the ripples indicate the direction in which sound would be reflected in the actual building.

Pl. VII (*c*) shows a ripple photograph for the auditorium section already referred to in connection with the geometrical and the sound-pulse analyses. While, on account of the subsidiary wavelets which accompany the main wave, the ripple photograph is not so clear as a sound-pulse photograph, nevertheless the general similarity of result is striking, not only as regards the main ceiling reflection, but as regards the extent and direction of reflection from subsidiary surfaces.

In cases where the shape is such that a study of sections is unlikely to reveal all the essential reflecting characteristics of the boundaries of a building, a three-dimensional model may be made, and the general directions of reflections studied by observing the reflection of light from small mirrors suitably placed on the bounding surfaces. Pl. VII, p. 257, shows a skeleton three-dimensional model employed by the writer at the National Physical Laboratory for a study of this kind. A small light 'gun,' mounted upon bearings graduated in altitude and in azimuth, is placed in the model auditorium at the position of the speaker, and directs light in any required direction. By dividing the hemisphere above the speaker's head into 100 equally distributed regions, it

* W. C. Sabine, *Collected Papers on Acoustics*.

† A. H. Davis, *Phys. Soc., Proc.*, 38, 234, 1926.

is possible to trace out all probable echoes and repeated reflections in the hall.

Reverberation. The difficulty most frequently encountered in auditoriums is excessive reverberation, sounds being reflected to and fro without sufficient weakening. Consequently, the estimation of the reverberant condition of rooms, and the invention and test of absorbent building materials suitable for acoustic corrections, form an important phase of the acoustics of buildings.

It is frequently thought that defective acoustics in a hall can be cured by stretching wires to and fro near the ceiling. It is, however, very problematical whether they have any value at all —certainly they do not reduce reverberation. Neither is the introduction of electrical amplifying equipment a universal cure for excessive reverberation. Used with discrimination—say, in a case of reverberation due to very high ceilings—it has a value which arises from the directive action of the loud-speakers, which, in projecting the sound downwards upon the audience-covered floor, are in fact directing it towards what is usually the most absorbent area in the auditorium. The equipment, however, does nothing to hasten the decay of sound which, already emitted, persists sufficiently to cause confusion. Indeed, in a highly reverberant hall this generally diffused sound affects the microphone and tends to cause the apparatus to 'sing'—the most troublesome difficulty encountered in speech amplification, and which, in fact, is overcome by reducing reverberation, suppressing echoes which reach the microphone, and, if necessary, reducing amplification.

Experience shows that reverberation in an auditorium tends to have a preferred value for good acoustics. If it is excessive, the ear is confused by hearing at any instant a number of sounds which have been emitted successively during the few seconds preceding. On the other hand, if it is insufficient, the loudness is often inadequate and the sounds appear to be unnaturally 'dead.' The formulæ given earlier (p. 149) have shown that the intensity of sound built up in a room is proportional to the reverberation period of that room, and Knudsen calculating from the manner in which the intelligibility of speech depends upon loudness and upon reverberation (p. 256), has found that in the optimum condition a moderate degree of reverberation is present. The larger the room, the greater the desirable degree of reverberation. As a matter of experience, it is generally agreed that for

halls of moderate size up to, say, 40,000 cu. ft. in volume, which are to be used for both speech and music, a standard period of about one second (calculated by the Sabine formula) represents the optimum condition when the audience is present. For a hall of 200,000 cu. ft. the preferred period is apparently about $1\frac{1}{2}$ seconds, and for very large halls of 1,000,000 cu. ft. a period approaching 2 seconds is indicated. Acceptable halls which are used for music alone appear to have reverberation periods some 25 per cent. greater than the values indicated above, and where speech alone is concerned, the periods may with advantage be less. Excessive reverberation is the more serious defect where speech is concerned; insufficient reverberation is unacceptable for music.

From a recording or broadcasting point of view it is necessary to arrange the reverberation in the recorded music to correspond with that which is usual for the type of sound recorded. Normally large orchestras perform in halls where the reverberation is greater than in rooms devoted to chamber music, and so on. Thus the degree of studio reverberation required when recording speech is about $\frac{3}{4}$ second, whereas it is about 1 second for a singer, $1\frac{1}{2}$ seconds for an octet or small orchestra, and 2–$2\frac{1}{2}$ seconds or more for a large orchestra.

The above figures for optimum reverberation relate to a sound of frequency 512 cycles per second, corresponding to a pitch in the middle of the audible range at which most measurements have been made. Naturally, however, reverberation at other frequencies needs some control, but there appears to be no very definite agreement as to the manner in which reverberation should vary with frequency. MacNair * suggests that the loudness of all components should die away at the same rate; Knudsen,† that all frequencies should die away to inaudibility in the same time. In both cases the effect would be to make the reverberation period rather greater at low frequencies than at medium pitch—a result which is, in fact, attained to some extent in normal auditoriums, because an audience is less absorbent to low notes than to higher. As regards speech, however, Knudsen ‡ finds that intelligibility depends but little upon the frequency characteristic of the reverberation.

* W. A. MacNair, *Acous. Soc. Am. J.*, 1, 242, 1930.
† V. O. Knudsen, *Acous. Soc. Am. J.*, 2, 424, 1930.
‡ V. O. Knudsen, *Architectural Acoustics*, p. 387, 1932.

The Control of Reverberation. Formulæ for calculating the period of reverberation of a hall are given earlier. From them * it may be seen that reverberation may be reduced by keeping the volume V of the hall low, and by increasing the absorbing power a_1, etc., of its surfaces.

The table on p. 286 presents some average values for the absorbing power of unit areas of various types of absorbing surface, as determined from large samples and for a frequency of 512 cycles per second. Almost invariably the absorption is less at lower frequencies, but differences at higher frequencies are not so consistent. It will be seen that audience usually contributes the greater part of the absorbing power of an auditorium, and that furniture is helpful. The acoustic plasters and acoustic tiles mentioned have a spongy texture. Fibrous boards and tiles have also been developed for sound absorption, and are often partially perforated or slotted to increase the absorbing power. Various soft materials, such as hair felt, eel-grass, asbestos, or slag-wool may be applied in various ways in panels, behind a screen of canvas or rep if desired. If the covers are perforated with small holes they may be distempered without seriously reducing the absorbing power of the combination.

Excluding Noise from Buildings.† The sound-proofing of enclosures with a view to the isolation of noise is of importance but of some complexity. The complexity arises from the variety of paths by which unwanted sounds may effect their entrance, and one of the first considerations is to recognise clearly the nature of those paths.

If the sound originates in the air outside the room it may gain entrance through inadequately sound-proof partitions, or through open windows or ventilating apertures. If it orginates in the structure of the building or in its foundations it may travel in the structure itself and manifest itself unexpectedly in rooms very remote from the source, particularly if it reaches a point where resonance of a partition, or of an air column, is favourable to the enhancement of the sound emitted.

Isolation from Air-borne Sound. To prevent the entry (or escape) of air-borne sound it is necessary to have walls sufficiently massive and rigid, to avoid openings for pipes and ventilators,

* The Sabine equation is usually satisfactory, except for very absorbent rooms or for very large ones.

† See Davis and Kaye, *Acoustics of Buildings*.

TABLE XIV

Table of Approximate Absorption Coefficients for Frequency 500 Cycles per Second

Material	Absorption Coefficient (a)
Ordinary Wall and Ceiling Surfaces—	
Open window	1·0 per unit area
Brick, marble glass, ordinary plaster, etc.	0·01–0·03 „ „
Varnished wood	0·03–0·08 „ „
Wood panelling on studs .	0·1–0·2 „ „
Porous breeze blocks, unplastered .	0·4 „ „
Fibre-board panelling (ordinary) .	0·2–0·3 „ „
„ „ „ (2–3 cm. thick, perforated)	0·4–0·7 „ „
Curtains, cretonne . . .	0·15 „ „
„ medium weight . .	0·2–0·4 „ „
„ heavy, in folds . .	0·5–1·0 „ „
Floor Coverings—	
Wood floor	0·03–0·08 „ „
Linoleum, rubber carpet . .	0·1 „ „
Carpet	0·15 „ „
„ heavy pile on thick underfelt	0·3–0·5 „ „
Audience as ordinarily seated . .	0·96 „ „
Special Absorbents—	
Acoustical plasters and tiles . .	0·2–0·35 „ „
Felt, about 2·5 cm. thick, with muslin cover, distempered and perforated.	0·75 „ „
Fibre-board tiles, $\frac{7}{8}$ in.–$1\frac{1}{4}$ in. thick, perforated or slotted.	0·5–0·65 „ „
Slag wool, wood wool, loose felts, etc., 1 in. thick.	0·55–0·8 „ „
Individual Objects	**Absorbing Power per Object**
Wood seats for auditoriums, per seat .	0·1–0·2 sq. ft. of complete absorption
Upholstered seats, per seat . .	1–2 „ „
Upholstered chairs, per chair . .	3 „ „
Audience, per person . . .	4·7 „ „

Absorption coefficients depend upon the method of fixing—for instance, upon the distance of separation between the wall and the material. For this reason, materials are normally mounted for test in the manner which will be employed in practice.

and to exercise discretion in the location of doors and windows. In extreme cases doors must be double, or even triple, separate frames being employed. In windows heavy glass and small well-braced panes are best. Double windows are effective when they are in separate insulated frames, the panes being at least 4 in. apart, but not specially so otherwise. Doors and windows must shut completely, for the sound entering through cracks may easily be large relative to that entering by other paths. For instance, a crack $\frac{1}{4}$ in. wide under a wooden door 2 in. thick would admit, say, four times as much sound of medium frequency as the door itself. Clearly ventilation arrangements require special attention, and it must always be realised how they may nullify elaborate schemes of sound proofing. Where isolation is important, ducts for individual rooms should not be branches communicating at once with a common main, but should communicate independently with the exterior or with a distant chamber. It is advantageous to introduce felt or other absorbent into the duct as a lining or as baffles; it is not difficult, with lined ducts, to attain an average reduction of 1 db. per foot-run. Fans should be of slow speed, the tip speed not exceeding 55 ft. per second for medium or large sizes; they should not be in direct metallic connection with the duct, but should have an insulating section of canvas duct interposed. The air speed in ducts should not exceed some 20 ft. per second if the moving air is to be noiseless.*

Sound-resisting Partitions and Floors.† Research on the transmission of sound through partitions from the air of one room to the air of another has shown that for single masonry walls and sheet materials the sound reduction factor depends somewhat upon the pitch of the note. This is illustrated by the results for some simple varied panels which were tested at the National Physical Laboratory ‡ (fig. 95). It is seen that, in general, low-frequency sounds are the most difficult to exclude, but resonances of the panel modify results at particular frequencies.

* V. O. Knudsen, *Heating, Piping, and Air Conditioning*, **3**, 1055, 1931.

† For a full bibliography of work at various laboratories see F. R. Watson, *Acous. Soc. Am. J.*, **2**, 14, 1931. Papers by R. Berger, P. E. Sabine, E. A. Eckhardt, V. L. Chrisler, W. F. Snyder, V. O. Knudsen, H. Kreuger, E. Meyer, F. R. Watson, A. H. Davis, T. S. Littler, and others are mentioned; see also *Building Rch. Bull.*, No. 14, H. Bagenal and P. W. Barnett, H.M. Stationery Office, 1933.

‡ A. H. Davis and T. S. Littler, *Phil. Mag.*, **7**, 1050, 1929.

However, when results are averaged for sounds of different pitch, it is found that the average reduction factor for a single panel is determined mainly by the weight of the panel ; thus results for sailcloth, boards, and a brick wall all lie approximately upon a

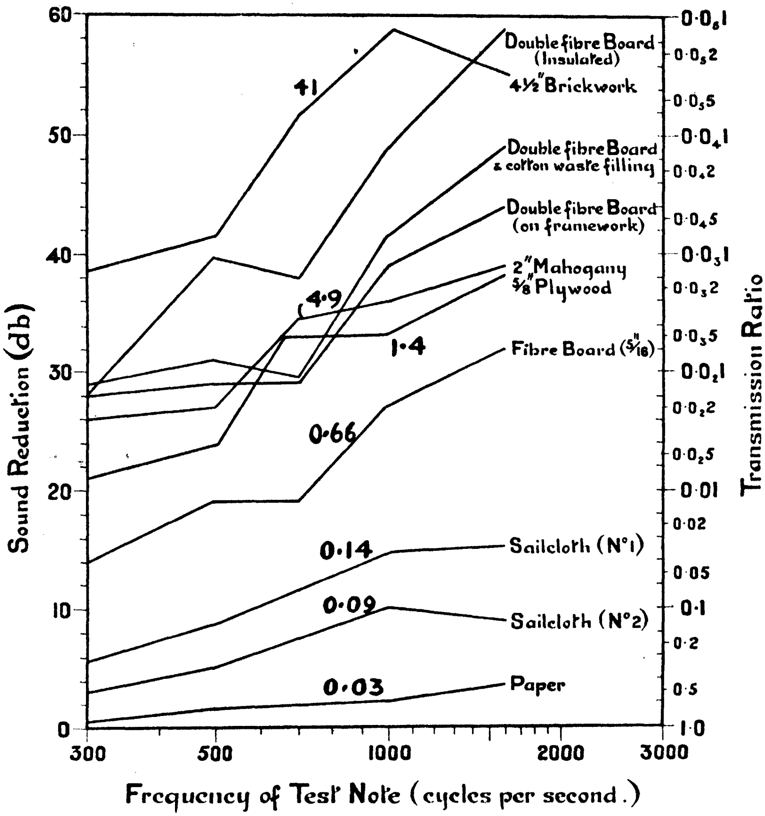


FIG. 95.—Sound transmission through various types of partition

(The weights of the single partitions are indicated, in lb. per sq. ft., by the numbers on the curves)

single weight curve. A large number of simple building constructions tested at the National Physical Laboratory also lie close to the same line. The line (but not actual points on it) is shown in fig. 96, together with comparison curves obtained by other laboratories. Curves are limited to single panels (including plastered panels, hollow tile panels, etc.), and do not apply to double panels of the type referred to later.

In interpreting figures for single partitions we may perhaps

make use of the following tentative grouping, assuming a panel to be used between two fairly bare rooms, in one of which conversation is supposed to be in progress, whilst in the other an observer is listening. The conversation is intelligible to the listener if the average reduction factor of the panel is less than

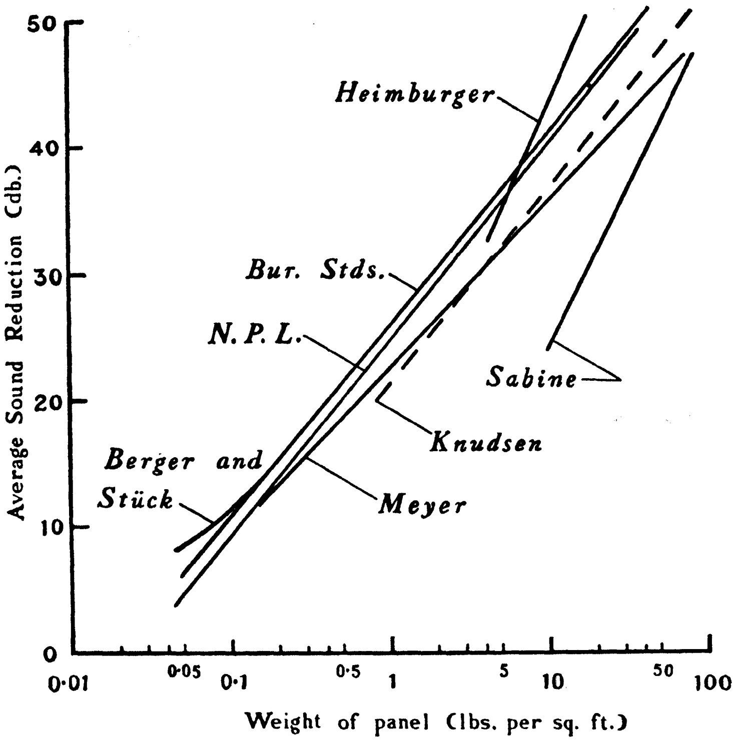


FIG. 96.—Transmission of sound through simple partitions

(Results averaged over a series of frequencies)

40 db., and comprehensible with difficulty if the factor lies between 40 and 50 db. If the reduction is above 50 db. the conversation is unintelligible and becomes inaudible if the reduction exceeds 60 db. If the rooms contained heavily absorbent rugs, draperies, etc., the panels would apparently be more effective than when tested between bare rooms.

Hollow tile masonry, simple ply constructions, lath and plaster wood-stud partitions (whether plastered or covered with building boards), etc., have no special advantage over a simple

homogeneous panel of the same weight. Filling the interspace in wood-stud partitions with soft fillers increases the weight and slightly increases the insulation, but experiments have revealed that filling materials placed in the space between a double wall are of little or no value for walls weighing, say, 10 lb. per sq. ft. or more.

To obtain satisfactory sound insulation by means of a homogeneous wall would entail considerable (and often prohibitive) weight. Double wall constructions—if the two walls are completely isolated from each other *—enable the results to be obtained with much less weight. In practice of course complete isolation is difficult to obtain, but P. E. Sabine † found improvements above simple walls of the same weight for staggered or separated wood or steel-stud partitions, and for plastering over an insulating base. The Bureau of Standards ‡ found plastered wood-stud partitions to be improved when, say, ½-in. fibre board was used as an insulating base (Panels 122) for the plaster or, to a lesser extent, as a base for a finishing board. Such a panel —with plastered faces and of weight 15 lb. per sq. ft.—was found to give sound reduction equivalent to that of a solid panel of six times the weight. Similarly plastered masonry walls give fairly good results when wood furring strips are used to take the plaster base—whether fibre board or paper and metal lath—and finally the plaster finish (Panels 73, 74, 71). As evidence of this a 4-in. hollow block with a plaster coating on both sides, fixed ¾ in. from the masonry by means of wood furring strips on 16-in. centres, gave a higher insulating value (57 db.) than an 8-in. wall weighing almost three times as much.

Where more elaborate insulation is required, as between adjacent rooms in talking-picture studios, separated double walls are desirable. For instance, in a double wall, composed of an 8-in. concrete wall and a completely separated multifaced wood construction, Knudsen found an average reduction factor of about 70 db.

In the case of floors the chief difficulty is not air-borne sound but contact noises due to impacts of footsteps, etc., upon the

* If the faces are not separated by a few inches, the air space between them acts as a film with stiffness calculable from the adiabatic variation of Boyle's Law, and communicates increased pressures to the second face with a corresponding increase of transmitted sound.

† P. E. Sabine, *Acous. Soc. Am. J.*, 1, 181, 1930.

‡ V. L. Chrisler and W. F. Snyder, *Bur. Stds. J. Rch.*, 2, 541, 1929.

floor. A number of ceiling structures have been tested for impact sounds by the Bureau of Standards,* and the provision of the following features is favourable for the prevention of sound transmission through floors.

(*a*) An inner floor floating on a soft insulating material upon the structural floor, and isolated from the walls. (*b*) A ceiling (preferably upon different joists from the floor), and insulated from the joists by, say, slag wool slabs, or by being suspended in an insulating manner which gives a break in the structural contact between the suspending members and the floor itself.

In insulating complete rooms each room should be constructed more or less as a complete inner box, floating upon suitable insulation on the structural floor or joists, and insulated from the structural ceiling.

Where almost complete suppression of entrant sound is desirable, double masonry walls upon separate foundations with an air space between them should be adopted (fig. 54, p. 159).

Isolation from Structure-borne Noise. To isolate a room from structure-borne sounds, such as those arising from motors, elevators, and attached machinery generally, it is necessary to break up the continuity of the solid conducting path, say, by interposing layers of felt-like material in the path of sound. It is essential, however, that no rigid members should pass from structure to structure through the layer, or the sound will pass along the bridge so formed and render the insulation valueless. Bolts can be insulated by bushing, and by using insulating washers under metal ones.

Hot-water pipes, ventilation ducts, and similar continuous metal systems may carry sound from pumps or fans to considerable distances; † they should be insulated from the structure in some way, and should not pass through important partition walls. In some cases, as in ventilation ducts, the continuity can be broken by the insertion of a flexible section. Lift shafts should be insulated from the main structure and should be surrounded with corridors or stairs, not with rooms.

Acoustical laboratories often present in an extreme degree the necessity for exclusion of extraneous noise. They thus serve to illustrate the principle of the exclusion of structure-borne vibration and air-borne sound. In the acoustical building at the

* V. L. Chrisler and W. F. Snyder, *Bur. Stds. J. Rch.*, **2**, 541, 1929.
† See K. W. Wagner, *V.D.I.*, **77**, 1, 1933.

National Physical Laboratory each room has been built upon a 14-in. ferro-concrete floor, the floor being supported at its corners, where it rests upon masonry piers which have a layer of cork incorporated in them. In this way earth-borne vibration is minimised (pp. 29, 159). Over the whole chamber an outer 9-in. masonry shell has been built, on separate foundations, and completely isolated at all points from the inner enclosure. In one case the inner and outer shells have each been provided with a steel door of such thickness ($2\frac{1}{2}$ to $3\frac{1}{2}$ inches) that the superficial weight corresponds to that of the wall in which the door is placed. In other cases three solid wooden doors (3 in.) in separate frames have been employed. A large aperture for light and ventilation has been provided in each ceiling, together with heavy steel shutters for covering the apertures and excluding extraneous sound when necessary.

Elastic Supports for Isolating Machinery. Motors and machinery may be insulated from the floor by substantial layers of materials such as cork, felt, or rubber. Alternatively, machinery may be mounted upon spring supports which can be obtained in compact pedestal forms for loads in the range 40–10,000 lb. per spring. With springs of the girder type almost any load can be carried. Springs are said to be suitable for mounting under all types of machines except those which, on account of their high acceleration forces, require anchorage to heavy foundations. They are useful for machines which work with heavy and sudden impacts such as lifts, linotypes, and mixing machines, for machines of high rotative speed; for machines of excessive humming noises as transformer and circular saws, and for smaller sizes of reciprocating engines, compressors, printing-presses, etc. Machines which require a heavy base, or which cause considerable horizontal vibrations, may be bolted to a massive block of concrete floated upon a suitable insulating layer of leather, asbestos, felt, or cork. Gravel and sand have proved to be moderately good insulators in these circumstances, and there are special combinations of cork and felt, etc., obtainable commercially for the purpose. Layers of insulation are normally adequate for the isolation of machinery situated in basements.

The essential principle of isolation is to mount machines upon springs of such stiffness that the natural period of oscillation of the machine upon the springs is definitely lower than the rate of revolution of the machine when it is running, and indeed lower

than the frequency of any disturbing forces which then arise in the machine (p. 25).* Two further points in connection with the isolation of machinery vibration require notice. The first is that if metal springs are used as supports they need to be bedded upon felt, cork, rubber or other insulating pads, in order to prevent transmission of sound in the manner in which stretched wires or rods transmit sound. The second is that machines are sometimes capable of types of vibration other than simple movement in a vertical direction—for instance, torsional oscillation about an axis is liable to occur with some machines. It is then necessary to apply elastic restraints to these other modes of vibration, using the principle that the frequency of the machine under its elastic restraints should be made much lower than the frequency of the disturbing forces.

* C. R. Soderburg, *Elec. J.*, 21, 161, 1924.

CHAPTER XVII

RECORDING AND REPRODUCTION OF SOUND*

Gramophone Recording. The earlier method of gramophone recording was essentially to record directly upon wax or other material the vibrations of a diaphragm moving under the action of incident sound. A horn, at the base of which the diaphragm was mounted, was used to collect the sound over a fairly large area. The diaphragm was attached to one end of a short lever, the other end of which carried a cutting stylus which inscribed on a rotating soft wax cylinder or disc a record of the diaphragm vibrations. A re-traverse of the 'record' by the stylus resulted in a reproduction of the original sounds. In practice of course the actual wax record was not usually played; but electrotype moulds were prepared, and copies were made in thermoplastic material for playing purposes. This acoustic recording had various disadvantages. In the first place, resonances of the horn and diaphragm distorted the record from a true representation of the pressure changes in the sound wave. The energy available for cutting was limited to that supplied by the sound itself, and in consequence performers needed to be very close to the horn—practically an impossibility with even a moderate orchestra.

The introduction of electrical recording has resulted in immense improvements. For receiving the sound the old horn and diaphragm have been replaced by a high-quality microphone,

* The following references give a review of the subject of sound recording:—

(1) Papers by J. C. Steinberg, E. C. Wente, H. A. Frederick, D. MacKenzie, H. M. Stoller, E. O. Scriven, and H. B. Santee, *Soc. Motion Picture Engineers, Trans.*, 12, 633–643 and 657–741, 1928; and *Bell Sys. Tech. J.*, 8, 159–208, 1929. (2) *Recording for Sound Motion Pictures*, a collection of papers prepared for the Academy of Motion Picture Arts and Sciences: M'Graw-Hill Book Co. Inc., 1931. (3) *Physics in Sound Recording*, A. Whitaker. Report of a lecture given before the Institute of Physics, November 1931. (4) "Recording and Reproduction of Sound," A. G. D. West, *Roy. Soc. Arts, J.*, 79, 992, etc., 1931.

with a consequent avoidance of complications due to horn resonances. By means of amplifiers the electrical output of the microphone, similar in wave-form to the incident sounds, is amplified to a degree necessary for operating a stylus, and it is no longer necessary to crowd performers near the microphone. Any reasonable degree of distortion introduced by the microphone, amplifier, and associated equipment can be corrected by introducing an electrical 'attenuation equaliser' into the circuit.

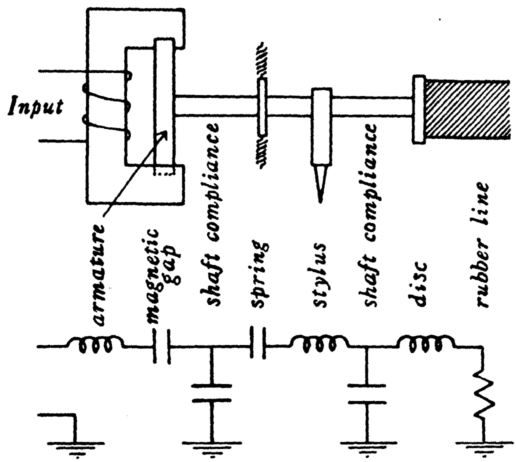


FIG. 97.—Diagram illustrating Maxfield and Harrison's recorder, with equivalent electrical circuit

Further, the electric current from the microphone can be telephoned to a distance—to headquarters if necessary—so that recording of performances in concert halls can be arranged.

The actual mechanism operating the recording stylus is usually of electromagnetic type. In design the mechanical system has been regarded as analogous to an electrical band-pass filter circuit, which may be arranged to transmit a certain range of frequencies without loss, whilst disregarding all other frequencies.* The adoption of this principle revolutionised gramophone recording. Fig. 97 gives a diagrammatic representation of Maxfield and Harrison's recorder, together with its equivalent electrical circuit. Essentially the recorder consists of a soft iron armature pivoted

* J. P. Maxfield and H. C. Harrison, *Bell Sys. Tech. J.*, 5, 493, 1926. See also A. Whitaker, *Physics in Sound Recording*. Report of lecture read before the Institute of Physics, November 1931.

between the pole pieces of an electromagnet. Currents through the armature coils cause it to move to and fro. To the armature is attached a shaft which carries the cutting stylus. The provision of a pure mechanical resistance for terminating the mechanical filter—an essential for proper action—proved troublesome, because most mechanical resistances have values which are functions of frequency or amplitude, and they are often associated with a mass or stiffness reactance. The solution adopted in designing the recorder was to introduce the damping effect of rubber; the end of the shaft is therefore firmly held in a rod

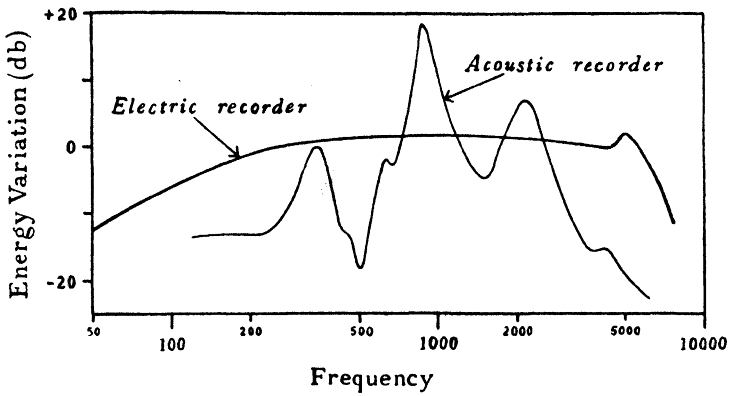


FIG. 98.—Response curve of Maxfield and Harrison's electric recorder (Whitaker)

of gum rubber, 25 cm. long, which provides the non-reflecting terminating resistance. The load imposed by the 'wax' is variable but is relatively small. Torsional vibrations are transmitted along the rubber at about 3000 cm. per sec., but are substantially dissipated by the time they have travelled to the end and back. The rod acts as a pure mechanical resistance of 2500 mechanical ab. ohms, referred to the stylus point as its point of application. The response curve of the recorder is shown in fig. 98 in comparison with one for an old mechanical recorder. The relative levels are quite arbitrary. The superiority of the electrical recorder is evident.

Below 300 cycles per second and above 5000 cycles per second the response of the electrical recorder falls off. Provided compensation can be arranged in the reproducer there is some advantage in the falling off below 300 cycles, for low-frequency notes have considerable amplitude, and, in lateral cut records, full

reproduction of bass notes would cause adjacent record grooves to run into each other unless a prohibitive distance were allowed between them. An electrical gramophone reproducer makes it possible to introduce electrical circuits to correct for the falling off in bass notes due to the recorder. In practice such a correction is often unnecessary owing to the characteristics of the pick-up used.

Gramophone Reproduction. Reference has already been made to the acoustical gramophone reproducer (p. 22). Whilst acoustic reproducers have an adequate output for many purposes, the output is nevertheless limited. For increasing the output it would appear that needles much stiffer than steel are necessary and record materials adequate to withstand them. Again, with an acoustic reproducer the volume of sound cannot be varied without interfering with the frequency characteristic, and even needles of reduced stiffness are not strictly permissible. For these and other reasons electrical reproducers with controllable amplification have a distinct advantage.

Certain problems in connection with the gramophone needle have not yet been overcome. The needle is assumed to follow the axis of the groove accurately, but it does not always do so. If—as is almost inevitable—the needle possesses any resonance in the plane at right angles to its normal mode of motion, then, owing to the non-vertical side of the walls of the groove, the needle is liable to ride up the groove walls at this frequency and give rise to distortion and 'buzz.' It is found that the suppression of this defect calls for small record amplitudes and other conditions which together appear to restrict the output and to conflict with other requirements of acoustic reproducers. In electrical pick-ups the conditions for suppression can be satisfied more easily.

In practice it is found that calculated mechanical vibratory systems do not behave exactly as their electrical equivalents, probably because stiffness and mass are not in practice entirely separated or 'lumped' as in electrical theory. Consequently arbitrary adjustments are necessary. Convenient methods of measuring mechanical impedance would be of great assistance in this connection, and in elucidating the dynamics of the needle-groove system.

Optical Methods of Recording Sound. Great strides have been made recently in connection with 'talking pictures' and the reproduction of sound by optical means.

Sounds are often recorded photographically for this purpose by means of changes of intensity of a beam of light, which affects the density of blackening of a continuously moving photographic film, in a manner which corresponds exactly to the pressure variations of the sound wave being recorded. In some cases, however, they are recorded upon a film as a trace of constant intensity but variable width. Pl. VIII, p. 272, taken from a paper by E. C. Wente,* illustrates the two types.

Generally speaking, the recording process involves a microphone-amplifier apparatus for converting the incident sound waves into their electrical equivalent without distortion, arrangements for controlling the width or intensity of a beam of light in a manner corresponding exactly with the electrical wave-form obtained, and a uniformly moving film for receiving the fluctuating image of the light.

In the variable area system the sound record is produced by a narrow beam of light of constant intensity which, reflected from the mirror of an oscillograph of the Duddell type, moves to and fro across a slit of fixed dimensions. In Germany some success has been obtained with a method which employs a cathode-ray oscillograph. The resulting sound track has the appearance of a serrated edge of uniform intensity adjacent to a transparent area.

In variable density recording various methods have been employed to cause the intensity of a beam of light to fluctuate in conformity with the wave-form of electrical speech currents. In one the currents alter the brightness of a special type of lamp, and the light from the lamp passes through a fixed slit practically in contact with the moving film. The variations in brightness cause corresponding variations in the density of the film. The illuminating source developed by T. W. Case for this purpose is a gaseous discharge tube filled with an inert gas, such as helium, at very low pressure.

In a more widely used form of variable density recording a lamp of constant brilliance is employed, but the light passes to the film through a slit of which the width varies in accordance with the speech currents. One form of variable slit, or 'light valve,' due to Wente, is essentially an electromagnetic shutter on the principle of the string galvanometer, and consists of a narrow stretched loop of duralumin tape formed into a slit in a

* E. C. Wente, *Soc. Motion Picture Engineers, Trans.*, 12, 657, 1928.

plane at right angles to a magnetic field.* Sound currents from the microphone and amplifier flow in the loop and cause it to open and close in accordance with the current variations. Viewed against the light the slit is normally 2 mils wide (1 mil = 0·001 in.). When one-half of the sound wave opens the valve to 4 mils and the other half closes it completely, full modulation of the aperture is accomplished. The natural frequency of the light valve is about 7000 cycles per second. In use, when the valve is interposed between a light source and a photographic film, the source (ribbon filament) is focused on the plane of the valve. The light passed by the valve is then focused on the photographic film. The normal slit focuses on this as a line 1 mil wide, its length being at right angles to the direction of motion of the film. The width of this line fluctuates with the sound currents supplied to the valve, so that the film receives a varying exposure.

Other methods of variable density recording which give good results utilise optical polarisation effects to control the intensity of the light transmitted. In one the Kerr effect is used, and a cell of suitable medium (usually liquid) is placed in the beam of light between crossed Nicol prisms. No light passes through the system until a potential is applied to electrodes in the cell. In another method the similar effect due to a magnetic field—the Faraday effect—is employed. Both systems have practical disadvantages.

Sound Film Reproducers. For reproducing speech or music from a sound film the record, which consists of a track running along one side of the picture, is converted by the following means into corresponding electrical current for actuating a loud-speaker system. Light from a bright filament lamp is focused as a narrow 1 mil line upon the film, on the other side of which a light sensitive cell is situated. Thus at any instant the cell is illuminated to a degree depending upon the photographic density or width of the sound film record at the illuminated region. Selenium cells may be used as the light sensitive element, but they exhibit at high frequencies an attenuation which requires extreme correction in the electrical amplifiers. Photoelectric cells are free from this disadvantage. The essential property of a photoelectric cell is that, when it is polarised by a suitable voltage, the current through it is proportional to the incident light. The

* See D. MacKenzie, *Bell Sys. Tech. J.*, 8, 159, 1929.

appropriate circuit, illustrated in fig. 99,* is arranged so that an e.m.f., proportional to the current in the cell, is applied to an amplifier valve. Owing to the fact that the cell is of very high impedance, it is necessary for this amplifier to be very near the cell. It is therefore incorporated with the cell in a heavy metal box or shield, the whole being mounted on a shock-proof mounting.

When sounds are *recorded* by film processes, the film, travelling at 90 ft. per minute, is maintained at uniform speed—to, say,

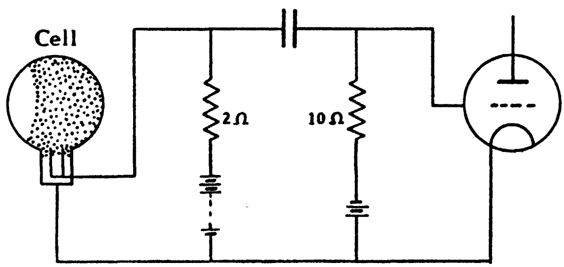


FIG. 99.—Photoelectric cell circuit

1/10 per cent.—by flywheel arrangements in the driving mechanism. As is well known, when cinema film is exhibited in an ordinary projector the film moves into the field of projection in a series of jerks, and the result seen on the screen is a series of momentary photographs viewed at a speed of about 16 per second. When sound films are *projected*, therefore, the sound 'gate' in the projector is located, some 15 in. below the picture 'gate,' in a region where the film is just leaving the steadily rotating film-drum and where the motion of the film is continuous. This necessitates some displacement on the film between the picture record and the corresponding sound record.

Electromagnetic Method of Recording and Reproducing Sound. In an electromagnetic method of recording,† which has the convenience that the record can be reproduced practically as fast as it is produced, acoustic signals are recorded upon a thin steel wire or ribbon. The wire is passed between the poles of an electromagnet, the magnetic field of which is varying in accordance with the fluctuations of speech currents which are passing through its coils. The magnetic field results in local transverse magnetisation

* E. O. Scriven, *Bell Sys. Tech. J.*, 8, 159, 1929.
† C. Stille, *E.T.Z.*, 51, 449, 1930.

of the wire, and at any point the degree of residual magnetisation depends upon the strength of the magnetic field at the moment the point passes between the poles of the magnet. Thus, after the recording operation, the transverse magnetisation of the wire fluctuates along the length of the wire in accordance with the variations which occurred in the speech currents during the recording operation.

To reproduce the sound the wire is passed between the poles of an electromagnetic reproducer. The flux through the reproducer varies in accordance with the fluctuations in the magnetisation of the moving-wire record, and this sets up corresponding speech currents in the coils of the reproducer. These currents are amplified and reproduced. The magnetic reproducer may be situated within a few centimetres of the recording magnet, so that the wire, as the record is made, passes immediately through the reproducer. The record may thus be inspected as fast as it is made. It suffers no deterioration. If a mistake is made in the record the wire need not be destroyed: a saturating magnetic field obliterates the trace and leaves the strip ready for use again. Great progress has been made in developing an extremely homogeneous steel tape of high magnetic coercivity, and the quality of reproduction is almost as good as that from gramophone discs.

Limitations in the Reproduction of Sound. The data due to Fletcher and Snow (p. 258) indicate that in music a frequency range from about 40–15,000 cycles per second is covered, and that loss within this range is detectable by ear. The range for a gramophone reproduction is about 50–5000 cycles per second, a limitation imposed partly by the recorder. In the reproducer the upper-frequency range is limited by the flats which become ground on the needle as it moves in the groove, for these become comparable with the wave-length in the groove at about 5000 cycles per second. Consequently, if the mechanical system of the recorder is to be improved, very fine, hard needles will be necessary in the reproducer. There is promise that both lateral cut and hill-and-dale * systems may be made capable of dealing with an extended frequency range. Film reproduction begins to fail at about 7000 cycles per second, because of the finite width (1 mil) of the illuminating slit—a width which would cover

* H. A. Frederick, *Soc. Motion Picture Eng., J.*, 18, 141, 1932 (*Bell Sys. Monograph*, B 651).

nearly half a wave-length at 7000 cycles per second if the film speed was 90 ft. per second.

Another limitation in the reproduction of sound concerns the intensity range which can be reproduced. The output of a large orchestra, in changing from pianissimo to fortissimo playing, may vary from 4 microwatts to 70 watts (p. 259), a range of 73 decibels. At present no recording system will deal with more than about half this range. An upper limit to the maximum amplitude is imposed in the case of gramophone recording by the separa-

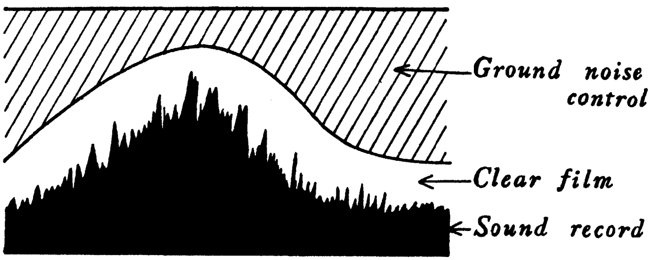


FIG. 100.—Diagram illustrating method of suppressing ground noise in variable area film recording

tion of adjacent grooves on the record, in the case of film records by the width of the sound track or the limits of strict proportionality between exposure and photographic density in the emulsion, and in the case of magnetised wire records by failure of proportionality in the magnetic effects. A lower limit to the amplitude that can usefully be recorded is imposed by irregularities in the recording or reproducing medium—defects which reveal themselves through record 'scratch' * with disc records, and through 'ground noise' in the case of film and wire records. Great reductions in gramophone surface noise or 'scratch' have been made in recent years, as a result of an investigation of the extent to which the noise is introduced in the original wax record, in the electrotype copy, and in the final record material. Ground noise in films has also been reduced. In this case, the noise increases with the amount of light which passes through the film. Without some form of control in variable area recording this amount would be considerable for soft sounds, where normally only a narrow region of the film is blackened by the sound trace. Consequently a shutter is arranged which

* G. Buchmann and E. Meyer (*E.N.T.*, 8, 218, 1931) have analysed needle 'scratch.'

limits the area of unexposed film when the sound amplitudes are small; ground noise is thereby reduced (fig. 100). The control is automatic, being due to a circuit which operates to an extent determined by the intensity of the sound, and with a lag of only about one-hundredth of a second. Any sudden increase of loudness causes distortion for a moment whilst the shutter is operating. To control ground noise in the case of variable intensity recording, soft passages are automatically recorded at a greater mean density of film than is permissible with loud ones. The mean density can be controlled by automatically altering the average width of the slit of the light valve as the loudness varies in the passage recorded.

CHAPTER XVIII

APPLICATIONS OF ACOUSTICAL MEASUREMENTS

In previous chapters various types of fundamental acoustical measurement have been described and certain important special cases dealt with. The following pages deal with the application of the methods to certain particular cases which are of interest in themselves, and which serve to illustrate further the principles involved in acoustical measurements.

Telephone Transmitters.* Frequency characteristics of telephone transmitters are obtained by following the general procedure adopted in calibrating microphones; the transmitters are calibrated in a lagged enclosure by substitution for a calibrated condenser microphone, or by exposing them to the sound from a calibrated source. The records obtained indicate the order of performance only, because carbon transmitters are very inconstant owing to a tendency for the carbon granules to pack closely under the action of diaphragm movement. A record taken with a rising frequency usually differs from one taken with a falling frequency, and resonances differ with time.

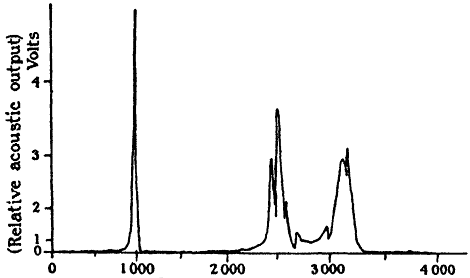
Generally speaking, response curves of commercial transmitters show pronounced resonances in the region 1000–2000 cycles per second, often in the form of two irregular adjacent peaks. The lower is due to the diaphragm resonance, the higher to acoustic resonance of the air in front of the diaphragm and in the mouthpiece. Recent transmitters have considerably lighter moving parts than those usual a few years ago. As they are consequently less sharply resonant, the resonance region is wider and extends from, say, 900 to over 3000 cycles per second.

Telephone Receivers.† As shown by MacGregor Morris and

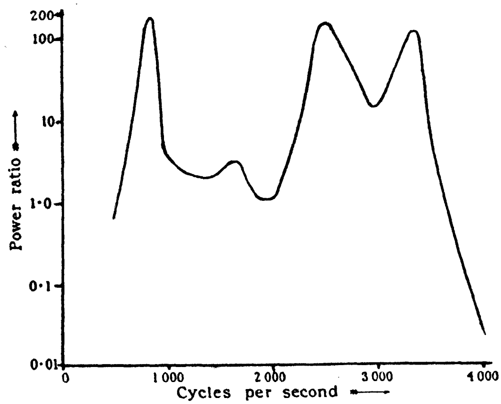
* Cohen, Aldridge, and West, *I.E.E., J.*, 64, 1023, 1926; Aldridge, Barnes, and Foulger, *P.O.E.E., J.*, 22, 185, 1929; B. S. Cohen, *I.E.E., J.*, 66, 165, 1928; H. R. Harbottle, *I.E.E., J.*, 71, 605, 1932.

† See Cohen, Aldridge, and West, *loc. cit.*; also Cohen, *loc. cit.*, and Harbottle, *loc. cit.*

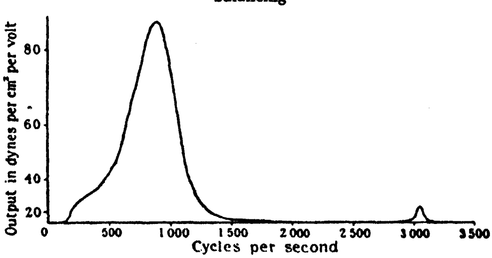
Mallett,* the nature of the resonances of a telephone diaphragm may be revealed by an experiment in which fine sand is sprinkled over the diaphragm when it is held in a horizontal position. The ear-cap of the receiver is removed for the purpose, only the clamping ring being retained. On exciting the diaphragm by electric current of various frequencies sand figures are produced.†



(a) Characteristic of Bell receiver with ear-cap normal



(b) Frequency-power characteristic of Bell receiver by aural balancing



(c) Characteristic of Bell receiver held to an artificial ear

FIG. 101.—Characteristics of various Bell telephone receivers

In connection with the testing of ordinary telephone receivers, it should be noted that under working conditions a receiver is held to the ear, and that its resonant frequency is generally lowered by some 200 cycles per second when it is removed from the ear and exposed freely to the air. An average change for Bell receivers is to lower the resonance from about 1150 cycles per second to about 950 cycles per second. It follows that change in the adjustment of the receiver to the ear alters its

* J. T. MacGregor Morris and E. Mallett, *I.E.E.*, *J.*, 61, 1134, 1923; 62, 278, 1924.

† Sand figures have been discussed recently by E. N. da C. Andrade and D. H. Smith, *Phys. Soc.*, *Proc.*, 43, 405, 1931.

characteristics, and some standardisation is necessary in actual tests. Fig. 101 (*a*) * shows the frequency-response curve of a Bell receiver placed 17 cm. from a measuring microphone. Judging from sand figures obtained with the ear-cap removed except for the clamping ring, it would appear that the resonance about 2500 is probably largely due to resonances of the air-chamber in front of the diaphragm. The fundamental diaphragm resonance occurs at about 1000 cycles per second and the nodal circle at about 3000 cycles per second. The single-line node and double-line node vibrations (which would occur at about 1700 and 2500 cycles per second respectively) are naturally small owing to the central position of the pole-pieces.

Actual estimates of the loudness of the output of the receiver at different frequencies may be made by alternately applying to the receiver a voltage of standard frequency and another voltage of the particular frequency under study, the magnitude of the latter being varied until the two sounds, although of different pitch, appear to have the same loudness. Average results for 60-ohm Bell receivers, converted to ratio of power dissipated in the receiver, are shown in fig. 101 (*b*).

To obtain closer information concerning the response of the receiver when it is held to the ear, experiments have been conducted in which the pressure set up by the diaphragm is transmitted *via* an artificial ear-canal (p. 248) to a calibrated condenser microphone. Fig. 101 (*c*) was obtained in this way by Cohen, Aldridge, and West * for a certain receiver (not the same as those referred to above) which, in free air, had resonances at 750 and 3300 cycles per second. It may be stated that the artificial ear alters the frequencies of resonance to an extent similar to a real ear.

Since telephone receiver coils may be wound to different electrical impedances, a simple statement of the acoustical output in terms of either the applied voltage or applied current gives only an incomplete idea of sensitivity. It is necessary to know also the electrical impedance; then the sensitivity may be fairly inferred from the ratio of the acoustical pressure set up in the ear to the square root of the electrical watts supplied.

Articulation tests (p. 252) are freely used in the tests of telephone systems. In order to obtain satisfactory results—free from the personal element which arises from the varying degree of quality and of loudness of enunciation by different individuals—it appears

* Cohen, Aldridge, and West, *loc. cit.*

to be necessary to use trained testing crews. These crews should employ standard methods of testing, and should preferably be calibrated on a standard or 'reference' transmission circuit. The use of trained crews increases the consistency of results, and thus reduces the length of time necessary in obtaining useful results.

Electrical Deaf Aid Sets. An electrical deaf aid set consists essentially of a microphone for receiving the sound, suitable electrical arrangements, and an ear-piece for communicating the amplified sound to the ear. A complete test of the system should include an overall determination, at a series of frequencies, of the acoustical output from the ear-piece resulting from a given acoustical input into the microphone. The known input to the microphone may be obtained, for instance, by placing the microphone in a previously explored sound field as in microphone testing (p. 143). The output from the ear-piece may be tested—as telephone receivers are tested—by communicating the pressure through an artificial ear canal to the diaphragm of a calibrated condenser microphone. Incidentally, as the microphone of a deaf aid set is usually of the carbon granule type, no great constancy is to be expected, and it is well to adopt a technique which will yield average values.

Articulation tests of deaf aid sets are also of value. H. Fletcher* has described a simplified articulation test in which a list of 100 monosyllabic words is used instead of a series of disconnected syllables. Fifty words such as bat, bait, bet, bit, and bite are employed, which differ only in their vowel sound, and fifty such as die, fie, guy, in which the changes occur only in the consonants. There are necessarily some repetitions, and a list is given below.

Vowels: bike, bat, bite, boot, beat, boat, bake, buck, bout, bit, book, bait, but, bout, bit, book, back, balk, back, bike, boot, bet, bought, bat, bite, boot, beak, boat, bake, buck, bout, beat, boat, bait, but, bout, bit, book, beck, balk, bake, bit, book, bet, bought, back, bite, boot, beak, boat.

Consonants: vie, by, high, wing, thy, wiz, wig, win, shy, why, wish, which, wick, pie, thigh, by, high, wing, thy, wiz, with, die, lie, wry, tie, which, wick, pie, thigh, whiff, thigh, fie, my, sigh, vie, die, will, wry, tie, whip, wit, guy, nigh, shy, why, fie, whim, sigh, vie, sigh.

* H. Fletcher, *Bell Laboratories Record,* October 1927. See also H. Fletcher and J. C. Steinberg, *Bell Sys. Tech. J.*, 8, 806, 1929; *Acous. Soc. Am. J.*, 1, Supplement, pp. 40–42, 1930.

The procedure during the articulation test is to write each of the hundred words on a separate card and to shuffle them before each test, thereby eliminating any possibility of memory affecting the result. The testing crews should, however, be familiar with the words in the lists. The words are pronounced in a natural voice at about 3 ft. from the hearing aid, the strength of the voice being kept as constant as possible and the rate of enunciation being slow enough to allow one word to be written by the hearer before the next is called. In determining the vowel percentage only the vowel part of the word is considered. If 'bat' were interpreted as 'pat' it would be rated correct since the vowel was correct. Similarly, when considering consonants, only the consonant part of the word is used for rating. A figure of merit expressing for any individual the value to him of any particular hearing aid, is obtained by multiplying the percentage of vowel sounds he hears correctly by the percentage of correct consonant sounds and again by the percentage of correct consonant sounds. Consonants contribute more to intelligibility than vowels, and this makes it necessary to give them more weight in the final rating. Much has been done to determine just how much more important the consonants are.

If V_w is the vowel articulation determined as above and expressed as a ratio, and if C_w is the consonant articulation ratio, the standard syllable articulation S expressed as a ratio (p. 252) is given by *

$$S = 1 - (1 - V_w C_w^2)^{0 \cdot 9}$$

Loud-speakers. In order to specify the behaviour of a loud-speaker it is important to know the variation of acoustic output with frequency, the extent to which non-linear distortion occurs with loud sounds (p. 245), and whether the output is concentrated in certain directions or uniformly distributed. This latter characteristic is of lesser importance when loud-speakers are employed in rooms, since reflections from walls tend to obliterate directional effects. Another characteristic of interest is the damping of the instrument, and its capacity to follow transient impulses without superimposing a note of its own natural frequency.

In the past greatest attention appears to have been paid to

* H. Fletcher and J. C. Steinberg, *Bell Sys. Tech. J.*, 8, 806, 1929; *Acous. Soc. Am. J.*, 1, Supplement, pp. 40–42, 1930.

the determination of the variation of the acoustical output with frequency. Since loud-speakers are used with valve amplifiers, and since the output depends largely upon the extent to which the impedances of the output valve and of the loud-speaker are matched, it is probably best for a loud-speaker to be tested in conjunction with the associated valve, a relation being obtained between the voltage supplied to the grid of this valve and the acoustic output of the loud-speaker. Alternatively the variation of the electrical impedance of the loud-speaker with frequency may be determined, and the acoustical output studied when either the input current or the input voltage are maintained constant as the frequency is varied; the acoustical output may conveniently be plotted in terms of the electrical power consumed.

There are also a number of acoustical considerations which affect tests of loud-speakers, even under ideal conditions.* In the first place, the sounds near the loud-speaker arising from different parts of the mouth of the horn or of the radiating diaphragm may interfere and cause concentration of sound in some regions in the air and diminution at others. The locations of these regions change with frequency. It will be recalled (p. 61) that for a piston diaphragm (diam. D) vibrating in a large rigid wall with frequency n there is a succession of maxima and minima along the axis out to a distance $D^2n/4500$ ft. Beyond this limit these fluctuations disappear and pressure varies inversely as distance. It is true that a loud-speaker is not accurately similar to a vibrating piston, but the above criterion is useful for predicting suitable measuring conditions. Naturally the fundamental requirement is to obtain data at the typical listening distance. If this is greater than the above limit, response-frequency measurements would be applicable to any greater distance. If the typical listening distance is less than the limit prescribed, the response characteristic will have irregularities due to interference, but since these irregularities would be heard they should be attributed to the loud-speaker, and such a curve would be indicative of such a performance. In this case additional response measurements at distances greater than the limit would be valuable for design purposes to distinguish between ordinary resonances and variations due to interference.

Measurements are best made out-of-doors under conditions where reflections from neighbouring surfaces are avoided. One

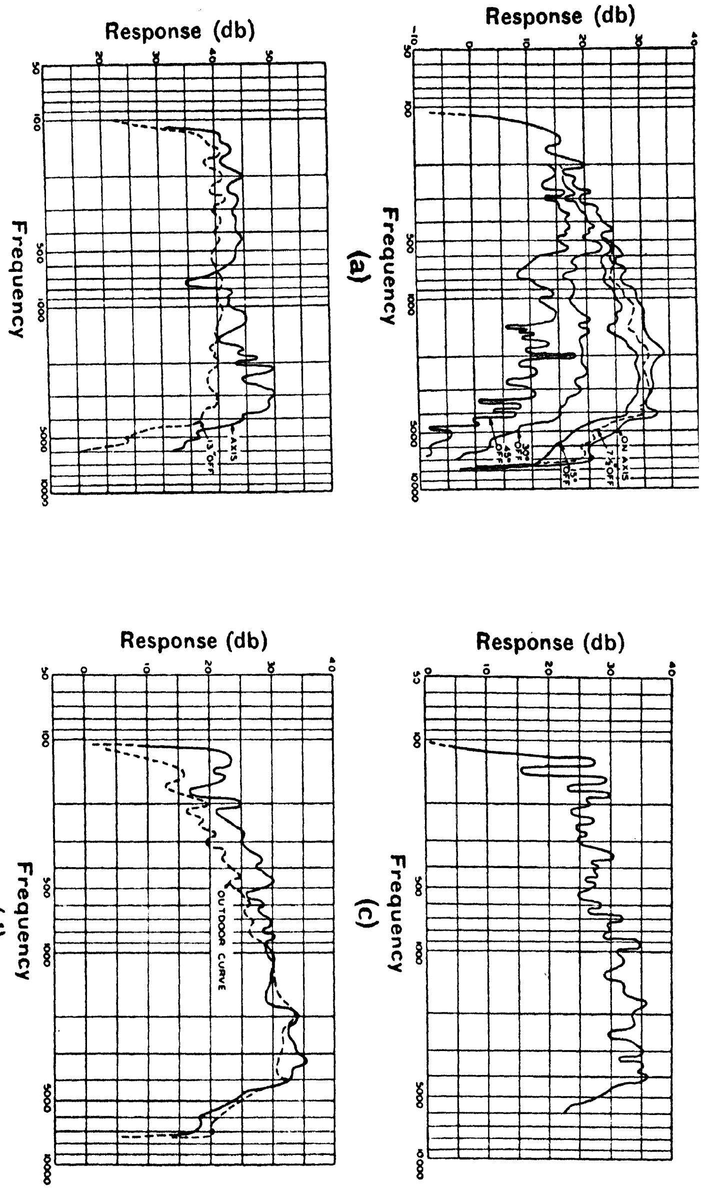
* See L. G. Bostwick, *Bell Sys. Tech. J.*, 8, 135, 1929.

method is to work on top of a building with a flat roof, the loud-speaker being directed outwards over the parapet. Alternatively, the loud-speaker may be directed out of an upper window.* In each case the microphone must be suspended in front of the loud-speaker, and a suitable support is necessary. Where reflections from neighbouring surfaces are anticipated, these may often be avoided by arranging in front of them large oblique reflectors which will throw the sound away from the microphone. An arrangement employed at the National Physical Laboratory is shown in Pl. VIII, p. 272.

Often, however, it is necessary to make measurements indoors. In a room standing waves will exist, and the sound pressure at any fixed position may vary greatly with frequency, although the acoustic output of the loud-speaker may be constant. However, by using a room having all its dimensions large compared with the distance between the measuring microphone and the loud-speaker—a distance which is determined by the considerations given above—and thoroughly lagging the walls with sound absorbent, it is possible at all frequencies, except the lowest, to reduce to negligible proportions the intensity of the reflected sound which reaches the microphone. The response of the loud-speaker at any frequency can then be measured by averaging the squares of the pressures throughout a suitable volume. The use of a large room has the additional advantage that the room exerts less reaction upon the loud-speaker itself. Generally speaking, this reaction is not great, but in small measuring rooms at low frequencies it may become important.

Bostwick gives some interesting curves for the output of a Wente and Thuras loud-speaker movement used in conjunction with an exponential horn of 30 in. diameter, having a cut-off frequency of 115 cycles per second. The upper curve of fig. 102 (*a*) shows the frequency-response curve obtained out-of-doors with a condenser microphone on the axis at a distance 12 ft. from the mouth, which was greater than the limiting distance referred to above. Except for variations near the cut-off frequency of the horn there are no pronounced irregularities, but the curve falls off somewhat in the low-frequency region. Fig. 102 (*b*), showing the characteristics measured on the axis at a distance of only 2 in. from the horn, differs considerably from the results obtained at 12 ft. Firstly, a marked depression occurs at 750 cycles

* F. Trendelenburg, *Zeits. f. tech. Phys.*, 5, 236, 1924.



(*a*) Measured outdoors at 12 ft. from the horn mouth
(*b*) Measured outdoors at 2 in. from horn mouth
(*c*) Measured in highly absorbent room at 12 ft. from the horn mouth, on axis
(*d*) Measured in highly absorbent room at average distance of 12 ft. from horn mouth, a moving microphone being employed

FIG. 102.—Response-frequency characteristics of loud-speaker (Bostwick)

per second. This is found to agree in position with the first interference minimum, as calculated for a piston radiator of 30 in. diameter; also, after allowance is made for a slight reduction of the radiating surface as the frequency is increased, the irregularities at the higher frequencies are similarly explicable. Secondly, the average curve below 1000 cycles is nearly parallel to the axis, instead of sloping as in fig. 102; this effect would be expected if there were an increasing concentration of the sound about the axis as the frequency increased. Curves taken off the axis (fig. 102 (*a*)) show that there was such concentration, the response obtained off the axis being markedly lower at high frequencies than that obtained on the axis.

The same loud-speaker was also studied indoors in a room some 34 ft. by 18 ft. by 9 ft. in size, lagged with ½-in. felt and heavy cloth curtains draped loosely over the walls. Fig. 102 (*c*) shows the curve obtained indoors at a distance of 12 ft. The irregularities, compared with the outdoor curve, are due to stationary waves in the room. To avoid these, measurements were also taken over a region instead of at a point. The condenser microphone was rotated in a circle of 69 in. diameter inclined at 45° to the horizontal, a suitable mechanism being arranged to keep the plane of the microphone diaphragm always perpendicular to the loud-speaker axis. The indicating instrument involved a thermocouple, and, as the transmitter was rotated, the average deflection obtained was proportional to the average of the squares of the acoustical pressures on the microphone. Fig. 102 (*d*) shows the frequency-response curve so obtained at a distance of 12 ft. in comparison with the outdoor curve (dotted) obtained at the same distance at a point on the axis. The fact that at low frequencies the response in rooms is higher than in open air may probably be attributed to failure of the lagging on the walls of the room to absorb low-frequency sound, and a consequent raising of its intensity level by reverberation of the reflected sound.

Grützmacher and Meyer,* and also Cohen,† have described devices for automatically taking frequency-response curves of telephones and loud-speakers. Each consists essentially of a heterodyne oscillator giving constant output for actuating the loud-speaker. The frequency of the note is varied continuously

* M. Grützmacher and E. Meyer, *E.N.T.*, 4, 203, 1927.
† B. S. Cohen, A. J. Aldridge, and W. West, *I.E.E., J.*, 64, 1023, 1926.

by rotation of the control condenser of the oscillator. This condenser is rigidly coupled to a drum camera, upon the film of which a spot of light indicates at any moment the intensity of the sound as registered by a microphone, and thus as the drum rotates the spot of light automatically registers the frequency-response curve of the loud-speaker. A cathode-ray tube may also be used to record frequency-response characteristics (p. 105).

Distortion in a loud-speaker due to non-linear response may be tested at each frequency by noting whether the acoustical pressure set up by the loud-speaker is proportional to the input voltage or current. Such proportionality may exist over a very wide range with moving-coil instruments (p. 41). However, when the coil excursions are so great that the coil moves out of the uniform part of the magnetic field, non-linear distortion will occur. It will occur for other reasons in other types of loud-speakers. The effect of non-linear response is the introduction of extraneous harmonics. Such impurities may be detected by ear if pure sine-wave current is used to excite the loud-speaker, and are particularly easy to detect in a room if one ear of the observer is stopped up and the other is placed near a node (silence point) for the fundamental.

The testing of the overall output of gramophones may follow the general lines of the tests on loud-speakers. In this case the gramophone is excited by playing suitable records, such as the calibrated constant-note records referred to in an earlier chapter.

Electrical Gramophone 'Pick-ups.' An electrical gramophone pick-up may be tested by measuring the voltage developed at its terminals when it is played either on a series of constant-note records or on a calibrated gliding-note record. A Moullin voltmeter is often a suitable instrument for measuring the output, since electromotive forces of the order of a few volts are usual. The pick-up should also be tested for non-linear distortion, which is liable to occur. Since records giving varied needle displacement at constant frequency are not available, the most convenient test of non-linear distortion is to analyse the output when the pick-up is used with a pure-note record, or to note the wave form of the output—say by means of a cathode-ray oscillograph. Non-linear distortion is revealed by the presence of harmonics which are not present when a non-distorting pick-up is employed.

Transmission of Sound through Partitions. In the arrangements adopted at the National Physical Laboratory for measuring transmission of sound through panels of building material, the test panel covers an aperture in a sound-proof wall between two rooms, which are isolated even to their foundations. On one side sound falls upon the partition, and at points on the other side measurements are made of the sound transmitted. Comparison with the transmission through the uncovered aperture gives the transmission ratio for the partition. The walls may be heavily lagged to absorb reflections, and in that case the sound is directed obliquely upon the partition to avoid any reflection of sound back to the source—for otherwise some reaction upon the acoustic output of the source might result. Some nodes and antinodes occur in the room, because the lagging upon the walls, etc., is not sufficient to suppress reflections entirely. In the new acoustical laboratory at the National Physical Laboratory rooms with non-parallel walls have been provided to reduce resonances.* In practice, to reduce errors, an average of measurements at several points is taken, for several positions of the loud-speaker source. The movement of the microphone and loud-speaker can be automatic and continuous, and warble notes can be used (p. 50) to assist in completing the mixing of sound in the rooms. Alternatively the two rooms can be used in a reverberant condition (with the lagging removed). This is the practice normally adopted at the American Bureau of Standards. Measurements are made, at a number of points in the source room and in the transmission room, of the steady intensity of sound set up both before and after the test panel is placed in position. Points near the source or test panel are, however, avoided, for the sound intensity in these positions is rather abnormal.

Naturally it is possible to make overall measurements of sound transmission from room to room in an actual building. For the purpose a gramophone with a warble-note record is a convenient source of sound, and with a portable microphone equipment the sound may be measured in the two rooms. Knowler † has described a simple method in which the microphone is dispensed with, and aural balancing of two sounds is employed. Two loud-speakers are employed, one in each of the two rooms. The observer, in one room, switches on the loud-speakers alternately.

* *N.P.L. Annual Report*, p. 58, 1932.
† A. E. Knowler, *Phil. Mag.*, **12**, 1039, 1931.

The sound from the one loud-speaker is reduced by the partition; that from the other is reduced electrically in a measurable ratio until it appears to the observer to be of equal intensity. By interchanging the loud-speakers any difference between their activities is allowed for. The experiment is repeated with the observer in the other room—a procedure which eliminates the effect of different absorbing powers in the two rooms.

W. C. Sabine * measured the transmission of sound through partitions by a method which involved measuring the duration of audibility of sound after the source was stopped, the measurements being made both in the source room and in the room separated from the source by the test panel. Broadly the method consisted in producing, by means of a source in the larger room, a volume of sound of which the intensity in terms of threshold audibility was known, and then, by stopping the source, allowing the sound to decay at the rate natural to the room until it ceased to be audible on the other side of the partition. The intensity of the sound at this instant was estimated from reverberation measurements in the source room, and was taken to be numerically equal to the 'reduction factor' of the partition. In later work carried out in the specially designed Wallace Clement Sabine Laboratory of Acoustics, P. E. Sabine † determined simultaneously the duration, after the source had ceased, of the residual sound in the source chamber and in the adjacent test chamber, and computed from these results the ratio of intensity on the two sides of the partition.

Davis † and Buckingham † have discussed the theory of measurements of sound transmission by observation of the decay of reverberation, and have given equations expressing the transmission coefficient of the panel in terms of audibility measurements. On the basis of the equations referred to, V. O. Knudsen ‡ has attempted to reconcile measurements of sound transmission made by Sabine's decay method with those made by more direct observations at the other laboratories. The discrepancy to be explained is illustrated in fig. 96 (p. 289).

It will be realised that the methods of measurement referred to above provide tests of the degree to which the test panel,

* W. C. Sabine, *Brickbuilder*, 24, 31, 1915; *Collected Papers*, p. 237.

† See A. H. Davis and G. W. C. Kaye, *Acoustics of Buildings*, p. 179, 1927.

‡ V. O. Knudsen, *Acous. Soc. Am. J.*, 2, 129, 1930.

which may be representative of wall, floor, or ceiling, excludes sound which reaches it through the air. To test its capacity to exclude noises due to impacts on the exterior face—such as arise by footsteps upon a floor, etc.—some other form of measurement is necessary. In this case the test panel is subjected to a series of blows by mechanically driven hammers, and the sound emitted by the panel is measured.

APPENDICES

I. Mathematical Formulæ

$$e = 2{\cdot}718; \qquad \log_e(x) = 2{\cdot}303 \log_{10}(x)$$

$$f(x+h) = f(x) + \frac{h}{1!}f'(x) + \frac{h^2}{2!}f''(x) + \ . \ . \ .$$

$$f(h) = f(0) + \frac{h}{1!}f'(0) + \frac{h^2}{2!}f''(0) + \ . \ . \ .$$

$$(1+x)^m = 1 + mx + \frac{m(m-1)x^2}{2!} + \ . \ . \ .$$

$$(1+x)^{-1} = 1 - x + x^2 - \quad + \ . \ . \ .$$

$$(1-x)^{-1} = 1 + x + x^2 + \quad + \ . \ . \ .$$

$$(1+x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}x - \frac{1.1}{2.4}x^2 + \quad - \ . \ . \ .$$

$$(1+x)^{-\frac{1}{2}} = 1 - \tfrac{1}{2}x + \frac{1.3}{2.4}x^2 - \quad + \ . \ . \ .$$

$$e^{\pm x} = 1 \pm \frac{x}{1!} + \frac{x^2}{2!} \pm \frac{x^3}{3} + \ . \ . \ .$$

$$\log_e(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \ . \ . \ .$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ . \ . \ .$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ . \ . \ .$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ . \ . \ .$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ . \ . \ .$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} \quad \text{approx. } (x \text{ small})$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \quad + \ . \ . \ .$$

$$e^{x} = \cosh x + \sinh x$$
$$e^{-x} = \cosh x - \sinh x$$
$$e^{ix} = \cos x + i \sin x$$
$$e^{-ix} = \cos x - i \sin x$$
$$\cos ix = \cosh x; \qquad \cosh ix = \cos x$$
$$\sin ix = -i \sinh x; \qquad \sinh ix = -i \sin x$$
$$1 = \sin^2 x + \cos^2 x = \cosh^2 x - \sinh^2 x$$
$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$
$$\sin (x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos (x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\sinh (x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh (x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y)$$
$$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y)$$
$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y)$$
$$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y)$$
$$\sin nx = n \cos^{n-1} x \sin x - \frac{n}{3} \cos^{n-3} x \sin^3 x + \ldots$$
$$\cos nx = \cos^n x - \frac{n}{2} \cos^{n-2} x \sin^2 x + \frac{n}{4} \cos^{n-4} x \sin^4 x + \ldots$$
$$\mp \sin x = \cos\left(\frac{\pi}{2} \pm x\right) = \sin (\pi \pm x)$$
$$\cos x = \sin\left(\frac{\pi}{2} \pm x\right) = -\cos (\pi \pm x)$$
$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 6^2} + \ldots$$
$$J_1(2x) = x\left[1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.2.3} - \frac{x^6}{1.2.3.2.3.4} + \ldots\right]$$
$$J_1(x) = -J_0'(x) = -J_1(-x)$$
$$J_n(x) = \frac{x^n}{2^n . n!}\left\{1 - \frac{x^2}{2(2n+2)} + \frac{x^4}{2.4(2n+2)(2n+4)} + \right\}$$

Every $J_n(x)$ satisfies Bessel's differential equation

$$\frac{d^2 f}{dx^2} + \frac{1}{x}\frac{df}{dx} + \left(1 - \frac{n^2}{x^2}\right) f = 0$$

If n is integral,

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos (x \sin \phi - n\phi) d\phi$$
$$K_1(x) = \frac{2}{\pi}\left(\frac{x^3}{3} - \frac{x^5}{3^2.5} + \frac{x^7}{3^2.5^2.7} - \ldots\right)$$
$$= \frac{2x}{\pi} \text{ (approx.) if } x \text{ is large.}$$

$$\frac{1}{\pi}\int_0^\pi a\sin x dx=\frac{2}{\pi}a=0{\cdot}64a$$

$$\frac{1}{\pi}\int_0^\pi a^2\sin^2 x dx=\tfrac{1}{2}a^2=(0{\cdot}71a)^2$$

If m and n be unequal integers,

$$\int_0^\pi \sin mx \sin nx dx=\int_0^\pi \cos mx\cos nx dx=0$$

Complex Quantities

If

$$z=x\pm yi=r(\cos\theta\pm i\sin\theta)=re^{\pm i\theta}$$

then

$$r=|z|=\sqrt{x^2+y^2} \quad\text{and}\quad \tan\theta=\frac{y}{x}$$

also

$$\frac{1}{z}=\frac{1}{r}e^{\mp i\theta} \qquad x-yi=re^{-i\theta}$$

$$z_1z_2=r_1r_2e^{i(\theta_1+\theta_2)}$$

The real part of $(x\pm yi)e^{in}$

$$=x\cos n\mp y\sin n=\sqrt{x^2+y^2}\{\cos(n\pm\theta)\}$$

Combinations of Harmonic Terms

$$\sum_{k=1}^{k=p} a_k\cos(nt+\epsilon_k)=r\cos(nt+\theta)$$

where $r^2=(\Sigma a_k\cos\epsilon_k)^2+(\Sigma a_k\sin\epsilon_k)^2$

and $\tan\theta=\dfrac{\Sigma a_k\sin\epsilon_k}{\Sigma a_k\cos\epsilon_k}$

$$\sum_{k=1}^{k=m}(a\cos nt+k\epsilon)=r\cos(nt+\theta)$$

where $r=ma\sin\theta/\theta$

and $2\theta=m\epsilon=$ phase difference between the first and last terms of the summation

$$a_1\cos(n_1t+\epsilon_1)+a_2\cos(n_2t+\epsilon_2)=r\cos(nt+\theta) \quad \text{if } n_1-n_2 \text{ is small}$$

where $r^2=a_1{}^2+a_2{}^2+2a_1a_2\cos\{(n_1-n_2)t+(\epsilon_1-\epsilon_2)\}$

and $\tan\theta=\dfrac{a_1\sin\epsilon_1+a_2\sin(n_1-n_2+\epsilon_2)}{a_1\cos\epsilon_1+a_2\cos(n_1-n_2+\epsilon_2)}$

$$(a\cos p+b\cos q)^2=\tfrac{1}{2}(a^2+b^2)+\tfrac{1}{2}a^2\cos 2p+\tfrac{1}{2}b^2\cos 2q$$
$$+ab\cos(p+q)+ab\cos(p-q)$$

$$(a\cos p+b\cos q)^3=(\tfrac{3}{4}a^3+\tfrac{3}{2}ab^2)\cos p+(\tfrac{3}{4}b^3+\tfrac{3}{2}a^2b)\cos q$$
$$+\tfrac{1}{4}a^3\cos 3p+\tfrac{1}{4}b^3\cos 3q$$
$$+\tfrac{3}{4}a^2b\cos(2p\pm q)+\tfrac{3}{4}ab^2\cos(p\pm 2q)$$

Fourier Series

$$f(x)=\tfrac{1}{2}a_0+\sum_{k=1}^{\infty}(a_k\cos kx+b_k\sin kx)$$

$$a_k=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos kx\,.\,dx \qquad b_k=\frac{1}{\pi}\int_0^{2\pi}f(x)\sin kx\,.\,dx$$

If this series converges it represents a periodical function $f(x)$ of period 2π.

II. Fourier Analysis * by Tabular Calculations

It is well known that any continuous periodic curve can be expressed as the sum of a series of harmonic curves (*i.e.* curves of sines or cosines) which are such that the sum of their ordinates for any value of the abscissa will be equal to the ordinate of the given curve for that value of the abscissa. In some cases an infinite series of such harmonic curves is necessary. Where, however, harmonics higher than a specified limit may be neglected, it is possible to deduce the magnitude of the lower harmonics by fairly simple tabular calculations.† Perry indicated the principle, Runge dealt with the general case, Thomson reduced the labour by grouping ordinates and effected considerable simplification by dealing solely with odd harmonics—the case in which the two halves of the wave-form are alike.

In a solution due to Runge, where only harmonics up to the sixth ‡ are effectively involved, the procedure is as set out below. A complete fundamental wave-length of the curve to be analysed is taken, and twelve equidistant ordinates are drawn through points $x_0, x_1, x_2, \ldots x_{11}$, the points being $\frac{1}{12}$ of the fundamental wave-length apart. The corresponding values of y may be denoted $y_0, y_1, \ldots y_{11}$. From these 12 pairs of values 12 equations may be obtained which yield the 12 coefficients A_0, B_1, etc., of the equation

$$y=A_0+A_1\cos x+A_2\cos 2x+\ldots+A_6\cos 6x$$
$$+B_1\sin x+B_2\sin 2x+\ldots+B_5\sin 5x$$

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* A bibliography of references to mechanical analysers is given by F. W. Kranz, *Frank. Inst. J.*, 204, 245, 1927.

† See A. Eagle, *Practical Treatise on Fourier's Theorem and Harmonic Analysis*, p. 83, 1925; Perry, *Electrician*, 28, 362, 1892; C. Runge, *Zeits. f. Math. and Phys.*, 48, 443, 1903; 52, 117, 1905; S. P. Thomson, *Phys. Soc., Proc.*, 19, 443, 1904; 23, 334, 1911; A. E. Clayton, *I.E.E., J.*, 59, 491, 1921.

‡ A graphical method of analysis in this case is given by A. Eagle, *loc. cit.*

which expresses the analysed curve in terms of its harmonic components. To find the values of A_0, A_1, etc., a tabulated scheme of great convenience is available. The proof is given in textbooks, but the procedure is as follows. First the measured ordinates are written in two rows, as set out below. They are then added and subtracted, whence a_0, etc., and b_1, etc., are obtained as indicated.

	y_0	y_1	y_2	y_3	y_4	y_5	y_6
		y_{11}	y_{10}	y_9	y_8	y_7	
Sums .	a_0	a_1	a_2	a_3	a_4	a_5	a_6
Differences		b_1	b_2	b_3	b_4	b_5	

The a's and b's are now rearranged as below, and by addition and subtraction c's and d's are found.

	a_0	a_1	a_2	a_3		b_1	b_2	b_3
	a_6	a_5	a_4			b_5	b_4	
Sums .	c_0	c_1	c_2	c_3		d_1	d_2	d_3
Differences	c_0'	c_1'	c_2'			d_1'	d_2'	

The c's and d's are now arranged in two tabular forms, and these yield the values of A's and B's when the appropriate sums and additions are carried out.

Evaluation of A_0, A_1, etc.

	A_0 and A_6		A_1 and A_5		A_2 and A_4		A_3
	c_0	c_1	c_0'	$\frac{\sqrt{3}}{2}c_1'$	c_0	$\frac{1}{2}c_1$	$c_0'-c_2'$
	c_2	c_3	$\frac{1}{2}c_2'$		$-\frac{1}{2}c_2$	$-c_3$	...
Sums: 1st Column .	...		...		...		...
Sums: 2nd Column .	...		...		...		...
Sum of Cols. .	$12A_0$		$6A_1$		$6A_2$		$6A_3$
Diff. of Cols. .	$12A_6$		$6A_5$		$6A_4$		...

Thus, to find A_0 and A_6 take the division headed A_0, A_6; add the terms in the first column, getting c_0+c_2; add those in the second, getting c_1+c_3; then adding these two results yields $(c_0+c_2)+(c_1+c_3)$, the value of $12A_0$, while the difference $(c_0+c_2)-(c_1+c_3)$ is the value of $12A_6$. The other coefficients are found in a similar manner.

Evaluation of B_1, B_2, etc.

	B_1 and B_5		B_2 and B_4		B_3
	$\frac{1}{2}d_1$ d_3	$\frac{\sqrt{3}}{2}d_2$	$\frac{\sqrt{3}}{2}d_1'$	$\frac{\sqrt{3}}{2}d_2'$	$d_1 - d_3$
Sums { 1st Column .	...		...		...
Sums { 2nd Column .	...		...		...
Sum of Cols. . .	$6B_1$		$6B_2$		$6B_3$
Diff. of Cols. . .	$6B_5$		$6B_4$		...

III. Linear Differential Equations

The equation $m\ddot{\xi}+r\dot{\xi}+s\xi=0$ is a *homogeneous linear differential* equation of the *second* order, with constant coefficients. It is linear because, regarding ξ and its derivatives $\dot{\xi}$, $\ddot{\xi}$ as the elements of the equation, products or squares or higher powers of these elements do not enter. It is homogeneous, since every significant term of the equation contains one of the elements to the same power, namely, the first power. It is said to be of the second order, because the highest order of derivative which occurs is the second order.

It may be noted that—

(1) The sum of two or more solutions of a linear homogeneous equation is a solution of the equation; that is, the solutions are additive.

(2) If (in any way) a solution of a linear, homogeneous equation of the n^{th} order is found, the solution, if it contains n independent arbitrary constants, is the most general solution, or the complete integral of the equation.

(3) If, in the case of any linear differential equation with real coefficients, we seek a solution of the type $y = Ce^{\lambda x}$, the imaginary values of λ will occur in conjugate pairs of the form $m \pm in$.

IV. Solution of Differential Equations by Operational Calculus

The solution of differential equations representing conditions in electrical networks or in analogous acoustical arrangements, may be written down directly by the aid of the expansion theorems of Heaviside's operational calculus. These expansion theorems are expressed below in terms of electrical circuits, but the procedure is applicable to acoustical problems. Firstly, the differential equation between voltage

and current which is to be solved is written down, and then is rewritten in operational form by writing p for $\frac{d}{dt}$. Thus for a circuit with an e.m.f. E in series with an inductance L, a resistance R, and a capacity C, the differential equation for the current i is

$$E = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt \tag{1}$$

which is written in the form

$$E = \left(Lp + R + \frac{1}{Cp}\right)i \tag{2}$$

The equation is now regarded as a simple algebraic one, and the quantity in brackets, being the ratio between applied e.m.f. and current, is known as the generalised impedance of the system. For convenience it is denoted by Z_p.

Another example will illustrate the procedure where simultaneous equations are involved. For instance, the electro-mechanical equations for a telephone receiver (p. 75) may be written in operational terms as

$$Lpi + Ri + Mpu = e \tag{3}$$

$$mp^2u + rpu + su - Mi = 0 \tag{4}$$

Regarding these as simple algebraical equations, either i or u may be eliminated, and we obtain a single equation. Eliminating u, it is

$$i = \frac{e(s + rp + mp^2)}{(s + rp + mp^2)(R + Lp) + pM^2} = \frac{e}{(Z_p)} \tag{5}$$

The relation between u and e could also be written down.

When the generalised impedance Z_p for the case under study has been written down, the solution of the differential equation is obtained by simple algebraic processes from the following expansion formulæ. Both steady-state and transient solutions are yielded by the procedure.

First Expansion Formula. (Applied steady e.m.f.'s.) For a given electrical network, and a steady voltage applied suddenly at any point in the circuit, the current is given by the following expression:—

$$i = \underbrace{\frac{E}{Z(p)_{p=0}}}_{\text{Steady state}} + \underbrace{E\sum_{n=1}^{n=m} \frac{e^{p_n t}}{p_n\left[\frac{\partial Z(p)}{\partial p}\right]_{p=p_n}}}_{\text{Transient component}} \tag{6}*$$

where $Z(p)$ is the generalised impedance of the system, $Z(p)_{p=0}$ is the value of $Z(p)$ when $p=0$, and the summation term extends to all the m roots of the equation $Z(p)=0$.

If, as in (5), $Z(p)$ is a quotient of two expressions, it is desirable to

* O. Heaviside, *Collected Papers*, 2, 371; L. Cohen, *Frank. Inst. J.*, 194, 763, 1922.

differentiate by the rule for quotients. Then one of the terms is zero for $p=p_n$ and the algebra is greatly simplified.

Second Expansion Formula. Applied Alternating E.m.f.'s. If the e.m.f. impressed on the given circuit be represented by the real part of $Ee^{j\omega t}$, *i.e.* E cos ωt, we have,

$$i=\frac{Ee^{j\omega t}}{Z(p)_{p=j\omega}}-E\sum_{n=1}^{n=m}\frac{e^{p_n t}}{(j\omega-p_n)\left[\frac{\partial Z(p)}{\partial p}\right]_{p=p_n}} \qquad (7)^*$$

Of this equation we have to take only real parts. In the case of the summation term the imaginary part is readily eliminated and we obtain

$$i=\frac{Ee^{j\omega t}}{Z(p)_{p=j\omega}}+E\sum_{n=1}^{n=m}\frac{p_n e^{p_n t}}{(p_n^2+\omega^2)\left[\frac{\partial Z(p)}{\partial p}\right]_{p=p_n}} \qquad (8)$$

In this the steady-current component is still complex, and only the real part has to be taken.

It is important to notice from this that for alternating current problems the **steady state** *solution may be written down directly* when the generalised impedance (Z_p) has been found, by writing $j\omega$ for p in the operational equation of types (2) or (5), or *by writing* $j\omega$ *for* d/dt *in the original differential equation* (1). As an illustration, (5) should be compared with the solution 28, p. 75.

If the applied e.m.f. is given by E cos $(\omega t+\theta)$ instead of E cos ωt, then in place of $Ee^{j\omega t}$ we have $Ee^{j(\omega t+\theta)}=Ee^{j\theta}e^{j\omega t}$, and in the final formula E is to be multiplied by $e^{j\theta}$. This, when the imaginary part of the summation term is neglected, gives

$$i=\frac{Ee^{j(\omega t+\theta)}}{Z(p)_{p=j\omega}}+E\sum_{n=1}^{n=m}\frac{\cos(\phi_n+\theta)e^{p_n t}}{\sqrt{p_n^2+\omega^2}\left[\frac{\partial Z(p)}{\partial p}\right]_{p=p_n}} \qquad (9)$$

$$\tan\phi_n=\frac{\omega}{p_n} \qquad (10)$$

Subsidence of Currents in Circuits. The above formulæ apply to the case in which a voltage (steady or alternating) is suddenly applied to the given circuit. The expansion also gives correctly—except for a reversal of sign—the subsidence of current in the same circuit when the applied voltage is suddenly short-circuited. For suppose the circuit had been closed sufficiently long to ensure the establishment of the steady state—the transient component being now zero—the current is then given by the term $i=E/Z(p)_{p=0}$ [or $E/Z(p)_{p=j\omega}$ if alternating].

The effect of short-circuiting the applied voltage can be attained

* This equation applies to a suddenly impressed force $e^{\lambda t}$, if λ is written for $j\omega$.

by suddenly introducing another voltage of exactly the same character and magnitude but of opposite sign. We then have two equal voltages of opposite sign acting upon the circuit, one of which produces the steady term referred to, the other produces a reversed steady term and a reversed transient. The resultant current is the sum of the two currents or

$$i = -\mathrm{E}\sum \frac{e^{pt}}{p\dfrac{\partial Z(p)}{\partial p}} \tag{11}$$

The summation term in the expansion formula thus gives the subsidence current obtained on short-circuiting the voltage.

V. Electrical Data and Impedances

Capacity.

Units. 1 cm. (E.S. unit) = 1·11 micro-microfarads approx. (written 1·11 $\mu\mu$F).

1 pint jar = 0·0015 μF. 1 gal. jar = 0·003 μF.

Parallel Plate Condenser.

$C = KA/4\pi d$ cm., $C = 0{\cdot}0885 \times 10^{-6} KA/d$ microfarads,

where A = surface area of each plate,
d = thickness of dielectric (cm.),
K = dielectric constant (K = 1 (air), 2–2·5 (paper, paraffin wax), 5–10 (glass), 6–7 (mica)).

Discharge of Condenser through Resistance R.

Voltage falls to half value in time 0·691 RC, where R is in ohms and C in farads.

Inductance. (All dimensions in cm.) 1 henry = 10^6 μH = 10^9 c.g.s. E.M. units.

Circular Coil. $L = 0{\cdot}0126an^2\{2{\cdot}30 \log_{10}(16a/d) - 1{\cdot}75\}$, μH.

a = radius of wire (fine) of coil, d = diameter of coil,
n = number of wires.

Single-layer Coil or Solenoid. $L = 0{\cdot}0395a^2n^2k/b$, μH.

a = radius of the coil, measured to centre of wire,
n = number of turns, b = length of coil,
$k = f\,(2a/b) = 1$ if b is great,
$= 0{\cdot}69$ when $2a = b$,
$= 0{\cdot}20$ when $2a = 10b$.

To obtain maximum inductance from a given length of wire, the diameter of the solenoid should be 2·45 times the length b of the coil.

Toroidal Coils (n turns).

Rectangular Cross-section. $L = 0{\cdot}0046 n^2 h \log_{10}(R_2/R_1)$, μH.

R_1 = inner radius (cm.), R_2 = outer radius,
h = axial depth of toroid.

Circular Cross-section. $L = 0{\cdot}0126 n^2 [R - \sqrt{R^2 - a^2}]$, μH.

R = mean radius of ring,
a = radius of cross-section of winding.

Multilayer Coils of Rectangular Cross-section.

$$L = 0{\cdot}0395 \frac{R^2 n^2}{b + 1{\cdot}5t + R} F_1 F_2, \quad \mu\text{H}.$$

R = mean radius of coil, n = number of turns,
b = length of coil, t = depth of winding,

$$F_1 = \frac{10b + 13t + 2R}{10b + 10{\cdot}7t + 1{\cdot}4R}, \qquad F_2 = 1{\cdot}15 \log_{10}\left(100 + \frac{14R + 7t}{2b + 3t}\right).$$

The distance between the wires is assumed small compared with wire diameter.

Iron Cores in Coils. Iron cores increase the inductance of the coil to an extent depending upon the magnetic properties of the iron. The flux-density in the core is given by $B = \mu H$, where H is the magnetising force, and μ the permeability of the iron at the flux-density concerned. μ varies with the degree of saturation of the iron. Further, H is not uniform across the section, so that B and μ also vary, and only approximate estimates can be made of the effect of the introduction of an iron core.

Coil with Iron Core in nearly closed Circuit with small air-gap.

$$L = \frac{0{\cdot}4 N^2 \mu A}{l_g} \text{ henries,}$$

A = area normal to flux,
N = number of turns,
l_g = length of air-gap.

Open Core Coil.

$$L = \frac{N^2 A \mu k}{l} \text{ henries,}$$

A = cross-section of iron core,
l = length of iron core,
k = experimental factor.

Impedance Operators (z).

$$z = (\text{e.m.f.})/(\text{current}) = e/i$$

$z = R + jX$ may be written as $Z\angle\phi$, where $Z = \sqrt{R^2 + X^2}$ and $\tan\phi = \frac{X}{R}$.

If $e = e_1 \cos \omega t$,

$$i = (e_1/Z) \cos(\omega t - \phi)$$

The following are the values of the impedance operators for

certain circuit arrangements of resistance R, inductance L, and capacity C:—

Circuit	z	Z
•—C—•	$\frac{1}{j\omega C} = -\frac{j}{\omega C}$	$1/\omega C$
•—L—•	$j\omega L$	ωL
•—L—R—C—•	$R + j\left(\omega L - \frac{1}{\omega C}\right)$	$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
•— L—R C —•	$\frac{R + j\omega[L(1 - \omega^2 CL) - CR^2]}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}$	$\sqrt{\frac{R^2 + \omega^2 L^2}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}}$
•— L—R R_1 —•	$\frac{R_1\{[R(R + R_1) + \omega^2 L^2] + j\omega L R_1}{(R + R_1)^2 + \omega^2 L^2}$	$R_1\sqrt{\frac{R^2 + \omega^2 L^2}{(R + R_1)^2 + \omega^2 L^2}}$

Reactances.

Condenser, $1/\omega C$. Inductance, ωL.

At 1000 cycles per second 1 microfarad has a reactance of 160 ohms.
1 henry has a reactance of 6280 ohms.

Resonance Frequency (n) of Series Circuit.

$$4\pi n^2 = \omega^2 = 1/CL$$

When C is expressed in microfarads and L in henries,

$$n = 159/\sqrt{CL}$$

VI. Electrical Ladder Networks

In the theory of electrical transmission lines ladder networks as illustrated in fig. 103 are of importance. In transmission of sound through chains of mechanical acoustical members, the acoustical circuit is often reducible to an analogous electrical line of the same type. It

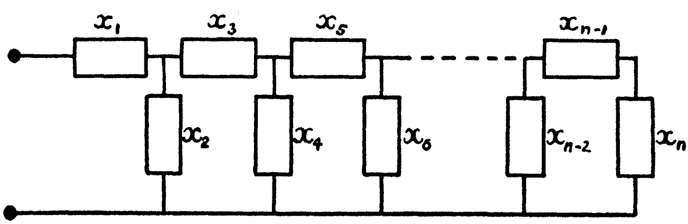

Fig. 103.—Ladder network

is desirable therefore to note that the distribution of current and voltage throughout such a line may be put into compact form by using

the notation of continuants.* Let fig. 103 represent a ladder network constructed of n impedances $x_1, x_2, x_3, \ldots x_n$. The impedance of this network at the input terminals can be written down by the ordinary rules for combining series and shunt impedances and is given by the continued fraction

$$x_1+\cfrac{1}{\cfrac{1}{x_2}+\cfrac{1}{x_3+\cfrac{1}{\cfrac{1}{x_4}+\ldots}}} \quad \text{or} \quad a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+\cfrac{1}{a_4+\ddots+\cfrac{1}{a_n}}}}$$

if the impedances of the elements x_1, x_3, x_5, etc., be written as a_1, a_3, a_5, etc., and the impedances of the shunt elements x_2, x_4, etc., be written as $\frac{1}{a_2}, \frac{1}{a_4}$, etc.

In the notation of continuants this is equal to

$$\frac{\mathrm{K}(a_1, a_2, \ldots a_n)}{\mathrm{K}(a_2 \ldots a_n)}$$

The current through any series member x_{2r+1} is equal to the input current multiplied by

$$\frac{\mathrm{K}(a_{2r+2} \ldots a_n)}{\mathrm{K}(a_2 \ldots a_n)}$$

Let the current in the series member x_{2r+1} due to a voltage v impressed at the sending end be v/Z_{2r+1}, so that Z_{2r+1} can be termed the impedance of x_{2r+1} with respect to the input terminals; Z_{2r+1} is given by

$$\mathrm{Z}_{2r+1}=\frac{\mathrm{K}(a_1 \ldots a_n)}{\mathrm{K}(a_{2r+2} \ldots a_n)}$$

Again, the voltage across the shunt element x_{2r} is equal to the input voltage multiplied by

$$\frac{\mathrm{K}(a_{2r+1} \ldots a_n)}{\mathrm{K}(a_1 \ldots a_n)}$$

and the ratio of the voltage across x_{2r} to the voltage across x_{2s} is

$$\frac{\mathrm{K}(a_{2r+1} \ldots a_n)}{\mathrm{K}(a_{2s+1} \ldots a_n)}$$

* See A. C. Bartlett's *The Theory of Electrical Artificial Lines and Filters*, chap. iii (Chapman & Hall), 1930; Chrystal's *Algebra*, chap. xxxiv. The function K is defined thus:

If $m > 2$

$$\mathrm{K}(a_1, a_2, \ldots a_m) = a_m\mathrm{K}(a_1, a_2, \ldots a_{m-1}) + \mathrm{K}(a_1, a_2, \ldots a_{m-2})$$

also

$$\mathrm{K}(0)=1; \quad \mathrm{K}(a_1)=a_1; \quad \mathrm{K}(a_1, a_2)=a_2\mathrm{K}(a_1)+\mathrm{K}(0)$$

If Z_{2r} is the impedance of the shunt element x_{2r} with respect to the input terminals, it follows that

$$Z_{2r} = \frac{1}{a_{2r}} \frac{K(a_1 \ . \ . \ . \ a_n)}{K(a_{2r+1} \ . \ . \ . \ a_n)}$$

VII. The Theory of Electrical Filters

Let an infinite line of series and shunt impedances be arranged as in fig. 104. Let the currents in the various sections be denoted by

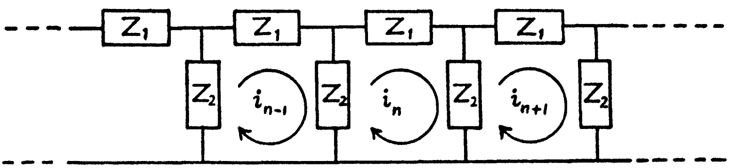

FIG. 104.—Electrical filter circuit

i_{n-1}, i_n, i_{n+1}, etc., and the e.m.f.'s across the shunt impedances Z_2 be denoted by e_{n-1}, e_n, etc.

Then we have

$$Z_2(i_n - i_{n-1}) + Z_1 i_n + Z_2(i_n - i_{n+1}) = 0$$

i.e. $$Z_2 i_{n-1} - i_n(2Z_2 + Z_1) + Z_2(i_{n+1}) = 0 \tag{1}$$

As the filter consists of an infinite number of sections the impedance to the right of *any* section may be represented by Z_∞. We may then write

$$e_n = i_n(Z_1 + Z_\infty)$$
$$e_{n+1} = i_{n+1}(Z_1 + Z_\infty)$$

Whence

$$\frac{e_n}{e_{n+1}} = \frac{i_n}{1_{n+1}} \tag{2}$$

But also from considerations of the currents in the shunt impedances Z_2 at the respective sections

$$e_n = Z_2(i_{n-1} - i_n)$$
$$e_{n+1} = Z_2(i_n - i_{n+1})$$

Consequently we have another ratio for e_n/e_{n+1}, and may combine them both thus

$$\frac{i_n}{i_{n+1}} = \frac{e_n}{e_{n+1}} = \frac{i_{n-1} - i_n}{i_n - i_{n+1}} \tag{3}$$

Whence by the properties of ratios

$$\frac{i_n}{i_{n+1}} = \frac{i_{n-1}}{i_n} = e^\lambda \text{ (say)} \tag{4}$$

and the attenuation per section is thus constant. Hence (1) may be rewritten in the form

$$Z_2e^{-\lambda} - (2Z_2 + Z_1) + Z_2e^{\lambda} = 0$$

or

$$\left(2 + \frac{Z_1}{Z_2}\right) = e^{\lambda} + e^{-\lambda} = 2\cosh\lambda \tag{5}$$

This enables the attenuation per section to be calculated. In general λ will be complex. To determine λ therefore write $\lambda = -(\alpha + i\phi)$, *i.e.* $e^{\lambda} = e^{-\alpha}e^{-i\phi}$, where α and ϕ are both real. Here α represents an attenuation, and ϕ a phase change. The determination of α and ϕ from equation (5) is facilitated by noting that

$$\cosh\lambda = \cosh\alpha\cos\phi + i\sinh\alpha\sin\phi$$

If λ is a pure imaginary, e^{λ} is a pure circular function, and p_{n+1} and p_n differ only in phase, and there is no progressive attenuation in the filter circuit. The condition for this is that $\cosh\lambda$ (*i.e.* $\cos i\lambda$) shall be between $+1$ and -1. That is to say, there is no attenuation but only phase shift if

$$1 > 1 + \tfrac{1}{2}(Z_1/Z_2) > -1 \tag{6}$$

The region of no attenuation is thus bounded by the limits $Z_1/Z_2 = 0$ and $Z_1/Z_2 = -4$. The frequencies at which these limiting values are attained are the 'cut-off' frequencies of the filter. Outside these limits λ is a complex number of the form $-(\alpha + i\phi)$, and attenuation occurs.

The characteristic impedance of the filter depends upon the point at which successive sections are divided, *i.e.* upon the point at which the filter begins. Simplest relations are associated with symmetrical sections, and these are obtained when the division between adjacent sections is in the middle of the series impedances Z_1, or when the shunt impedances Z_2 are each replaced by two impedances $2Z_2$ in parallel, and the division is between them. These arrangements lead to the T and π filters respectively. The characteristic impedance Z_0 of either type is simply calculated by replacing the filter to the right of one division by its equivalent Z_0, and then expressing the e.m.f. at the junction, firstly in terms of the currents and impedances in the sections to the left of the sections, and secondly in terms of the current and impedance Z_0 to the right. Values of Z_0 so calculated for the two types are given on p. 184, but it is to be noted that, for convenience in applying results to concrete filters, Z_1 and Z_2 have there a slightly different significance. In the present notation the impedances Z_0 become

$$\sqrt{\tfrac{1}{4}Z_1^2 + Z_1Z_2} \text{ (T circuit)} \quad \text{and} \quad \frac{Z_1Z_2}{\sqrt{\tfrac{1}{4}Z_1^2 + Z_1Z_2}} \text{ (π circuit)}$$

VIII. The Analogy between Ripples and Cylindrical Sound Waves

The writer * has shown that if the media concerned are inviscid, the differential equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+k^2\phi=0 \tag{1}$$

where ϕ is the velocity potential at a point and k is equal to 2π/wave-length, is satisfied, for harmonic disturbances, by ripples upon a liquid surface as well as by cylindrical sound waves.

The general theory of functions of this type shows that the potential ϕ_P at any point P within a region to which the equation applies is determined by the values of ϕ and of $\partial\phi/\partial n$ at the boundary. The formula is

$$\phi_P=-\tfrac{1}{4}\int D_0(kr)\frac{\partial\phi}{\partial n}ds+\tfrac{1}{4}\int\phi\frac{\partial}{\partial n}D_0(kr)ds \tag{2}$$

where the integrations are performed around the boundary and where

$$D_0(z)=(2/\pi)\int_0^\infty e^{-iz\cosh u}du$$

Now, in considering equation (2) we may note that for a family of boundaries of similar shape but of different size (r) the directions of the normals ∂n at corresponding points will be the same; further, at rigid boundaries $\partial\phi/\partial n$ has the value zero both for sound waves and for ripples. The values of kr at corresponding points within the different boundaries will be the same provided r varies as $1/k$—that is, provided r varies as λ; or, in other words, if the linear dimensions of the model boundaries are proportional to the wave-length of the disturbance set up within them.

Consequently, if we have two similar regions in which this scale relation is satisfied and to which equation (1) applies—say a region of sound waves and a region of ripples—and if we arrange the disturbance at the boundaries of the two regions to be such that values of ϕ and $\partial\phi/\partial n$ correspond, then the disturbance ϕ_P at all corresponding points within the region will correspond exactly.

Having established this analogy in terms of velocity potential, it may be shown that within a certain limitation the height h of ripples may be taken as a measure of the condensation s of air at the corresponding point in a sound field. The complete analogy is between hk and s.

We have for cylindrical sound waves

$$s=\frac{1}{c^2}\frac{\partial\phi}{\partial t} \tag{3}$$

* A. H. Davis, *Phys. Soc.*, *Proc.*, 38, 234, 1926; 40, 90, 1928.

As regards ripples, it may be shown, for harmonic disturbances, that

$$hk=\frac{1}{c^2}\frac{\partial\phi}{\partial t} \tag{4}$$

provided tanh $kl=1$, as is approximately the case when the depth l of liquid exceeds half the wave-length. Thus for ripples of a given wave-length (since $k=2\pi/\lambda$)—the height of a ripple is a measure of the condensation of air at the corresponding point in the acoustic field.

Viscosity and other Effects. The effect of moderate viscosity is of the same type for sound waves and for surface waves; it is most marked for short wave-lengths, and it decreases amplitudes without appreciably altering the wave-length. The decrease in amplitude follows an exponential law $a=a_0e^{-t/\tau}$, where t is the time which has elapsed since the commencement of free oscillations, or, for maintained oscillations, since the disturbance concerned left the source. τ depends upon the kinematical viscosity and is equal to $3/2\nu k^2$ in the case of sound waves and $1/2\nu k^2$ in the case of ripples. For surface waves further damping occurs if the depth is less than half a wave-length, and the apparent viscosity may be appreciably increased by contamination of the surface by means of a film of oil of excessive thinness. Viscous effects are generally negligible in the case of sound unless the wave-length is very short, but they are appreciable for the water waves that would usually be used in a small ripple tank not greater than, say, 10 ft. in size. Mercury waves are much less affected by viscous damping than water waves.

Experiments conducted with model obstacles in a ripple tank showed that the effects of the meniscus around the obstacle and of the amplitude of the source were not important. When the water becomes stale on exposure—and presumably contaminated—the relative distribution of ripples around obstacles is appreciably modified, if not to an extent which would correspond to any marked change in the loudness of a sound.

When ripples are studied in narrow conduits, the meniscus at the boundary of the conduit gives optical distortion of the wave image upon the screen.

The following technique minimises this difficulty. The boundaries of the model conduit are constructed of teak, and firmly cemented in proper position on a sheet of plate-glass. The upper surface is then planed to ensure an accurately flat surface. The model thus assembled is laid upon levelling wedges in the ripple tank, adjusted to be level and thus parallel with the water surface. The depth of water is adjusted so that the surface of the model is barely emergent. With this adjustment the meniscus effect is eliminated when the water is undisturbed, but occurs to some extent when ripples are passing. A photograph shows the ripples, but gives defective definition of the edges of the conduit. Clear outlines of the model are obtained with a second photograph after the depth of water has been increased until the model is submerged.

IX. Wave Propagation and Vibrating Systems

Velocity of Sound (c).

$$c = \sqrt{E/\rho}, \qquad \rho = \text{density of medium.}$$

In Fluids. E = adiabatic volume elasticity (for gases $E = \gamma P$, where P is the pressure of the gas and γ the ratio of its specific heats).

In Solid Bars. E = Young's modulus of elasticity.

Velocity of Waves in Wires (c).

Torsional Waves. $c = \sqrt{n/\rho}$, where n is the modulus of rigidity of the material of the wire.

Transverse Waves. $c = \sqrt{f/m}$, where m is the mass per unit length and f the stretching force (in dynes).

Velocity of Ripples upon Liquid Surface (c).

$$c^2 = \frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda},$$

where T = surface tension of liquid,
g = acceleration due to gravity,
λ = wave-length of wave,
ρ = density of liquid.

Natural Frequencies of Vibration (n).

String of length l and mass m per unit length, stretched by force f.

$$n = \frac{p}{2l}\sqrt{\frac{f}{m}}, \text{ where } p \text{ is an integer.}$$

For a stiff string write $\{f + (p^2\pi^3 r^4 E/8l^2 f)\}$ for f,
where r = radius of cross-section of the string,
E = Young's modulus of elasticity for the material.

Membranes and Plates. See pp. 67, 70, 71.

Bars of length l.

$$n = \frac{cK}{2\pi l^2} m^2,$$

where c = velocity of longitudinal waves in the bar ($= \sqrt{E/\rho}$),
K = radius of gyration of the rod about an axis perpendicular to the plane of bending,
$= t/2\sqrt{3}$ for rectangular bars of thickness t,
$= r/\sqrt{2}$ for circular rods of radius r.

Free-free bar (*i.e.* free at both ends),
$m = 3{\cdot}01\pi/2, \quad 5\pi/2, \quad 7\pi/2, \quad 9\pi/2, \ldots$

Clamped free bar (*cf.* tuning-fork, p. 31),
$m = 1{\cdot}19\pi/2, \quad 3\pi/2, \quad 5\pi/2, \quad 7\pi/2, \ldots$

Rectangular Cavities, gas-filled, having sides of lengths l_1, l_2, l_3,

$$n^2 = \frac{c^2}{4}\left(\frac{p^2}{l_1^2} + \frac{q^2}{l_2^2} + \frac{r^2}{l_3^2}\right), \text{ where } p, q, r \text{ are integers.}$$

Spherical Cavities, gas-filled,

$$n = \frac{c}{2a}p.$$

In radial modes, $p = 1{\cdot}43$, $2{\cdot}45$, $3{\cdot}47$ . . .
Other transverse modes, $p = 0{\cdot}66$, $1{\cdot}89$, $2{\cdot}93$, $3{\cdot}95$ . . .

Cylindrical Cavities, gas-filled,

$$n = \frac{c}{2a}p.$$

In radial modes, $p = 1{\cdot}21$, $2{\cdot}23$, $3{\cdot}24$ . . .
Other transverse modes, $p = 0{\cdot}59$, $1{\cdot}70$, $2{\cdot}72$ . . .

Law of Linear Dimensions. Bodies of geometrically similar form, made of the same material, vibrating in the same manner and differing only in dimensions, have natural periods of vibration proportional to their linear dimensions.

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Names of persons are printed in italics